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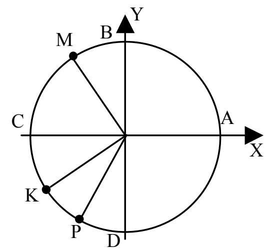
Домашняя работа по алгебре и началам анализа за 10 класс

к задачнику «Алгебра и начала анализа. Задачник для 10-11 кл. общеобразовательных учреждений А.Г. Мордкович, Л.О. Денищева, Т.А. Корешкова, Т.Н. Мишустина, Е.Е. Тульчинская — М.: «Мнемозина», 2001 г.»

Глава 1. Тригонометрические функции

§ 1. Введение

В задачах 1-8требуется найти длину дуги. Она находится по формуле: $\ell = \alpha \cdot r$, где α - радианная мера дуги, r - радиус окружности. Так как рассматривается единичная окружность, то $r = 1$, т.е. $\ell = \alpha$. Переход от градусной к радианной мере: $\alpha = \frac{\beta}{180} \cdot \pi$, где β - мера угла в градусах.

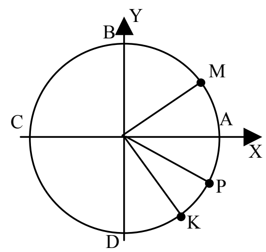


1. $\overset{\smile}{AM}$ =90°+45°=135°, $\overset{\smile}{BK}$ = 90° + 30° = 120°

$\overset{\smile}{MP}$ = 45° + 60°= 105°, $\overset{\smile}{DC}$ = 270°,

$\overset{\smile}{KA}$ = 150°, $\overset{\smile}{BP}$ = 150°,

$\overset{\smile}{CB}$ = 270°, $\overset{\smile}{BC}$ = 90°.

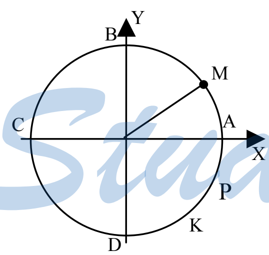


2. $\overset{\smile}{AM}$ = 45°, $\overset{\smile}{BD}$ = 180°;

$\overset{\smile}{CK}$ = 120°, $\overset{\smile}{MP}$ = 285°;

$\overset{\smile}{DM}$ = 135°, $\overset{\smile}{MK}$ = 360° – 105° = 255°;

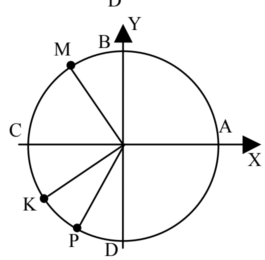
$\overset{\smile}{CP}$ = 150°; $\overset{\smile}{PC}$ = 210°.



3.

$\overset{\smile}{AM}$ = 36°, $\overset{\smile}{MB}$ = 54°

$\overset{\smile}{DM}$ =126°, $\overset{\smile}{MC}$ = 144°

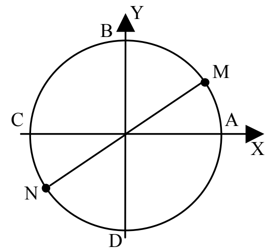


4.

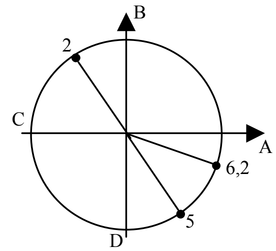
$\overset{\smile}{CP}$ = 15°, $\overset{\smile}{PD}$ = 75°

$\overset{\smile}{AP}$ = 105°

5. $\overset{\smile}{AN}=225°$



6. а) Да б) Да в) Да г) Нет, т.к. длина всей окружности $\ell = 2\pi r = 2 \cdot 3{,}1415\ldots < 6{,}3$



7. $\angle AOM = 60°$; $MN = 2 \cdot \sin\angle AOM = 2 \cdot \dfrac{\sqrt{3}}{2} = \sqrt{3}$;

$\overset{\smile}{AM}=60°$, $\overset{\smile}{MB}=30°$, $\overset{\smile}{AN}=300°$, $\overset{\smile}{NA}=60°$.

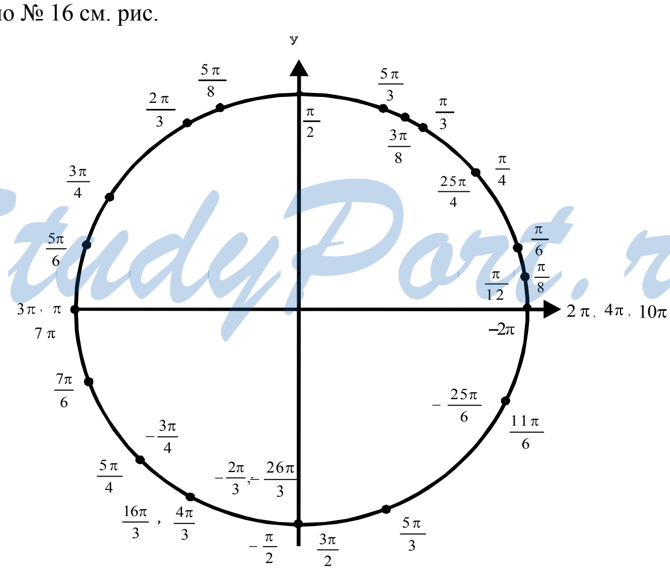
8. Воспользуемся результатами задачи №7.

$\overset{\smile}{AM}=\overset{\smile}{NA}=60°$, $\overset{\smile}{MB}=\overset{\smile}{DN}=30°$.

Аналогично $\overset{\smile}{BP}=\overset{\smile}{QD}=30°$, $\overset{\smile}{PC}=\overset{\smile}{CQ}=60°$

Окончательно получаем: $\overset{\smile}{AM}=\overset{\smile}{MP}=\overset{\smile}{PC}=\overset{\smile}{CQ}=\overset{\smile}{QN}=\overset{\smile}{NA}=60°$, ч.т.д.

с № 9 по № 16 см. рис.



§ 2. Числовая окружность

9. а) $\frac{\pi}{2}$; (0; 1). б) π; (– 1; 0). в) $\frac{3\pi}{2}$; (0; – 1). г) 2π; (1; 0).

10. а) 7π; (– 1; 0). б) 4π; (1; 0). в) 10π; (1; 0). г) 3π; (-1; 0).

11. а) $\frac{\pi}{3}$; $(\frac{1}{2};\frac{\sqrt{3}}{2})$. б) $\frac{\pi}{4}$; $(\frac{\sqrt{2}}{2};\frac{\sqrt{2}}{2})$. в) $\frac{\pi}{6}$; $(\frac{\sqrt{3}}{2};\frac{1}{2})$.

г) $\frac{\pi}{8}$ $\quad \cos^2\frac{\pi}{8}=\frac{1+\cos\frac{\pi}{4}}{2}$; $\cos\frac{\pi}{8}=\sqrt{\frac{1+\frac{\sqrt{2}}{2}}{2}}=\frac{\sqrt{2+\sqrt{2}}}{2}$;

$\sin^2\frac{\pi}{8}=\frac{1-\sin^2\frac{\pi}{4}}{2}$; $\sin\frac{\pi}{8}=\sqrt{\frac{2-\sqrt{2}}{2}}$; $(\sqrt{\frac{2-\sqrt{2}}{2}}; \sqrt{\frac{2-\sqrt{2}}{2}})$.

12. а) $\frac{2\pi}{3}$, $(-\frac{1}{2};\frac{\sqrt{3}}{2})$; б) $\frac{3\pi}{4}$, $(-\frac{\sqrt{2}}{2};\frac{\sqrt{2}}{2})$;

в) $\frac{5\pi}{6}$, $(-\frac{\sqrt{3}}{2};\frac{1}{2})$; г) $\frac{5\pi}{4}$, $(-\frac{\sqrt{2}}{2};-\frac{\sqrt{2}}{2})$.

13. а) $\frac{4\pi}{3}$, $(-\frac{1}{2};-\frac{\sqrt{3}}{2})$; б) $\frac{5\pi}{3}$, $(\frac{1}{2};-\frac{\sqrt{3}}{2})$;

в) $\frac{7\pi}{6}$, $(-\frac{\sqrt{3}}{2};-\frac{1}{2})$; г) $\frac{11\pi}{6}$, $(\frac{\sqrt{3}}{2};-\frac{1}{2})$.

14. а) $\frac{\pi}{12}$; $\quad \cos^2\frac{\pi}{12}=\frac{1+\cos\frac{\pi}{6}}{2}$; $\quad \cos\frac{\pi}{12}=\frac{\sqrt{2+\sqrt{3}}}{2}$

$\sin^2\frac{\pi}{12}=\frac{1-\cos\frac{\pi}{6}}{2}$; $\sin\frac{\pi}{12}=\frac{\sqrt{2-\sqrt{3}}}{2}$; $(\frac{\sqrt{2+\sqrt{3}}}{2};\frac{\sqrt{2-\sqrt{3}}}{2})$;

б) $\frac{5\pi}{12}$; $\frac{5}{12}\pi=\frac{\pi}{4}+\frac{\pi}{6}$; $\cos\left(\frac{\pi}{4}+\frac{\pi}{6}\right)=\cos\frac{\pi}{4}\cos\frac{\pi}{6}-\sin\frac{\pi}{4}\sin\frac{\pi}{6}=\frac{\sqrt{2}\ (\sqrt{3}-1)}{4}$

$\sin\frac{5\pi}{12}=\frac{\sqrt{2}\ (\sqrt{3}+1)}{4}$; $(\frac{\sqrt{2}(\sqrt{3}-1)}{4};\frac{\sqrt{2}(\sqrt{3}+1)}{4})$;

в) $\frac{3\pi}{8}$: $\cos\frac{3\pi}{8}=\frac{\sqrt{2-\sqrt{2}}}{2}$; $\sin\frac{3\pi}{8}=\frac{\sqrt{2+\sqrt{2}}}{2}$; $(\frac{\sqrt{2-\sqrt{2}}}{2};\frac{\sqrt{2+\sqrt{2}}}{2})$;

г) $\frac{5\pi}{8}$: $\cos\frac{5\pi}{8}=-\frac{\sqrt{2-\sqrt{2}}}{2}$; $\sin\frac{5\pi}{8}=\frac{\sqrt{2+\sqrt{2}}}{2}$; $(-\frac{\sqrt{2-\sqrt{2}}}{2};\frac{\sqrt{2+\sqrt{2}}}{2})$

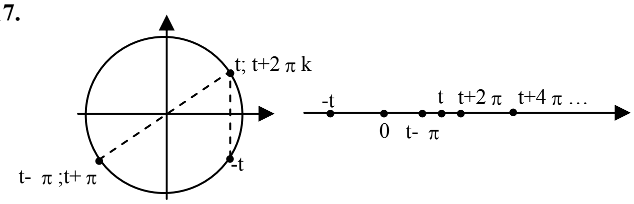
15. а) $-\frac{\pi}{2}$, (0; –1); б) $\frac{2\pi}{3}$, $(-\frac{1}{2};-\frac{\sqrt{3}}{2})$;

в) -2π, (1; 0); г) $-\frac{3\pi}{4}$, $(-\frac{\sqrt{2}}{2};-\frac{\sqrt{2}}{2})$.

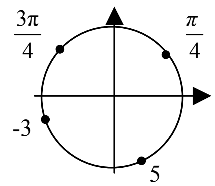
16. а) $\frac{25\pi}{4}$, $(\frac{\sqrt{2}}{2};\ \frac{\sqrt{2}}{2})$; б) $-\frac{26\pi}{3}$, $(-\frac{1}{2};-\frac{\sqrt{3}}{2})$;

в) $-\frac{25\pi}{6}$, $(\frac{\sqrt{3}}{2};-\frac{1}{2})$; г) $\frac{16\pi}{3}$, $(-\frac{1}{2};-\frac{\sqrt{3}}{2})$.

17.



а) t; – t. на прямой: симметрично относительно нуля
на окружности: симметрично относительно оси *x*.
б) t; t + 2 πk. на прямой: стоят с периодом 2πk
на окружности: совпадают.
в) t; t + π. на прямой: стоят на расстоянии в π
на окружности: диаметрально противоположны.
г) t + π, t – π. на прямой: стоят на расстоянии в 2π
на окружности: совпадают



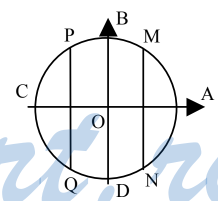
18. а) $M(\frac{\pi}{4}) = (\frac{\sqrt{2}}{2};\frac{\sqrt{2}}{2})$

б) M (5) = (0, 284, – 0,959)

в) $M(\frac{3\pi}{4}) = (-\frac{\sqrt{2}}{2};\frac{\sqrt{2}}{2})$

г) M (– 3) = (– 0,990; – 0,141).

19. а) A: 2πk; б) C: π + 2πk; в) A и C : πn



20. см. рис. к 19.

а) B: $\frac{\pi}{2} + 2\pi k$; б) D: $-\frac{\pi}{2} + 2\pi k$; в) B и D: $\frac{\pi}{2} + \pi n$

21. см. рис. к 19.
а) A: min положительное = 2π;
max отрицательное = 2π

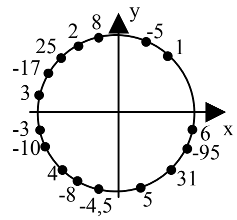
б) B: min положительное = $\frac{\pi}{2}$; max отрицательное = $-\frac{3}{2}\pi$

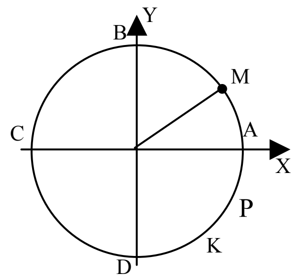
в) C: min положительное = π; max отрицательное = – π

г) и D: min положительное = $\frac{3\pi}{2}$; max отрицательное = $-\frac{\pi}{2}$

см. рис. (с № 22 по № 25)
22. а) 1: (0, 540; 0,841); б) – 5: (0,284; 0,950);
в) 4,5: (-0,211; -0,978); г) (– 3): (– 0,990; – 0,141).
23. а) 6: IV; б) 2: II; в) 3: II; г) 4: III.
24. а) 5: IV; б) – 5: I; в) 8: II; г) – 8: III.

25. а) 10: III; б) – I7: II; в) 31: IV; г) – 95: IV.





26. а) AM: $t \in (2\pi k;\ \frac{\pi}{4} + 2\pi k)$

б) CM: $t \in (-\pi + 2\pi k;\ \frac{\pi}{4} + 2\pi k)$

в) MA: $t \in (\frac{\pi}{4} + 2\pi k;\ 2\pi + 2\pi k)$

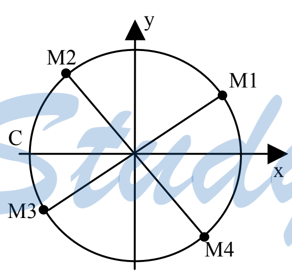
г) MC $t \in (\frac{\pi}{4} + 2\pi k;\ \pi + 2\pi k)$

27. см. рис. к 26

а) DM: $t \in (-\frac{\pi}{2} + 2\pi k;\ \frac{3\pi}{4} + 2\pi k)$

б) BD: $t \in (\frac{\pi}{2} + 2\pi k;\ \frac{3\pi}{2} + 2\pi k)$; в) MD: $t \in (\frac{3\pi}{4} + 2\pi k;\ \frac{3\pi}{2} + 2\pi k)$;

г) DB: $t \in (-\frac{\pi}{2} + 2\pi k;\ \frac{\pi}{2} + 2\pi k)$.



28.

а) M_1M_2: $t \in (\frac{\pi}{4} + 2\pi k;\ \frac{3\pi}{4} + 2\pi k)$

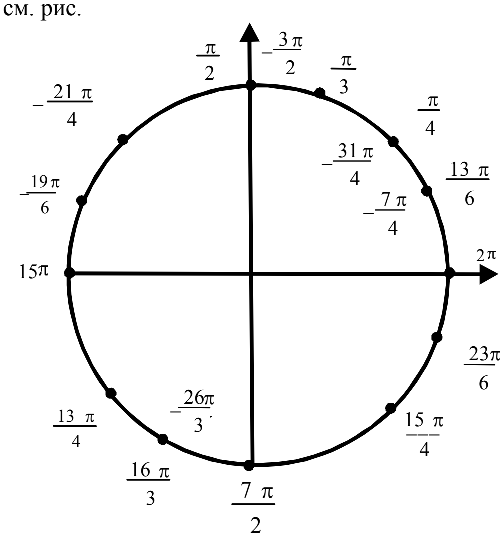
б) M_4M_1: $t \in (-\frac{\pi}{4} + 2\pi k;\ \frac{\pi}{4} + 2\pi k)$

в) M_3M_2: $t \in (-\frac{3\pi}{4} + 2\pi k;\ \frac{3\pi}{4} + 2\pi k)$

г) M_1M_3: $t \in (\frac{\pi}{4} + 2\pi k;\ \frac{5\pi}{4} + 2\pi k)$

§ 3. Числовая окружность на координатной плоскости

с № 29 по 32 см. рис.



29. а) $M\left(\frac{\pi}{4}\right)=\left(\frac{\sqrt{2}}{2};\frac{\sqrt{2}}{2}\right)$; б) $M\left(\frac{\pi}{6}\right)=\left(\frac{\sqrt{3}}{2};\frac{1}{2}\right)$;

в) $M\left(\frac{\pi}{3}\right)=\left(\frac{1}{2};\frac{\sqrt{3}}{2}\right)$; г) $M\left(\frac{\pi}{2}\right)=(0;1)$.

30. а) $M(2\pi)=(1;0)$; б) $M\left(-\frac{3\pi}{2}\right)=(0;1)$;

в) $M\left(\frac{7\pi}{2}\right)=(0;-1)$; г) $M(15\pi)=(-1;0)$.

31. а) $M\left(\frac{15\pi}{4}\right)=\left(\frac{\sqrt{2}}{2};-\frac{\sqrt{2}}{2}\right)$; б) $M\left(\frac{23\pi}{6}\right)=\left(\frac{\sqrt{3}}{2};-\frac{1}{2}\right)$;

в) $M\left(\frac{13\pi}{4}\right)=\left(-\frac{\sqrt{2}}{2};-\frac{\sqrt{2}}{2}\right)$; г) $M\left(\frac{16\pi}{3}\right)=\left(-\frac{1}{2};-\frac{\sqrt{3}}{2}\right)$.

32. а) $M\left(-\frac{19\pi}{6}\right)=\left(-\frac{\sqrt{3}}{2};\frac{1}{2}\right)$; б) $M\left(-\frac{31\pi}{4}\right)=\left(\frac{\sqrt{2}}{2};\frac{\sqrt{2}}{2}\right)$

в) $M\left(-\frac{21\pi}{4}\right)=\left(-\frac{\sqrt{2}}{2};\frac{\sqrt{2}}{2}\right)$; г) $M\left(-\frac{26\pi}{3}\right)=\left(-\frac{1}{2};-\frac{\sqrt{3}}{2}\right)$

33. а) $M\left(\frac{\sqrt{3}}{2};\frac{1}{2}\right)$: min положит.$=\frac{\pi}{6}$, max отрицат.$=-\frac{11\pi}{6}$

б) $M\left(-\frac{\sqrt{3}}{2};\frac{1}{2}\right)$: min положит.$=\frac{5\pi}{6}$, max отрицат.$=-\frac{7\pi}{6}$

в) M ($\frac{\sqrt{3}}{2}$; $-\frac{1}{2}$): min положит.= $\frac{11\pi}{6}$, max отрицат.= $-\frac{\pi}{6}$

г) M($-\frac{\sqrt{3}}{2}$; $-\frac{1}{2}$): min положит.= $\frac{7\pi}{6}$, max отрицат.= $-\frac{5\pi}{6}$

34. а) M ($\frac{1}{2}$; $\frac{\sqrt{3}}{2}$): min положит.= $\frac{\pi}{3}$, max отрицат.= $-\frac{5\pi}{3}$

б) M ($-\frac{1}{2}$; $\frac{\sqrt{3}}{2}$): min положит.= $\frac{2\pi}{3}$, max отрицат.= $-\frac{4\pi}{3}$

в) M ($-\frac{1}{2}$; $-\frac{\sqrt{3}}{2}$): min положит.= $\frac{4\pi}{3}$, max отрицат.= $-\frac{2\pi}{3}$

г) M ($\frac{1}{2}$; $-\frac{\sqrt{3}}{2}$): min положит.= $\frac{5\pi}{3}$, max отрицат.= $-\frac{\pi}{3}$

35. а) $y = \frac{\sqrt{2}}{2}$ $t = \frac{\pi}{4} + 2\pi n$; $\frac{3\pi}{4} + 2\pi n$;

б) $y = \frac{1}{2}$; $t = \frac{\pi}{6} + 2\pi n$; $\frac{5\pi}{6} + 2\pi n$; в) $y = 0$, $t = \pi n$;

г) $y = \frac{\sqrt{3}}{2}$, $t = \frac{\pi}{3} + 2\pi n$, $\frac{2\pi}{3} + 2\pi n$.

36. а) $y = -\frac{\sqrt{3}}{2}$, $-x = (-1)^{k+1}\frac{\pi}{3} + \pi k$; б) $y = 1$, $x = \frac{\pi}{2} + 2\pi k$;

в) $y = -\frac{\sqrt{2}}{2}$, $x = (-1)^{k+1}\frac{\pi}{4} + \pi k$; г) $y = -1$, $x = -\frac{\pi}{2} + 2\pi k$.

37. а) $x = \frac{\sqrt{3}}{2}$, $x = \pm\frac{\pi}{6} + 2\pi k$; б) $x = \frac{1}{2}$, $y = \pm\frac{\pi}{3} + 2\pi k$;

в) $x = 1$, $y = 2\pi n$; г) $x = \frac{\sqrt{2}}{2}$, $y = \pm\frac{\pi}{4} + 2\pi k$.

38. а) $x = 0$, $y = \frac{\pi}{2} + \pi n$; б) $x = -\frac{1}{2}$, $y = \pm\frac{2\pi}{3} + 2\pi k$;

в) $x = -\frac{\sqrt{3}}{2}$, $y = \pm\frac{5\pi}{6} + 2\pi k$; г) $x = -1$, $y = \pi + 2\pi n$.

39. а) да; б) нет; в) да; г) нет.

40. а) E (2) – +; б) K (4) – –; в) F (1) + +; г) L (6) + –.

41. а) M (12) + –; б) N (15) – +; в) P (49) + –; г) Q (100) + –.

42. а) $y = -\frac{\sqrt{3}}{2}$, $x < 0$ $\frac{4\pi}{3} + 2\pi n$; M($-\frac{1}{2}$; $-\frac{\sqrt{3}}{2}$)

б) $y = \frac{1}{2}$, $x < 0$ $\frac{5\pi}{6} + 2\pi n$; M($-\frac{\sqrt{3}}{2}$; $\frac{1}{2}$)

в) $y = \frac{1}{2}$, $x > 0$ $\frac{\pi}{6} + 2\pi n$; $M(\frac{\sqrt{3}}{2}; \frac{1}{2})$

г) $y = -\frac{\sqrt{3}}{2}$, $x > 0$ $-\frac{\pi}{3} + 2\pi n$; $M(\frac{1}{2}; -\frac{\sqrt{3}}{2})$

43. а) $x = \frac{\sqrt{3}}{2}$, $y > 0$ $\frac{\pi}{6} + 2\pi n$; $M(\frac{\sqrt{3}}{2}; \frac{1}{2})$

б) $x = -\frac{1}{2}$, $y < 0$ $-\frac{2\pi}{3} + 2\pi n$; $M(-\frac{1}{2}; -\frac{\sqrt{3}}{2})$

в) $x = \frac{\sqrt{3}}{2}$, $y < 0$ $-\frac{\pi}{6} + 2\pi n$; $M(\frac{\sqrt{3}}{2}; -\frac{1}{2})$

г) $y = -\frac{1}{2}$, $y > 0$ $\frac{2\pi}{3} + 2\pi n$; $M(-\frac{1}{2}; \frac{\sqrt{3}}{2})$

44. а) $x > 0$, $t \in (-\frac{\pi}{2} + 2\pi n; \frac{\pi}{2} + 2\pi n)$; б) $x < \frac{1}{2}$, $t \in (\frac{\pi}{3} + 2\pi n; \frac{5\pi}{3} + 2\pi n)$;

в) $x > \frac{1}{2}$, $t \in (-\frac{\pi}{3} + 2\pi n; \frac{\pi}{3} + 2\pi n)$; г) $x < 0$, $t \in (\frac{\pi}{2} + 2\pi n; \frac{3}{2}\pi + 2\pi n)$.

45. а) $x < \frac{\sqrt{2}}{2}$, $t \in (\frac{\pi}{4} + 2\pi k; \frac{7\pi}{4} + 2\pi k)$; б) $x > -\frac{\sqrt{2}}{2}$, $t \in (-\frac{3\pi}{4} + 2\pi k; \frac{3\pi}{4} + 2\pi k)$;

в) $x > \frac{\sqrt{2}}{2}$, $t \in (-\frac{\pi}{4} + 2\pi k; \frac{\pi}{4} + 2\pi k)$; г) $x < -\frac{\sqrt{2}}{2}$, $t \in (\frac{3\pi}{4} + 2\pi k; \frac{5\pi}{4} + 2\pi k)$.

46. а) $x \le \frac{\sqrt{3}}{2}$, $t \in (\frac{\pi}{6} + 2\pi k; \frac{11\pi}{6} + 2\pi k)$; б) $x \le -\frac{\sqrt{3}}{2}$, $t \in (\frac{5\pi}{6} + 2\pi k; \frac{7\pi}{6} + 2\pi k)$;

в) $x \ge \frac{\sqrt{3}}{2}$, $t \in (-\frac{\pi}{6} + 2\pi k; \frac{\pi}{6} + 2\pi k)$; г) $x \ge -\frac{\sqrt{3}}{2}$, $t \in (-\frac{5\pi}{6} + 2\pi k; \frac{5\pi}{6} + 2\pi k)$.

47. а) $y > 0$, $t \in (2\pi k; \pi + 2\pi k)$; б) $y < \frac{1}{2}$, $t \in (\frac{7\pi}{6} + 2\pi k; \frac{\pi}{6} + 2\pi k)$;

в) $y > \frac{1}{2}$, $t \in (\frac{\pi}{6} + 2\pi k; \frac{5\pi}{6} + 2\pi k)$; г) $y < 0$, $t \in (-\pi + 2\pi k; 2\pi k)$.

48. а) $y < \frac{\sqrt{2}}{2}$, $-\frac{5\pi}{4} + 2\pi k < t < \frac{\pi}{4} + 2\pi k$; б) $y > -\frac{\sqrt{2}}{2}$, $-\frac{\pi}{4} + 2\pi k < t < \frac{5\pi}{4} + 2\pi k$;

в) $y > \frac{\sqrt{2}}{2}$, $\frac{\pi}{4} + 2\pi k < t < \frac{3\pi}{4} + 2\pi k$; г) $y < -\frac{\sqrt{2}}{2}$, $\frac{5\pi}{4} + 2\pi k < t < \frac{7\pi}{4} + 2\pi k$.

49. а) $y \le \frac{\sqrt{3}}{2}$, $-\frac{4\pi}{3} + 2\pi k < t < \frac{\pi}{3} + 2\pi k$; б) $y \le -\frac{\sqrt{3}}{2}$, $\frac{4\pi}{3} + 2\pi k < t < \frac{5\pi}{3} + 2\pi k$;

в) $y \ge \frac{\sqrt{3}}{2}$, $\frac{\pi}{3} + 2\pi k < t < \frac{2\pi}{3} + 2\pi k$; г) $y \ge -\frac{\sqrt{3}}{2}$, $-\frac{\pi}{3} + 2\pi k < t < \frac{4\pi}{3} + 2\pi k$.

§ 4. Синус и косинус

50. а) $\sin 0 = 0$, $\cos 0 = 1$; б) $\sin \frac{\pi}{2} = 1$, $\cos \frac{\pi}{2} = 0$;

в) $\sin \frac{3\pi}{2} = -1$, $\cos = \frac{3\pi}{2} = 0$; г) $\sin \pi = 0$, $\cos \pi = -1$.

51. а) $\sin(-2\pi)=0$, $\cos(-2\pi) = 1$; б) $\sin(-\frac{\pi}{2}) = -1$, $\cos(-\frac{\pi}{2}) = 0$;

в) $\sin(-\frac{3\pi}{2}) = 1$, $\cos(-\frac{3\pi}{2}) = 0$; г) $\sin(-\pi) = 0$, $\cos(-\pi) = -1$.

52. а) $\sin\frac{5\pi}{6} = \frac{1}{2}$, $\cos\frac{5\pi}{6} = -\frac{\sqrt{3}}{2}$; б) $\sin\frac{5\pi}{4} = \frac{\sqrt{2}}{2}$, $\cos\frac{5\pi}{4} = -\frac{\sqrt{2}}{2}$;

в) $\sin\frac{7\pi}{6} = -\frac{1}{2}$, $\cos\frac{7\pi}{6} = -\frac{\sqrt{3}}{2}$; г) $\sin\frac{9\pi}{4} = -\frac{\sqrt{2}}{2}$, $\cos\frac{9\pi}{4} = \frac{\sqrt{2}}{2}$.

53. а) $\sin(-\frac{7\pi}{4}) = \frac{\sqrt{2}}{2}$, $\cos(-\frac{7\pi}{4}) = \frac{\sqrt{2}}{2}$; б) $\sin(-\frac{4\pi}{3}) = \frac{\sqrt{3}}{2}$, $\cos(-\frac{4\pi}{3}) = -\frac{1}{2}$;

в) $\sin(-\frac{13\pi}{4}) = \frac{\sqrt{2}}{2}$, $\cos(-\frac{13\pi}{4}) = -\frac{\sqrt{2}}{2}$; г) $\sin(-\frac{5\pi}{3}) = \frac{\sqrt{3}}{2}$, $\cos(-\frac{5\pi}{3}) = \frac{1}{2}$.

54. а) $\sin\frac{13\pi}{6} = \frac{1}{2}$, $\cos\frac{13\pi}{6} = \frac{\sqrt{3}}{2}$; б) $\sin(-\frac{8\pi}{3}) = -\frac{\sqrt{3}}{2}$, $\cos(-\frac{8\pi}{3}) = -\frac{1}{2}$;

в) $\sin\frac{23\pi}{6} = -\frac{1}{2}$, $\cos\frac{23\pi}{6} = \frac{\sqrt{3}}{2}$; г) $\sin(-\frac{11\pi}{4}) = -\frac{\sqrt{2}}{2}$, $\cos(-\frac{11\pi}{4}) = -\frac{\sqrt{2}}{2}$.

55. а) $\sin(-\frac{\pi}{4}) + \cos\frac{\pi}{3} + \cos(-\frac{\pi}{6}) = -\frac{\sqrt{2}}{2} + \frac{1}{2} + \frac{\sqrt{3}}{2} + = \frac{\sqrt{3}+1-\sqrt{2}}{2}$;

б) $\cos\frac{\pi}{6}\cos\frac{\pi}{4}\cos\frac{\pi}{3}\cos\frac{\pi}{2} = 0$;

в) $\sin(-\frac{\pi}{2})\cos(-\pi) + \sin(-\frac{3\pi}{2}) = -1 + 1 + 1 = 1$;

г) $\sin\frac{\pi}{6}\sin\frac{\pi}{4}\sin\frac{\pi}{3}\sin\frac{\pi}{2} = \frac{1}{2}\cdot\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2}\cdot 1 = \frac{\sqrt{6}}{6}$.

56. а) $\sin(-\frac{3\pi}{4}) + \cos(-\frac{\pi}{4}) + \sin\frac{\pi}{4}\cos\frac{\pi}{2} + \cos 0 \sin\frac{\pi}{2} = -\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} + 1 = 1$

б) $\cos\frac{5\pi}{3} + \cos\frac{4\pi}{3} + \sin\frac{3\pi}{2}\sin\frac{5\pi}{8}\cos\frac{3\pi}{2} = \frac{1}{2} - \frac{1}{2} = 0$

57. а) $\cos 2t$, $t = \frac{\pi}{2}$, $\cos\pi = -1$; б) $\sin\frac{t}{2}$, $t = -\frac{\pi}{3}$, $\sin(-\frac{\pi}{6}) = -\frac{1}{2}$;

в) $\sin 2t$, $t = -\frac{\pi}{6}$, $\sin(-\frac{\pi}{3}) = -\frac{\sqrt{3}}{2}$; г) $\cos\frac{t}{2}$, $t = -\frac{\pi}{3}$, $\cos(-\frac{\pi}{6}) = \frac{\sqrt{3}}{2}$.

58. а) $\sin^2 t - \cos^2 t$, $t = \frac{\pi}{3}$, $\sin^2\frac{\pi}{3} - \cos^2\frac{\pi}{3} = \frac{3}{4} - \frac{1}{4} = \frac{1}{2}$;

б) $\sin^2 t + \cos^2 t$, $t = \frac{\pi}{4}$, $\sin^2\frac{\pi}{4} + \cos^2\frac{\pi}{4} = \frac{1}{2} + \frac{1}{2} = 1$;

в) $\sin^2 t - \cos^2 t$, $t = \frac{\pi}{4}$, $\sin^2\frac{\pi}{4} - \cos^2\frac{\pi}{4} = 0$;

г) $\sin^2 t + \cos^2 t$, $t = \frac{\pi}{6}$, $\sin^2\frac{\pi}{6} + \cos^2\frac{\pi}{6} = \frac{1}{4} + \frac{3}{4} = 1$.

59. а) $f(t) = 2\sin t$ $\quad f_{max} = 2$, $\quad f_{min} = -2$.
б) $f(t) = 3 + 4\cos t$ $\quad f_{max} = 7$, $\quad f_{min} = -1$.
в) $f(t) = -3\cos t$ $\quad f_{max} = 3$, $\quad f_{min} = -3$.
г) $f(t) = 3 - 5\sin t$ $\quad f_{max} = 8$, $\quad f_{min} = -2$.

60. а) $1 - \sin^2 t = \cos^2 t$; б) $1 - \cos^2 t = \sin^2 t$;
в) $1 + \sin^2 t + \cos^2 t = 2$; г) $\sin t - \sin t\cos^2 t = \sin t\,(1 - \cos^2 t) = \sin^3 t$

61. а) $(\sin t - \cos t)^2 + 2\sin t\cos t = 1$;
б) $(\sin t + \cos t)^2 - 2\sin t\cos t = \sin^2 t + \cos^2 t + 2\sin t\cos t - 2\sin t\cos t = 1$.

62. а) $\sin^2 t\,(1{,}5+2\pi k) + \cos^2 1{,}5 + \cos\left(-\frac{\pi}{4}\right) + \sin\left(-\frac{\pi}{6}\right) = 1 + \frac{\sqrt{2}}{2} - \frac{1}{2} = \frac{\sqrt{2}+1}{2}$;

б) $\cos^2\left(\frac{\pi}{8} + 4\pi\right) + \sin^2\left(\frac{\pi}{8} - 44\pi\right) = 1$.

63. а) $\cos t = \frac{\sqrt{2}}{2}$, $t = \pm\frac{\pi}{4} + 2\pi n$; б) $\sin t = -\frac{1}{2}$, $t = (-1)^{n+1}\frac{\pi}{6} + \pi n$;

в) $\cos t = -\frac{1}{2}$, $t = \pm\frac{2\pi}{3} + 2\pi n$; г) $\sin t = \frac{\sqrt{2}}{2}$, $t = (-1)^{n+1}\frac{\pi}{4} + \pi n$.

64. а) $\sin t = \frac{\sqrt{3}}{2}$, $-t = (-1)^{k+1}\frac{\pi}{3} + \pi k$; б) $\cos t = \sqrt{3}$ решений нет $|\cos t| \le 1$;

в) $\cos t = -\frac{\sqrt{3}}{2}$, $t = \pm\frac{5\pi}{6} + 2\pi n$; г) $\cos t = -\frac{\pi}{3}$ решений нет, т.к. $|\cos t| \le 1$.

65. а) 0; б) $\frac{\pi}{2}$; в) $-\frac{\pi}{6}$; г) $\frac{\pi}{3}$.

66. а) $\frac{\pi}{2}$; б) 0; в) $\frac{2\pi}{3}$; г) $\frac{5\pi}{6}$.

67. а) $\sin t + 1 = 0$, $\quad \sin t = -1$, $\quad t = -\frac{\pi}{2} + 2\pi n$

б) $\cos t - 1 = 0$, $\quad \cos t = 1$, $\quad t = 2\pi n$

в) $1 - 2\sin t = 0$, $\quad \sin t = \frac{1}{2}$, $\quad t = (-1)^n\frac{\pi}{6} + \pi n$

г) $2\cos t + 1 = 0$, $\quad \cos t = -\frac{1}{2}$, $\quad t = \pm\frac{2\pi}{3} + 2\pi n$

68. а) $\frac{\sin t - 1}{\cos t}$, $\cos t \neq 0$, $t \neq -\frac{\pi}{2} + \pi n$

б) $\frac{\cos t + 5}{2\sin t - 1}$, $\sin t \neq \frac{1}{2}$, $t \neq (-1)^n \frac{\pi}{6} + \pi n$

в) $\frac{\cos t}{1 - \sin t}$, $\sin t \neq 1$, $t \neq \frac{\pi}{2} + 2\pi n$

г) $\frac{\sin t}{1 + \cos t}$, $\cos t \neq -1$, $t \neq \pm \pi + 2\pi n$.

69. а) $\sin \frac{4\pi}{7} > 0$; б) $\cos (-\frac{5\pi}{7}) < 0$; в) $\sin \frac{9\pi}{8} < 0$; г) $\sin (-\frac{3\pi}{8}) < 0$.

70. а) $\sin (-2) < 0$; б) $\cos 3 < 0$; в) $\sin 5 < 0$; г) $\cos (-6) > 0$.

71. а) $\sin 10 < 0$; б) $\cos (-12) > 0$; в) $\sin (-15) < 0$; г) $\cos 8 < 0$.

72. а) $\sin 1 \cos 2 < 0$; б) $\sin \frac{\pi}{7} \cos (-\frac{7\pi}{5}) < 0$;

в) $\cos 2 \sin (-3) > 0$; г) $\cos (-\frac{14\pi}{9}) \sin \frac{4\pi}{9} > 0$.

73. а) $\frac{\sin^2 t}{1 + \cos t} = \frac{1 - \cos^2 t}{1 + \cos t} = \frac{(1 - \cos t)(1 + \cos t)}{1 + \cos t} = 1 - \cos t$;

б) $\sin^4 t + \cos^4 t + 2 \sin^2 t \cos^2 t = (\sin^2 t + \cos^2 t)^2 = 1$;

в) $\frac{\cos^2 t}{1 + \sin t} + \sin t = 1 - \sin t + \sin t = 1$;

г) $\cos^4 t + \cos^2 t \sin^2 t - \cos^2 t + 1 = \cos^4 t + \cos^2 t (\sin^2 t - 1) + 1 = \cos^4 t - \cos^4 t + 1 = 1$.

74. а) $10 \sin t = \sqrt{75}$, $\sin t = \frac{5\sqrt{3}}{10} = \frac{\sqrt{3}}{2}$, $t = (-1)^n \frac{\pi}{3} + \pi n$

б) $\sqrt{8} \sin t + 2 = 0$, $\sin t = -\frac{\sqrt{2}}{2}$, $t = (-1)^{k+1} \frac{\pi}{4} + \pi k$

в) $8 \cos t - \sqrt{32} = 0$, $\cos t = \frac{\sqrt{2}}{2}$, $t = \pm \frac{\pi}{4} + 2\pi k$

г) $8 \cos t = -\sqrt{48}$, $\cos t = -\frac{\sqrt{3}}{2}$, $t = \pm \frac{5\pi}{6} + 2\pi k$

75. а) $\sin^2 \frac{\pi}{8} + \cos^2 \frac{\pi}{8} - \sin \sqrt{2}$, $\sin t = \frac{\sqrt{2}}{2}$, $t = (-1)^k \frac{\pi}{4} + \pi k$

б) $\sqrt{\frac{4}{3}} \cos t = \cos^2 1 + \sin^2 1$, $\cos t = \frac{\sqrt{3}}{2}$, $t = \pm \frac{\pi}{6} + 2\pi n$.

76. а) $|\sin t| = 1$, $\sin t = \pm 1$, $t = \frac{\pi}{2} + \pi n$

б) $\sqrt{1 - \sin^2 t} = \frac{1}{2}$, $|\cos t| = \frac{1}{2}$, $\cos t = \pm \frac{1}{2}$, $t = \pm \frac{\pi}{3} + 2\pi k$

в) $|\cos t| = 1$, $\cos t = \pm 1$, $t = \pi n$

г) $\sqrt{1-\cos^2 t} = \frac{\sqrt{2}}{2}$, $|\sin t| = \frac{\sqrt{2}}{2}$ $\sin t = \pm\frac{\sqrt{2}}{2}$, $t = \frac{\pi}{4} + \frac{\pi n}{2}$.

77. а) $\cos 1 + \cos(1+\pi) + \sin t\left(-\frac{\pi}{3}\right) + \cos\left(-\frac{\pi}{6}\right) = \cos 1 - \cos 1 - \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} = 0$

б) $\sin 2 + \sin(2+\pi) + \cos^2\left(-\frac{\pi}{12}\right) + \sin^2\frac{\pi}{12} = \sin 2 - \sin 2 + 1 = 1$.

78. а) $\sqrt{\sin 10{,}2\pi}$ – Да; б) $\sqrt{\cos 1{,}3\pi}$ – Нет;

в) $\sqrt{\sin(-3{,}4)\pi}$ – Да; г) $\sqrt{\cos(-6{,}9\pi)}$ – Нет.

79. а) $\cos 2\,(2x-1) < 0$, $x > \frac{1}{2}$;

б) $\cos 3 \cos 5\,(x^2-4) < 0$, $-x^2+4 < 0$; $x \in (-\infty; -2)\ (2; +\infty)$.

80. а) $(\cos t - 5)(3x-1) \ge 0$, $3x - 1 \le 0$, $x \le \frac{1}{3}$;

б) $(2 + \sin t)(9 - x^{2)} \ge 0$, $9 - x^2 \ge 0$, $x \in [-3; 3]$.

81. а) $a = \sin\frac{7\pi}{10}$, $b = \sin\frac{5\pi}{6}$, $a > b$; б) $a = \cos 2$, $b = \sin 2$, $a < b$;

в) $a = \cos\frac{\pi}{8}$, $b = \cos\frac{\pi}{3}$, $a > b$; г) $a = \sin 1$, $b = \cos 1$, $a > b$.

82. а) $\sin\frac{2\pi}{9} - \sin\frac{10\pi}{9} > 0$; б) $\sin 1 - \sin 1{,}1 < 0$;

в) $\sin\frac{15\pi}{8} - \cos\frac{\pi}{4} < 0$; г) $\cos 1 - \cos 0{,}9 < 0$.

83. а) $\sin\frac{4\pi}{3}$, $\sin\frac{7\pi}{6}$, $\sin\frac{\pi}{7}$, $\sin\frac{\pi}{5}$, $\sin\frac{2\pi}{3}$;

б) $\cos\frac{5\pi}{6}$, $\cos\frac{5\pi}{4}$, $\cos\frac{\pi}{3}$, $\cos\frac{7\pi}{4}$, $\cos\frac{\pi}{8}$.

84. а) cos 4, sin 3, cos 5, sin 2; б) cos 3, cos 4, cos 7, cos 6;

в) sin 4, sin 6, sin 3, sin 7; г) cos 3, sin 5, sin 4, cos 2.

85. а) $\sqrt{\sin^2 1 + \sin^2 2 - 2\sin 1 \sin 2} + \sqrt{\frac{1}{4} - \sin 1 + \sin^2 1} +$

$+ \sqrt{1 + \sin^2 2 - 2\sin 2} = |\sin 1 - \sin 2| + \left|\sin 1 - \frac{1}{2}\right| +$

$+ |\sin 2 - 1| = \sin 2 - \sin 1 + \sin 1 - \frac{1}{2} - \sin 2 + 1 = \frac{1}{2}$.

б) $\sqrt{\cos^2 6 + \cos^2 7 - 2\cos 6 \cos 7} + \sqrt{\frac{1}{4} - \cos 7 + \cos^2 7} +$

$+\sqrt{1+\cos^2 6-2\cos 6} = |\cos6-\cos7| + |\cos7-\frac{1}{2}| +$

$+|\cos6-1| = 1-\cos6+\cos6-\cos7+\cos7-\frac{1}{2} = \frac{1}{2}$

86. а) $\sin(\pi - t) = \sin t$, $\sin(\pi - t) = -\sin(-t) = \sin t$
б) $\sin(2\pi - t) = -\sin t$, $\sin(2\pi - t) = \sin(-t) = -\sin t$
в) $\cos(\pi - t) = -\cos t$, $\cos(\pi - t) = -\cos(-t) = -\cos t$
г) $\cos(2\pi - t) = \cos t$, $\cos(2\pi - t) = \cos(-t) = \cos t$

87. а) $\sin t > 0$, $t \in (2\pi k; \pi + 2\pi k)$; б) $\sin < \frac{\sqrt{3}}{2}$, $t \in (-\frac{4\pi}{3} + 2\pi k; \frac{\pi}{3} + 2\pi k)$;

в) $\sin t < 0$, $t \in (-\pi + 2\pi k; 2\pi k)$; г) $\sin t > \frac{\sqrt{3}}{2}$, $t \in (\frac{\pi}{3} + 2\pi k; \frac{2\pi}{3} + 2\pi k)$.

88. а) $\cos t > 0$, $t \in (-\frac{\pi}{2} + 2\pi k; \frac{\pi}{2} + 2\pi k)$; б) $\cos t < \frac{\sqrt{2}}{2}$, $t \in (\frac{\pi}{4} + 2\pi k; \frac{7\pi}{4} + 2\pi k)$;

в) $\cos t < 0$, $t \in (\frac{\pi}{2} + 2\pi k; \frac{3\pi}{2} + 2\pi k)$; г) $\cos t > \frac{\sqrt{2}}{2}$, $t \in (-\frac{\pi}{4} + 2\pi k; \frac{\pi}{4} + 2\pi k)$.

89. а) $\sin t < -\frac{1}{2}$, $t \in (\frac{7\pi}{6} + 2\pi k; \frac{11\pi}{6} + 2\pi k)$;

б) $\sin t > -\frac{\sqrt{2}}{2}$, $t \in (-\frac{\pi}{4} + 2\pi k; \frac{5\pi}{4} + 2\pi k)$; в) $\sin t > -\frac{1}{2}$, $t \in (-\frac{\pi}{6} + 2\pi k; \frac{7\pi}{6} + 2\pi k)$;

г) $\sin t < -\frac{\sqrt{2}}{2}$, $t \in (-\frac{5\pi}{4} + 2\pi k; \frac{7\pi}{4} + 2\pi k)$.

90. а) $\cos t > -\frac{\sqrt{3}}{2}$, $t \in (-\frac{5\pi}{6} + 2\pi k; \frac{5\pi}{6} + 2\pi k)$;

б) $\cos t < -\frac{1}{2}$, $t \in (\frac{2\pi}{3} + 2\pi k; \frac{4\pi}{3} + 2\pi k)$;

в) $\cos t < -\frac{\sqrt{3}}{2}$, $t \in (\frac{5\pi}{6} + 2\pi k; \frac{7\pi}{6} + 2\pi k)$;

г) $\cos t > -\frac{1}{2}$, $t \in (-\frac{2\pi}{3} + 2\pi k; \frac{2\pi}{3} + 2\pi k)$.

91. а) $\sin t \le \frac{1}{2}$, $t \in [-\frac{7\pi}{6} + 2\pi k; \frac{\pi}{6} + 2\pi k]$;

б) $\cos t \ge -\frac{\sqrt{2}}{2}$, $t \in [-\frac{3\pi}{4} + 2\pi k; \frac{3\pi}{4} + 2\pi k]$;

в) $\sin t \ge -\frac{1}{2}$, $t \in [-\frac{\pi}{6} + 2\pi k; \frac{7\pi}{6} + 2\pi k]$;

г) $\cos t \le \frac{\sqrt{2}}{2}$, $t \in [\frac{\pi}{4} + 2\pi k; \frac{7\pi}{4} + 2\pi k]$.

§ 5. Тангенс и котангенс

92. а) $\text{tg}\,\frac{5\pi}{4} = 1$; б) $\text{ctg}\,\frac{4\pi}{3} = \frac{\sqrt{3}}{3}$; в) $\text{tg}\,\frac{5\pi}{6} = -\frac{\sqrt{3}}{3}$; г) $\text{ctg}\,\frac{7\pi}{4} = -1$.

93. а) $\text{tg}\,(-\frac{5\pi}{4}) = -1$; б) $\text{ctg}\,(-\frac{\pi}{3}) = -\frac{\sqrt{3}}{3}$; в) $\text{tg}\,(-\frac{\pi}{6}) = -\frac{\sqrt{3}}{3}$; г) $\text{ctg}(-\frac{2\pi}{3}) = \frac{\sqrt{3}}{3}$

94. а) $\text{tg}\,\frac{\pi}{4} + \text{ctg}\,\frac{5\pi}{4} = 1 + 1 = 2$; б) $\text{ctg}\,\frac{\pi}{3} - \text{tg}\,\frac{\pi}{6} = \frac{\sqrt{3}}{3} - \frac{\sqrt{3}}{3} = 0$;

в) $\text{tg}\,\frac{\pi}{6} \cdot \text{ctg}\,\frac{\pi}{6} = 1$; г) $\text{tg}\,\frac{9\pi}{4} + \text{ctg}\,\frac{\pi}{4} = 1 + 1 = 2$.

95. а) $\text{tg}\,\frac{\pi}{4}\ \sin\frac{\pi}{3}\ \text{ctg}\,\frac{\pi}{6} = 1 \cdot \frac{\sqrt{3}}{2}\ \sqrt{3} = \frac{3}{2}$;

б) $2\sin\frac{\pi}{3}\ \cos\frac{\pi}{6} - \frac{1}{2}\,\text{tg}\,\frac{\pi}{3} = 2 \cdot \frac{\sqrt{3}}{2}\ \frac{\sqrt{3}}{2} - \frac{1}{2}\ \sqrt{3} = \frac{3-\sqrt{3}}{2}$;

в) $2\sin\pi + 3\cos\pi + \text{ctg}\,\frac{\pi}{2} = 0 - 3 + 0 = -3$;

г) $\text{tg}\,0 + 8\cos\frac{3\pi}{2} - 6\sin\frac{\pi}{3} = 0 + 0 - 6\ \frac{\sqrt{3}}{2} = 3\sqrt{3}$.

96. а) $\text{tg}\,\frac{\pi}{5} \cdot \text{ctg}\,\frac{\pi}{5} = 1$; б) $3\,\text{tg}\,2{,}3 \cdot \text{ctg}\,2{,}3 = 3$;

в) $\text{tg}\,\frac{\pi}{7} \cdot \text{ctg}\,\frac{\pi}{7} = 1$; г) $7\,\text{ctg}\,\frac{\pi}{12} \cdot \text{ctg}\,\frac{\pi}{12} = 7$.

97. а) $\text{tg}\,2{,}5 \angle \text{ctg}\,2{,}5 + \cos^2\pi - \sin^2\frac{\pi}{8} - \cos^2\frac{\pi}{8} = 1 + 1 - 1 = 1$.

б) $\sin^2\frac{3\pi}{7} - 2\,\text{tg}\,1 \cdot \text{ctg}\,1 + \cos^2(-\frac{3\pi}{7}) + \sin^2\frac{5\pi}{2} = 1 - 2 + 1 = 0$.

98. а) $\text{tg}\,\frac{6\pi}{7} < 0$; б) $\text{ctg}\,\frac{10\pi}{7} > 0$; в) $\text{tg}\,\frac{8\pi}{7} > 0$; г) $\text{ctg}\,\frac{11\pi}{7} < 0$.

99. а) $\sin t \cdot \text{ctg}\,t = \cos t$, $\sin t \cdot \frac{\cos t}{\sin t} = \cos t$; б) $\frac{\sin t}{\text{tg}\,t} = \cos t$, $\sin t \cdot \frac{\cos t}{\sin t} = \cos t$;

в) $\cos t \cdot \text{tg}\,t = \sin t$; г) $\frac{\cos t}{\text{ctg}\,t} = \sin t$, $\cos t \cdot \frac{\sin t}{\cos t} = \sin t$.

100. а) $\sin t \cdot \cos t \cdot \text{tg}\,t = \sin t \cdot \cos t \cdot \frac{\sin t}{\cos t} = \sin^2 t$;

б) $\sin t \cdot \cos t \cdot \text{ctg}\,t - 1 = \sin t \cdot \cos t \cdot \frac{\cos t}{\sin t} - 1 = \cos^2 t - 1 = -\sin^2 t$;

в) $\sin^2 t - \text{tg}\,t \cdot \text{ctg}\,t = \sin^2 t - 1 = -\cos^2 t$; г) $\frac{1-\cos^2 t}{1-\sin^2 t} = \frac{\sin^2 t}{\cos^2 t} = \text{tg}^2 t$.

101. $\dfrac{\sin\frac{\pi}{4}-\cos\pi-\operatorname{tg}\frac{\pi}{4}}{2\sin\frac{\pi}{6}-\sin\frac{3\pi}{2}} = \dfrac{\sqrt{2}}{4}$; $\quad \dfrac{\frac{\sqrt{2}}{2}+1-1}{1+1} = \dfrac{\sqrt{2}}{4}$.

102. а) $\cos\dfrac{5\pi}{9} - \operatorname{tg}\dfrac{25\pi}{18} < 0$; б) $\operatorname{tg} 1 - \cos 2 > 0$;

в) $\sin\dfrac{7\pi}{10} - \operatorname{ctg}\dfrac{3\pi}{5} > 0$; г) $\sin 2 - \operatorname{ctg} 5{,}5 > 0$.

103. а) $\sin 1 \cdot \cos 2 \cdot \operatorname{tg} 3 \cdot \operatorname{ctg} 4 > 0$; б) $\sin(-5)\cos(-6)\ \operatorname{tg}(-7)\operatorname{ctg}(-8) < 0$.

104. а) $1 + \operatorname{tg}^2 t = \cos^{-2} t$, $\quad 1 + \dfrac{\sin^2 t}{\cos^2 t} = \dfrac{1}{\cos^2 t} = \cos^{-2} t$;

б) $1 + \operatorname{ctg}^2 t = \sin^{-2} t$, $\quad 1 + \dfrac{\cos^2 t}{\sin^2 t} = \dfrac{1}{\sin^2 t} = \sin^{-2} t$;

в) $\sin^2 t\,(1 + \operatorname{ctg}^2 t) = 1$, $\ \sin^2 t \cdot \sin^{-2} t = 1$;

г) $\cos^2 t\,(1 + \operatorname{tg}^2 t) = 1$, $\ \cos^2 t \cdot \cos^{-2} t = 1$.

105. а) $\operatorname{tg}(\pi - t) = -\operatorname{tg} t$, $\quad \operatorname{tg}(\pi - t) = \operatorname{tg}(-t) = -\operatorname{tg} t$;

б) $\operatorname{tg}(2\pi + t) = \operatorname{tg} t$, $\quad \operatorname{tg}(2\pi + t) = \operatorname{tg}(\pi + t) = \operatorname{tg} t$;

в) $\operatorname{ctg}(\pi - t) = -\operatorname{ctg} t$, $\quad \operatorname{ctg}(\pi - t) = \operatorname{ctg}(-t) = -\operatorname{tg} t$;

г) $\operatorname{ctg}(2\pi + t) = \operatorname{ctg} t$, $\ \operatorname{ctg}(2\pi + t) = \operatorname{ctg} t$, $\ \operatorname{ctg}(2\pi + t) = \operatorname{ctg}(\pi + t) = \operatorname{ctg} t$.

106. а) $\cos^2 t \cdot \operatorname{tg}^2 t - \sin^2 t \cdot \cos^2 t = \sin^2 t\,(1 - \cos^2 t) = \sin^4 t$;

б) $1 - \cos^2 t + \operatorname{tg}^2 t \cdot \cos^2 t = \sin^2 t + \sin^2 t = 2\sin^2 t$;

в) $(1 - \sin^2 t)(\operatorname{tg}^2 t + 1) = \cos^2 t\ \dfrac{1}{\cos^2 t} = 1$;

г) $(1 - \cos^2 t)(\operatorname{ctg}^2 t + 1) = \sin^2 t\ \dfrac{1}{\sin^2 t} = 1$.

107. а) $\dfrac{\cos^2 t - \sin^2 t}{\cos t\ \sin t} = \dfrac{\frac{\cos^2 t}{\cos^2 t} - \frac{\sin^2 t}{\cos^2 t}}{\frac{\cos t \cdot \sin t}{\cos^2 t}} = \dfrac{1 - \operatorname{tg}^2 t}{\operatorname{tg} t}$.

б) $\dfrac{\cos^2 t - \sin^2 t}{\sin t\ \cos t} = \dfrac{\frac{\cos^2 t}{\sin^2 t} - \frac{\sin^2 t}{\sin^2 t}}{\frac{\cos t\ \cdot\ \sin t}{\sin^2 t}} = \dfrac{\operatorname{ctg}^2 t - 1}{\operatorname{ctg} t}$.

108. а) $\cos 1$, $\sin 1$, 1, $\operatorname{tg} 1$. б) $\operatorname{ctg} 2$, $\cos 2$, $\sin 2$, 2.

109. а) $\operatorname{ctg} 5\,(x - 1) \ge 0$, $\ -x + 1 \ge 0$, $\ x \le 1$;

б) $\dfrac{\operatorname{tg} 7\ \cdot \cos 1}{\sin 1}\,(2x^2 - 72) > 0$, $\ 2x^2 - 72 < 0$, $\ x^2 > 36$, $\ x \in (-6; 6)$;

в) $(\operatorname{tg} 2 \cdot \sin 5)(7 - 5x) \le 0$, $\ 7 - 5x \le 0$, $\ x \ge \dfrac{7}{5}$;

г) $\operatorname{tg} 1 \cdot \operatorname{ctg} 2 \cdot \operatorname{tg} 3 \cdot \operatorname{ctg} 4 \cdot (x^2 + 2) > 0$, $\ x^2 + 2 > 0$, $\ x \in R$.

110. а) $1 - \sin^2 t = \cos^2 t$; б) $\cos^2 t - 1 = -\sin^2 t$;
в) $1 - \cos^2 t = \sin^2 t$; г) $\sin^2 t - 1 = -\cos^2 t$.

111. а) $(1 - \sin t)(1 + \sin t) = 1 - \sin^2 t = \cos^2 t$;
б) $\cos^2 t + 1 - \sin^2 t = \cos^2 t + \cos^2 t = 2\cos^2 t$;
в) $(1 - \cos t)(1 + \cos t) = 1 - \cos^2 t = \sin^2 t$;
г) $\sin^2 t + 2\cos^2 t - 1 = 1 + \cos^2 t - 1 = \cos^2 t$.

112. а) $\frac{1}{\cos^2 t} - 1 = \frac{1-\cos^2 t}{\cos^2 t} = \mathrm{tg}^2 t$; б) $\frac{1-\sin^2 t}{\cos^2 t} = 1$;

в) $1 - \frac{1}{\sin^2 t} = \frac{\sin^2 t - 1}{\sin^2 t} = -\mathrm{ctg}^2 t$; г) $\frac{1-\cos^2 t}{1-\sin^2 t} = \frac{\sin^2 t}{\cos^2 t} = \mathrm{tg}^2 t$.

113. а) $\frac{(\sin t+\cos t)^2}{1+2\sin t\cos t} = \frac{1+2\sin t\cos t}{1+2\sin t\cos t} = 1$; б) $\frac{1-2\sin t\cos t}{(\cos t-\sin t)^2} = \frac{1-2\sin t\cos t}{1-2\sin t\cos t} = 1$.

114. а) $\frac{\cos^2 t}{1-\sin t} - \sin t = 1$, $\frac{1-\sin^2 t}{1-\sin t} - \sin t = \frac{1-\sin^2 t-\sin t+\sin^2 t}{1-\sin t} = 1$;

б) $\frac{\sin^2 t}{1+\cos t} + \cos t = 1$, $\frac{1-\cos^2 t}{1+\cos t} + \cos t = \frac{\sin^2 t+\cos t+\cos^2 t}{1+\cos t} = 1$.

115. а) $(\sin t + \cos t)^2 - 2\sin t\cos t = 1 + 2\sin t\cos t - 2\sin t\cos t = 1$;

б) $\frac{2-\sin^2 t-\cos^2 t}{3\sin^2 t+3\cos^2 t} = \frac{2-1}{3} = \frac{1}{3}$;

в) $\sin^4 t + \cos^4 t + 2\sin^2 t\cos^2 t = (\sin^2 t + \cos^2 t)^2 = 1$;

г) $\frac{\sin^4 t-\cos^4 t}{\sin^2 t-\cos^2 t} = \frac{(\sin^2 t-\cos^2 t)(\sin^2 t+\cos^2 t)}{\sin^2 t-\cos^2 t} = \sin^2 t + \cos^2 t = 1$.

116. а) $\sin t = \frac{4}{5}$, $\frac{\pi}{2} < t < \pi$, $\cos t = -\frac{3}{5}$, $\mathrm{tg}\, t = -\frac{4}{3}$, $\mathrm{ctg}\, t = -\frac{3}{4}$;

б) $\sin t = \frac{5}{13}$, $0 < t < \frac{\pi}{2}$, $\cos t = \sqrt{1-\frac{25}{169}} = \frac{12}{13}$, $\mathrm{tg}\, t = \frac{5}{12}$, $\mathrm{ctg}\, t = \frac{12}{5}$;

в) $\sin t = -0{,}6$, $-\frac{\pi}{2} < t < 0$, $\cos t = \frac{4}{5}$, $\mathrm{tg}\, t = -\frac{3}{4}$, $\mathrm{ctg}\, t = -\frac{4}{3}$;

г) $\sin t = -0{,}28$, $\pi < t < \frac{3\pi}{2}$, $\cos t = -\sqrt{1-\frac{43}{625}} = -\frac{24}{25}$, $\mathrm{tg}\, t = \frac{7}{24}$, $\mathrm{ctg}\, t = \frac{24}{7}$.

117. а) $\cos t = 0{,}8$, $0 < t < \frac{\pi}{2}$, $\sin t = \frac{3}{5}$, $\mathrm{tg}\, t = \frac{3}{4}$, $\mathrm{ctg}\, t = \frac{4}{3}$;

б) $\cos t = -\frac{5}{13}$, $\frac{\pi}{2} < t < \pi$, $\sin t = \frac{12}{13}$, $\mathrm{tg}\, t = -\frac{12}{5}$, $\mathrm{ctg}\, t = -\frac{5}{12}$;

в) $\cos t = 0{,}6$, $\frac{3\pi}{2} < t < 2\pi$, $\sin t = -\frac{4}{5}$, $\operatorname{tg} t = -\frac{4}{3}$, $\operatorname{ctg} t = -\frac{3}{4}$;

г) $\cos t = -\frac{24}{25}$, $\pi < t < \frac{3\pi}{2}$, $\sin t = -\frac{7}{25}$, $\operatorname{tg} t = \frac{7}{24}$, $\operatorname{ctg} t = \frac{24}{7}$.

118. а) $\operatorname{tg} t = \frac{3}{4}$, $0 < t < \frac{\pi}{2}$, $\operatorname{ctg} t = \frac{4}{3}$, $\sin t = \frac{3}{5}$, $\cos t = \frac{4}{5}$;

б) $\operatorname{tg} t = 2{,}4$, $\pi < t < \frac{3\pi}{2}$, $\operatorname{ctg} t = \frac{5}{12}$, $\cos t = -\frac{5}{13}$, $\sin t = -\frac{12}{13}$;

в) $\operatorname{tg} t = -\frac{3}{4}$, $\frac{\pi}{2} < t < \pi$, $\operatorname{ctg} t = -\frac{4}{3}$, $\sin t = \frac{3}{5}$, $\cos t = -\frac{4}{5}$;

г) $\operatorname{tg} t = -\frac{1}{3}$, $\frac{3\pi}{2} < t < 2\pi$, $\operatorname{ctg} t = -3$, $\frac{\cos t}{\sqrt{1-\cos^2 t}} = -3$, $\cos^2 t = 9 - 9\cos^2 t$,

$\cos t = \frac{3}{\sqrt{10}}$, $\sin t = -\sqrt{1-\frac{9}{10}} = -\frac{1}{\sqrt{10}}$.

119. а) $\operatorname{ctg} t = \frac{12}{5}$, $\pi < t < \frac{3\pi}{2}$, $\operatorname{tg} t = \frac{5}{12}$, $\sin t = -\frac{5}{13}$, $\cos t = -\frac{12}{13}$;

б) $\operatorname{ctg} t = \frac{7}{24}$, $0 < t < \frac{\pi}{2}$, $\operatorname{tg} t = \frac{24}{7}$, $\sin t = \frac{24}{25}$, $\cos t = \frac{7}{25}$;

в) $\operatorname{ctg} t = -\frac{5}{12}$, $\frac{3\pi}{2} < t < 2\pi$, $\operatorname{tg} t = -\frac{12}{5}$, $\cos t = \frac{5}{13}$, $\sin t = -\frac{12}{13}$;

г) $\operatorname{ctg} t = -\frac{8}{15}$, $\frac{\pi}{2} < t < \pi$, $\operatorname{tg} t = -\frac{15}{8}$, $\frac{\cos t}{\sqrt{1-\cos^2 t}} = -\frac{8}{15}$,

$\cos^2 t = \frac{64}{225} - \frac{64}{225}\cos^2 t$, $\cos t = -\frac{8}{17}$, $\sin t = \sqrt{1-\frac{64}{289}} = \frac{15}{17}$.

120. а) $f(x) = 1 - (\cos^2 t - \sin^2 t) = 2\sin^2 t$, $f_{max} = 2$, $f_{min} = 0$;

б) $f(t) = 1 - \sin t \cdot \cos t \cdot \operatorname{tg} t = 1 - \sin t \cdot \cos t \cdot \frac{\sin t}{\cos t} = \cos^2 t$, $f_{max} = 1$, $f_{min} = 0$.

в) $f(t) = \cos^2 t \cdot \operatorname{tg}^2 t + 5\cos^2 t - 1 = \cos^2 t \cdot \frac{\sin^2 t}{\cos^2 t} + 5\cos^2 t - 1 = \sin^2 t + 5\cos^2 t - 1 = 4\cos^2 t$,

$f_{max} = 4$, $f_{min} = 0$.

г) $f(t) = \sin t + 3\sin^2 t + 3\cos^2 t = \sin t + 3$, $f_{max} = 4$, $f_{min} = 2$.

121. а) $\operatorname{ctg} t - \frac{\cos t - 1}{\sin t} = \frac{\cos t - \cos t + 1}{\sin t} = \frac{1}{\sin t}$;

б) $\operatorname{ctg}^2 t - \left(\frac{1}{\sin^2 t} - 1\right) = \frac{\cos^2 t - 1 + \sin^2 t}{\sin^2 t} = 0$;

в) $\cos^2 t - (\operatorname{ctg}^2 t + 1)\sin^2 t = \cos^2 t - \cos^2 t - \sin^2 t = -\sin^2 t$;

г) $\frac{\sin^2 t - 1}{\cos^2 t - 1} + \operatorname{tg} t \cdot \operatorname{ctg} t = \operatorname{ctg}^2 t + 1 = \frac{\sin^2 t + \cos^2 t}{\sin^2 t} = \frac{1}{\sin^2 t}$.

122. а) $\frac{\sin t}{1+\cos t}+\frac{\sin t}{1-\cos t}=\frac{\sin t-\sin t\cos t+\sin t+\cos t\sin t}{1-\cos^2 t}=\frac{2\sin t}{\sin^2 t}=\frac{2}{\sin t}$;

б) $\operatorname{ctg}^2 t(\cos^2 t-1)+1=-\cos^2 t+1=\sin^2 t$;

в) $\frac{\cos t}{1+\sin t}+\frac{\cos t}{1-\sin t}=\frac{\cos t-\sin t\cos t+\cos t+\sin t\cos t}{1-\sin^2 t}=\frac{2\cos t}{\cos^2 t}=\frac{2}{\cos t}$;

г) $\frac{\operatorname{tg} t+1}{1+\operatorname{ctg} t}=\frac{\frac{\sin t+\cos t}{\cos t}}{\frac{\sin t+\cos t}{\sin t}}=\operatorname{tg} t.$

123. а) $(3\sin t+4\cos t)^2+(4\sin t-3\cos t)^2=9\sin^2 t+16\cos^2 t+$
$+24\sin t\cos t+16\sin^2 t+9\cos^2 t-24\sin t\cos t=25$;

б) $(\operatorname{tg} t+\operatorname{ctg} t)^2-(\operatorname{tg} t-\operatorname{ctg} t)^2=\operatorname{tg}^2 t+\operatorname{ctg}^2 t+2-\operatorname{tg}^2 t-\operatorname{ctg}^2 t+2=4$;

в) $\sin t\cdot\cos t(\operatorname{tg} t+\operatorname{ctg} t)=\sin t\cdot\cos t\,\frac{\sin^2 t+\cos^2 t}{\sin t\cos t}=1$;

г) $\sin^2 t\cdot\cos^2 t(\operatorname{tg}^2 t+\operatorname{ctg}^2 t+2)=\sin^2 t\cdot\cos^2 t(\operatorname{tg} t+\operatorname{ctg} t)^2=\sin^2 t\cdot\cos^2 t\,\frac{1}{\sin^2 t\cos^2 t}=1.$

124. а) $\frac{1-\sin^2 t}{1-\cos^2 t}+\operatorname{tg} t\cdot\operatorname{ctg} t=\frac{\cos^2 t}{\sin^2 t}+1=\operatorname{ctg}^2 t+1=\frac{1}{\sin^2 t}$;

б) $\frac{\cos^2 t-\operatorname{ctg}^2 t}{\sin^2 t-\operatorname{tg}^2 t}=\frac{\frac{\cos^2 t\cdot\sin^2 t-\cos^2 t}{\sin^2 t}}{\frac{\sin^2 t\cdot\cos^2 t-\sin^2 t}{\cos^2 t}}=\frac{\cos^2 t(\sin^2 t-1)\cos^2 t}{\sin^2 t\ \sin^2 t(\cos^2 t-1)}=\frac{-\cos^6 t}{-\sin^6 t}=\operatorname{ctg}^6 t$

125. а) $\frac{\operatorname{tg} t}{\operatorname{tg} t+\operatorname{ctg} t}=\sin^2 t$, $\frac{\operatorname{tg} t}{\frac{1}{\sin t\cos t}}=\frac{\sin t}{\cos t}=\sin t\cdot\cos t=\sin^2 t$

б) $\frac{1+\operatorname{tg} t}{1+\operatorname{ctg} t}=\operatorname{tg} t$, $\frac{\frac{\sin t+\cos t}{\cos t}}{\frac{\sin t+\cos t}{\sin t}}=\frac{\sin t}{\cos t}=\operatorname{tg} t$

в) $\frac{\operatorname{ctg} t}{\operatorname{tg} t+\operatorname{ctg} t}=\cos^2 t$, $\frac{\operatorname{ctg} t}{\frac{1}{\sin t\cos t}}=\operatorname{ctg} t\cdot\sin t\cdot\cos t=\cos^2 t$;

г) $\frac{1-\operatorname{ctg} t}{1-\operatorname{tg} t}=-\operatorname{ctg} t$; $\frac{\frac{\sin t-\cos t}{\sin t}}{\frac{-\sin t+\cos t}{\cos t}}=-\frac{\cos t}{\sin t}=-\operatorname{ctg} t.$

126. а) $1+\sin t=\frac{\cos t+\operatorname{ctg} t}{\operatorname{ctg} t}$, $\frac{\frac{\cos t\sin t+\cos t}{\sin t}}{\frac{\cos t}{\sin t}}=\frac{\cos t\sin t+\cos t}{\cos t}=\sin t+1$

б) $\frac{\sin t + \text{tg}\, t}{\text{tg}\, t} = 1 + \cos t$, $\sin t \cdot \frac{\cos t}{\sin t} + 1 = \cos t + 1$;

в) $\frac{1-\sin t}{\cos t} = \frac{\cos t}{1+\sin t}$, $\frac{1-\sin^2 t}{\cos t\,(1+\sin t)} = \frac{\cos t}{1+\sin t}$;

г) $\frac{\sin t}{1-\cos t} = \frac{1+\cos t}{\sin t}$, $\frac{\sin t(1+\cos t)}{1-\cos^2 t} = \frac{1+\cos t}{\sin t}$.

127. а) $\frac{(\sin t+\cos t)^2-1}{\text{ctg}\, t - \sin t\, \cos t} = 2\,\text{tg}^2\, t$, $\frac{2\sin t\cos t}{\cos t - \sin^2 t\cos t}\sin t = \frac{2\sin^2 t\cos t}{\cos t(1-\sin^2 t)} = 2\,\text{tg}^2\, t$.

б) $\sin^3 t\,(1 + \text{ctg}\, t) + \cos^3 t\,(1 + \text{tg}\, t) = \sin t + \cos t$,

$\sin^3 t \cdot \frac{\sin t + \cos t}{\sin t} + \cos^3 t\, \frac{\sin t + \cos t}{\cos t} = (\sin t + \cos t)\,(\sin^2 t + \cos^2 t) = \sin t + \cos t$.

в) $\frac{(\sin t+\cos t)^2}{\text{tg}\, t - \sin t\, \cos 2t} = 2\,\text{ctg}^2\, t = \frac{2\sin t\, \cos t}{\sin t - \sin t\, \cos^2 t}\cos t = \frac{2\sin t\cos^2 t}{\sin^3 t} = 2\,\text{ctg}^2\, t$.

г) $\frac{1-4\sin^2 t\cos^2 t}{(\sin t+\cos t)^2} + 2\sin t \cdot \cos t = 1$.

$\frac{(1-2\sin t\cos t)\,(1+2\sin t\cos t)}{1+2\sin t\cos t} + 2\sin t \cdot \cos t = 1 - 2\sin t \cdot \cos t + 2\sin t \cdot \cos t = 1$.

128. а) $\sin(4\pi + t) = \frac{3}{5}$, $0 < t < \frac{\pi}{2}$,

$\cos t = \frac{4}{5}$, $\text{tg}\, t = \frac{3}{4}$, $\text{tg}\,(-t) = -\frac{3}{4}$, $\text{tg}\,(\pi - t) = -\frac{3}{4}$;

б) $\cos(2\pi + t) = \frac{12}{13}$, $\frac{3\pi}{2} < t < 2\pi$,

$\sin t = -\frac{5}{13}$, $\text{ctg}\, t = -\frac{12}{5}$, $\text{ctg}\,(-t) = \frac{12}{5}$, $\text{ctg}\,(\pi - t) = \frac{12}{5}$.

129. а) $\cos t = -\frac{5}{13}$, $8{,}5\pi < t < 9\pi$, $\sin t = \frac{12}{13}$, $\sin(-t) = -\frac{12}{13}$;

б) $\sin t = \frac{4}{5}$, $\frac{9\pi}{2} < t < 5\pi$, $\cos t = -\frac{3}{5}$, $\cos(-t) = -\frac{3}{5}$,

$\sin(-t) = -\frac{4}{5}$, $\cos(-t) + \sin(-t) = -\frac{7}{5}$.

130. а) $\sin t + \cos t = 0{,}8$, $(\sin t + \cos t)^2 = \frac{16}{25}$,

$2\cos t \cdot \sin t = -\frac{9}{25}$, $\cos t \cdot \sin t = -\frac{9}{50}$;

б) $\sin t - \cos t = \frac{1}{3}$, $(\cos t - \sin t)^2 = \frac{1}{9}$, $-2\sin t \cdot \cos t = -\frac{8}{9}$, $9\sin t \cdot \cos t = 4$.

131. $\text{tg}\, t + \text{ctg}\, t = 2{,}3$, $(\text{tg}\, t + \text{ctg}\, t)^2 = \text{tg}^2 + 2\text{tg}\, t \cdot \text{ctg}\, t + \text{ctg}^2 t = 5{,}29$, $\text{tg}^2 t + \text{ctg}^2\, t = 3{,}29$

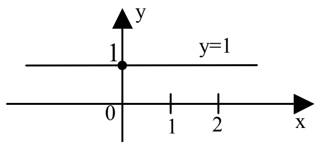
132. $\sin t\cos t = -\frac{1}{2}$, $\sin^4 t + \cos^4 t = 1 - 2\sin^2 t \cdot \cos^2 t = 1 - \frac{1}{2} = \frac{1}{2}$.

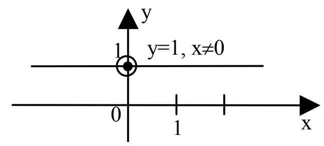
133. $\operatorname{tg} t - \frac{1}{\operatorname{tg} t} = -\frac{7}{12}$, $0 < t < \frac{\pi}{2}$, $12\operatorname{tg}^2 t + 7\operatorname{tg} t - 12 = 0$,

$\operatorname{tg} t = \frac{-7 \pm \sqrt{49 - 4\cdot 12(-12)}}{24} = \frac{-7 \pm 25}{24}$, $\operatorname{tg} t = -\frac{4}{3}$ не подходит, т.к. $0 < t < \frac{\pi}{2}$,

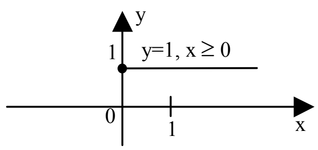
$\operatorname{tg} t = \frac{3}{4} \Rightarrow \cos t = \frac{4}{5}$, $\sin t = \frac{3}{5} \Rightarrow \sin t + \cos t = \frac{7}{5}$.

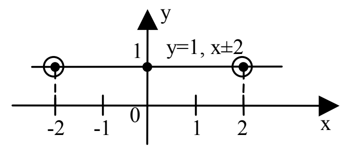
134. а) $y = \cos^2 t + \sin^2 t = 1$; б) $y = \cos^2 \frac{1}{x} + \sin^2 \frac{1}{x} = 1$ $(x \neq 0)$





в) $y=\sin^2 \sqrt{x} + \cos^2 \sqrt{x} = 1$ $(x \geq 0)$ г) $y=\sin^2 \frac{1}{x^2-4} + \cos^2 \frac{1}{x^2-4} = 1$ $(x \neq \pm 2)$





§ 7. Тригонометрические функции углового аргумента

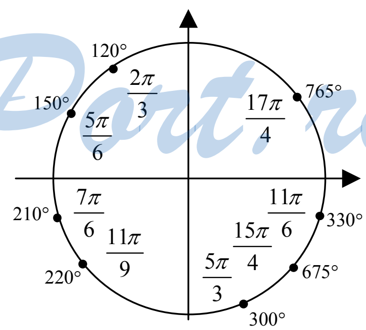
135 – 138 см. рис.

135. а) $120° = \frac{2\pi}{3}$; б) $220° = \frac{11\pi}{9}$;

в) $300° = \frac{5\pi}{3}$; г) $765° = \frac{17\pi}{4}$.

136. а) $210° = \frac{7\pi}{6}$; б) $150° = \frac{5\pi}{6}$;

в) $330° = \frac{11\pi}{6}$; г) $675° = \frac{15\pi}{4}$.



137. а) $\frac{3\pi}{4} = 135°$; б) $\frac{11\pi}{3} = 660°$;

в) $\frac{6\pi}{5} = 216°$; г) $\frac{46\pi}{9} = 920°$.

138. а) $\frac{5\pi}{8} = 112{,}5°$; б) $\frac{7\pi}{12} = 105°$;

в) $\frac{11\pi}{12} = 165°$; г) $\frac{47\pi}{9} = 940°$.

139. а) $\sin 90° = 1$, $\cos 90° = 0$, $\text{tg } 90°$ = не сущ., $\text{ctg } 90° = 0$
б) $\sin 180° = 0$, $\cos 180° = -1$, $\text{tg } 180° = 0$, $\text{ctg } 180°$ = не сущ.
в) $\sin 270° = -1$, $\cos 270° = 0$, $\text{tg } 270°$ = не сущ., $\text{ctg } 270° = 0$
г) $\sin 360° = 0$, $\cos 360° = 1$, $\text{tg } 360° = 0$, $\text{ctg } 360°$ = не сущ.

140. а) $\sin 30° = \frac{1}{2}$, $\cos 30° = \frac{\sqrt{3}}{2}$, $\text{tg } 30° = \frac{\sqrt{3}}{3}$, $\text{ctg } 30° = \sqrt{3}$;

б) $\sin 150° = \frac{1}{2}$, $\cos 150° = -\frac{\sqrt{3}}{2}$, $\text{tg } 150° = -\frac{\sqrt{3}}{3}$, $\text{ctg } 150° = -\sqrt{3}$;

в) $\sin 210° = -\frac{1}{2}$, $\cos 210° = -\frac{\sqrt{3}}{2}$, $\text{tg } 210° = \frac{\sqrt{3}}{3}$, $\text{ctg } 210° = \sqrt{3}$;

г) $\sin 240° = -\frac{\sqrt{3}}{2}$, $\cos 240° = -\frac{1}{2}$, $\text{tg } 240° = \sqrt{3}$, $\text{ctg } 240° = \frac{\sqrt{3}}{3}$.

141. sin 160°, sin 40°, sin 120°, sin 80°
142. cos 160°, cos 120°, cos 80°, cos 40°
143. sin 210°, sin 20°, sin 400°,sin 110°

144. а) $\text{tg } \alpha = \frac{x}{2}$, $x = 2 \text{ tg } \alpha$; б) $\cos \alpha = \frac{x}{4}$, $x = 4\cos \alpha$;

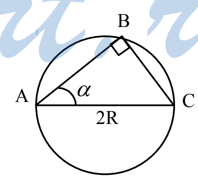
в) $\cos \alpha = \frac{3}{x}$, $x = \frac{3}{\cos \alpha}$; г) $\text{ctg } \alpha = x$.

145. а) $\sin 30° = \frac{2}{x}$, $x = \frac{2}{\frac{1}{2}} = 4$; б) $x = \frac{\sqrt{2}}{2}$

в) $\frac{\sqrt{3}}{2} = \frac{2}{x}$, $x = \frac{4}{\sqrt{3}}$; г) $\frac{x}{2} = \cos 60° = \frac{1}{2}$, $x=1$.

146. а) $c = 12$, $\alpha = 60°$, $a = 6$, $b = 6\sqrt{3}$, $S = 18\sqrt{3}$, $R = 6$;
б) $c = 6$, $\alpha = 45°$, $a = b = 3 \cdot \sqrt{2}$, $S = 9$, $R = 3$;
в) $c = 4$, $\alpha = 30°$, $a = 2$, $b = 2\sqrt{3}$, $S = 2 \cdot \sqrt{3}$, $R = 2$;
г) $c = 60$, $\alpha = 60°$, $a = 30$, $b = 30\sqrt{3}$, $S = 450 \cdot \sqrt{3}$, $R = 30$.

147. $\cos \alpha = \frac{AB}{AC}$,
$AB = AC \cdot \cos \alpha = 2R \cos \alpha$.

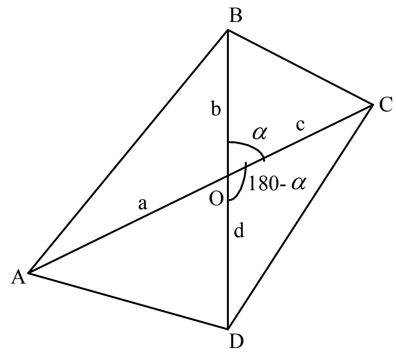


148.

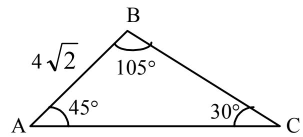
$$S_{ABCD} = \frac{1}{2} ab \sin \alpha + \frac{1}{2} bc \sin \alpha + \frac{1}{2} cd \sin \alpha + \frac{1}{2} da \sin \alpha =$$

$$= \frac{1}{2} \sin \alpha (ab + bc + cd + da) = \frac{1}{2} \sin \alpha (b (a + c) + d (c + a)) =$$

$$= \frac{1}{2} \sin \alpha (b + d) (a + c).$$



149.



$\frac{AB}{\sin C} = \frac{BC}{\sin A}$, $\frac{4\sqrt{2}}{1/2} = \frac{BC}{\sqrt{2}/2} \Rightarrow BC = 8$,

$\angle B = 180^\circ - 45^\circ - 30^\circ = 105^\circ$,

$\frac{AC}{\sin 105^o} = \frac{16}{\sqrt{2}}$,

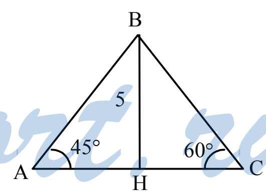
$AC = \frac{16}{\sqrt{2}} \cdot \sin 105^o$; $\sin 105^o = \sin 75^o = \sin(45^o + 30^o)$,

$AC = \frac{16}{\sqrt{2}} \left(\frac{\sqrt{2}}{2} \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \frac{1}{2} \right) = \frac{16}{\sqrt{2}} = \frac{\sqrt{6}+\sqrt{2}}{4} = 4\sqrt{3} + 4.$

$S = \frac{1}{2} \cdot AC \cdot AB \cdot \sin \angle C$; $S = \frac{1}{2} \cdot \frac{1}{2} (4\sqrt{3} + 4)\, 8 = 8(\sqrt{3} + 1).$

Ответ: $BC = 8$, $AC = 4\sqrt{3} + 4$, $S = 8(\sqrt{3} + 1)$ (см^2).

150.



$AH = BH = 5$ (т.к. $\angle A = \angle ABH = 45^\circ$)

$\text{tg}\, 60^\circ = \frac{5}{HC}$, $HC = \frac{5}{\text{tg}60^\circ} = \frac{5}{\sqrt{3}}$

$S = \frac{1}{2} \cdot 5 \cdot \left(5 + \frac{5}{\sqrt{3}}\right) = \frac{25(3+\sqrt{3})}{6}$

§ 8. *Формулы приведения*

151. а) $\sin\left(\frac{\pi}{2} - t\right) = \cos t$; б) $\cos(2\pi - t) = \cos t$;

в) $\cos\left(\frac{3\pi}{2} + t\right) = -\sin t$; г) $\sin(\pi + t) = -\sin t$.

152. а) $\sin(\pi - t) = \sin t$; б) $\cos\left(\frac{\pi}{2} + t\right) = -\sin t$;

в) $\cos(2\pi + t) = \cos t$; г) $\sin\left(\frac{3\pi}{2} - t\right) = -\cos t$.

153. а) $\cos(90^\circ - \alpha) = \sin\alpha$; б) $\sin(360^\circ - \alpha) = -\sin\alpha$;
в) $\sin(270^\circ + \alpha) = -\cos\alpha$; г) $\cos(180^\circ + \alpha) = -\cos\alpha$.

154. а) $\mathrm{tg}(90^\circ - \alpha) = \mathrm{ctg}\,\alpha$; б) $\mathrm{ctg}(180^\circ - \alpha) = -\mathrm{ctg}\,\alpha$;
в) $\mathrm{tg}(270^\circ + \alpha) = -\mathrm{ctg}\,\alpha$; г) $\mathrm{ctg}(360^\circ + \alpha) = \mathrm{ctg}\,\alpha$.

155. а) $\sin 240^\circ = -\sin 60^\circ = -\frac{\sqrt{3}}{2}$; б) $\mathrm{tg}\,300^\circ = \mathrm{tg}\,120^\circ = -\mathrm{tg}\,60^\circ = -\sqrt{3}$;

в) $\cos 330^\circ = \cos(-30^\circ) = \frac{\sqrt{3}}{2}$; г) $\mathrm{ctg}\,315^\circ = -\mathrm{ctg}\,45^\circ = -1$.

156. а) $\cos\frac{5\pi}{3} = \frac{1}{2}$; б) $\sin\left(-\frac{11\pi}{6}\right) = \frac{1}{2}$; в) $\sin\frac{7\pi}{6} = -\frac{1}{2}$; г) $\cos\left(-\frac{7\pi}{3}\right) = \frac{1}{2}$.

157. а) $\cos 630^\circ - \sin 1470^\circ - \mathrm{ctg}\,1125^\circ = -\frac{1}{2} - 1 = -\frac{3}{2}$;

б) $\sin(-7\pi) + 2\cos\frac{31\pi}{3} - \mathrm{tg}\frac{7\pi}{4} = 1 + 1 = 2$;

в) $\mathrm{tg}\,1800^\circ - \sin 495^\circ + \cos 945^\circ = -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} = -\sqrt{2}$;

г) $\cos(-9\pi) + 2\sin\left(-\frac{49\pi}{6}\right) - \mathrm{ctg}\left(-\frac{21\pi}{4}\right) = -1 - 1 + 1 = -1$.

158. а) $\sin(90^\circ - \alpha) + \cos(180^\circ + \alpha) + \mathrm{tg}(270^\circ + \alpha) + \mathrm{ctg}(360^\circ + \alpha) =$
$= \cos\alpha - \cos\alpha - \mathrm{ctg}\,\alpha + \mathrm{ctg}\,\alpha = 0$;

б) $\sin\left(\frac{\pi}{2} + t\right) - \cos(\pi - t) + \mathrm{tg}(\pi - t) + \mathrm{ctg}\left(\frac{5\pi}{2} - t\right) =$
$= \cos t + \cos t - \mathrm{tg}\,t + \mathrm{tg}\,t = 2\cos t$.

159. а) $\frac{\cos(180+\alpha)\cdot\cos(-\alpha)}{\sin(-\alpha)\cdot\sin(90+\alpha)} = \frac{-\cos\alpha\cos\alpha}{-\sin\alpha\ \cos\alpha} = \mathrm{ctg}\,\alpha$;

б) $\frac{\sin(\pi - t)\cos(2\pi - t)}{\mathrm{tg}(\pi - t)\cos(\pi - t)} = \frac{\sin t\cos t}{-\mathrm{tg}t\cdot(-\cos t)} = \cos t$;

в) $\frac{\sin(-\alpha)\,\mathrm{ctg}(-\alpha)}{\cos(360-\alpha)\,\mathrm{tg}(180+\alpha)} = \frac{\cos\alpha}{\cos\alpha\ \mathrm{tg}\,\alpha} = \mathrm{ctg}\alpha$;

г) $\frac{\sin(\pi+t)\ \sin(2\pi+t)}{\mathrm{tg}(\pi+t)\ \cos\left(\frac{3\pi}{2}+t\right)} = \frac{-\sin t\ \cos t}{\frac{\sin t}{\cos t}\cdot\sin t} = -\cos t$.

160. а) $\frac{\cos(\pi - t) + \cos\left(\frac{\pi}{2} - t\right)}{\sin(2\pi - t) - \sin\left(\frac{3\pi}{2} + t\right)} = \frac{-\cos t + \sin t}{-\sin t + \cos t} = -1$; б) $\frac{\sin^2(\pi - t) + \sin^2\left(\frac{\pi}{2} - t\right)}{\sin(\pi - t)}$;

161. а) $\frac{\mathrm{tg}(\pi - t)}{\cos(\pi + t)}\cdot\frac{\sin\left(\frac{3\pi}{2} + t\right)}{\mathrm{tg}\left(\frac{3\pi}{2} + t\right)} = \mathrm{tg}^2 t$, $\frac{-\mathrm{tg}\,t}{-\cos t}\cos\left(\frac{3\pi}{2} + t\right) = \mathrm{tg}^2 t$;

б) $\dfrac{\sin(\pi-t)}{\operatorname{tg}(\pi+t)}\cdot\dfrac{\operatorname{ctg}(\frac{\pi}{2}-t)}{\operatorname{tg}(\frac{\pi}{2}+t)}\cdot\dfrac{\cos(2\pi-t)}{\sin(-t)}=\sin t$.

$\dfrac{\sin t}{\operatorname{tg} t}\cdot\dfrac{\operatorname{tg} t}{-\operatorname{ctg} t}\cdot\dfrac{\cos t}{-\sin t}=\operatorname{tg} t\ \cos t=\sin t$.

162. а) $\dfrac{\cos^2(\pi-t)+\sin^2(\frac{\pi}{2}-t)+\cos(\pi+t)\ \cos(2\pi-t)}{\operatorname{tg}^2(t-\frac{\pi}{2})\ \operatorname{ctg}^2(\frac{3\pi}{2}+t)}=\cos^2 t$,

$\dfrac{\cos^2 t+\cos^2 t-\cos^2 t}{\operatorname{ctg}^2 t\ \operatorname{tg}^2 t}=\cos^2 t$;

б) $\dfrac{\sin^2(t-\frac{3\pi}{2})\ \cos(2\pi-t)}{\operatorname{tg}^2(t-\frac{\pi}{2})\ \cos(t-\frac{3\pi}{2})}=\cos t$, $\quad\dfrac{\cos^2 t\cos t}{\operatorname{ctg}^2 t\sin^2 t}=\cos t$.

163. а) $\dfrac{11\cos 287^\circ-25\sin 557^\circ}{\sin 17^\circ}=\dfrac{-11\cos 167^\circ+25\sin 17^\circ}{\sin 17^\circ}=$

$=\dfrac{11\sin 17^\circ+25\sin 17^\circ}{\sin 17^\circ}=36.$

б) $\dfrac{13\sin 469^\circ-8\cos 341^\circ}{\cos 19^\circ}=\dfrac{13\sin 109^\circ-8\cos 19^\circ}{\cos 19^\circ}=\dfrac{13\cos 109^\circ-8\cos 19^\circ}{\cos 19^\circ}=5.$

164. а) $\dfrac{2\cos\frac{11\pi}{5}+8\sin\frac{13\pi}{10}}{\cos\frac{\pi}{5}}=\dfrac{2\cos\frac{\pi}{5}+8\sin\frac{3\pi}{10}}{\cos\frac{\pi}{5}}=\dfrac{2\cos\frac{\pi}{5}-8\sin\frac{\pi}{5}}{\cos\frac{\pi}{5}}=-6.$

б) $\dfrac{5\sin\frac{5\pi}{7}+2\cos\frac{25\pi}{14}}{\sin\frac{2\pi}{7}}=\dfrac{5\sin\frac{2\pi}{7}-2\cos\frac{11\pi}{14}}{\sin\frac{2\pi}{7}}=\dfrac{5\sin\frac{2\pi}{7}+2\sin\frac{4\pi}{14}}{\sin\frac{2\pi}{7}}=7.$

165. а) $2\cos(2\pi+t)+\sin(\frac{\pi}{2}+t)=3$, $2\cos t+\cos t=3$, $\cos t=1$, $t=2\pi n$;

б) $\sin(\pi+t)+2\cos(\frac{\pi}{2}+t)=3$, $-\sin t-2\sin t=3$, $\sin t=-1$, $t=-\frac{\pi}{2}+2\pi n$;

в) $2\sin(\pi+t)+\cos(\frac{\pi}{2}-t)=-\frac{1}{2}$, $-2\sin t+\sin t=-\frac{1}{2}$, $\sin t=\frac{1}{2}$, $t=(-1)^k\ \frac{\pi}{6}+\pi k$;

г) $3\sin(\frac{\pi}{2}+t)-\cos(2\pi+t)=1$, $3\cos t-\cos t=1$, $\cos t=\frac{1}{2}$, $t=\pm\frac{\pi}{3}+2\pi n$.

166. а) $5\sin(\frac{\pi}{2}+t)-\sin(\frac{3\pi}{2}+t)-8\cos(2\pi-t)=1$,

$5\cos t+\cos t-8\cos t=1$, $\cos t=-\frac{1}{2}$, $t=\pm\frac{2\pi}{3}+2\pi n$;

б) $\sin(2\pi + t) - \cos(\frac{\pi}{2} - t) + \sin(\pi - t) = 1$,

$\sin t - \sin t + \sin t = 1$, $\sin t = 1$, $t = \frac{\pi}{2} + 2\pi n$.

167. а) $\sin^2(\pi + t) + \cos^2(2\pi - t) = 0$, $\sin^2 t + \cos^2 t = 0$ корней нет;
б) $\sin^2(\pi+t) + \cos^2(2\pi - t) = 1$, $\sin^2 t + \cos^2(2\pi - t) = 1$, $\sin^2 t + \cos^2 t = 1$, $t \in R$.

§ 9. *Функция y = sin x, ее свойства и график*

168. а) $\sin\pi = 0$; б) $\sin(-\frac{\pi}{2}) = -1$; в) $\sin\frac{2\pi}{3} = \frac{\sqrt{3}}{2}$; г) $\sin(-\frac{\pi}{3}) = -\frac{\sqrt{3}}{2}$.
169. а) $f(x) = \sin x$, $f(-x) = -\sin x$; б) $f(x) = \sin x$, $f(2x) = \sin 2x$;
в) $f(x) = \sin x$, $f(x + 1) = \sin(x + 1)$; г) $f(x) = \sin x$, $f(x) - 5 = \sin x - 5$.

170. а) $y = 2\sin(x - \frac{\pi}{6}) + 1$, $x = \frac{4\pi}{3}$, $y = 2\sin\frac{7\pi}{6} + 1 = -1 + 1 = 0$;

б) $y = -\sin(x + \frac{\pi}{4})$, $x = -\frac{\pi}{2}$, $y = -\sin(-\frac{\pi}{4}) = \frac{\sqrt{2}}{2}$;

в) $y = 2\sin(x - \frac{\pi}{6}) + 1$, $x = \frac{7\pi}{6}$, $y = 2\sin\pi + 1 = 1$;

г) $y = -\sin(x + \frac{\pi}{4})$, $x = -\frac{15\pi}{4}$, $y = -\sin(\frac{14\pi}{4}) = \sin\frac{7\pi}{2} = -1$.

171. а) $y = \sin x$, $\sin(-\frac{\pi}{2}) = -1$, $(-\frac{\pi}{2}; -1)$ принадлежит;

б) $y = \sin x$, $\frac{1}{2} \neq \sin\frac{\pi}{2}$, $(\frac{\pi}{2}; \frac{1}{2})$ не принадлежит;

в) $y = \sin x$, $1 \neq \sin\pi$, $(\pi; 1)$ не принадлежит.

г) $y = \sin x$, $-1 = \sin\frac{3\pi}{2}$, $(\frac{3\pi}{2}; -1)$ принадлежит.

172. а) $y = \sin(x + \frac{\pi}{6}) + 2 = -\sin\frac{\pi}{6} + 2 = \frac{3}{2}$, $(0; \frac{3}{2})$ принадлежит;

б) $y = -\sin(x + \frac{\pi}{6}) + 2 = -\sin\frac{\pi}{3} + 2 = -\frac{\sqrt{3}}{2} + 2$, $(\frac{\pi}{6}; -\frac{\sqrt{3}}{2} + 2)$ принадлежит;

в) $y = -\sin(x + \frac{\pi}{6}) + 2$, $\frac{3}{2} = -\sin\frac{5\pi}{6} + 2 = -\frac{1}{2} + 2$, $(\frac{2\pi}{3}; \frac{3}{2})$ принадлежит;

г) $y = -\sin(x + \frac{\pi}{6}) + 2$, $-\sin(4\pi + \frac{\pi}{6}) + 2 = -\frac{1}{2} + 2 \neq 2{,}5$, $(4\pi; 2{,}5)$ не принадлежит.

173. а) $y = \sin x$, $x \in [\frac{\pi}{4}; \frac{2\pi}{3}]$, $f_{max} = 1$, $f_{min} = \frac{\sqrt{2}}{2}$;

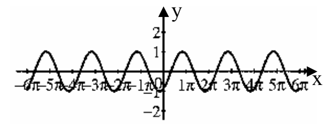
б) $y = \sin x$, $x \in [\frac{\pi}{4}; +\infty]$, $f_{max} = 1$, $f_{min} = -1$;

в) $y = \sin x,\ x \in [-\frac{3\pi}{2};\ \frac{3\pi}{4}]$, $f_{max} = 1$, $f_{min} = -1$;

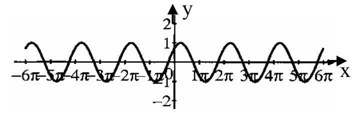
г) $y = \sin x,\ x \in [-\pi;\ \frac{\pi}{3}]$, $f_{max} = \frac{\sqrt{3}}{2}$, $f_{min} = -1$

174.

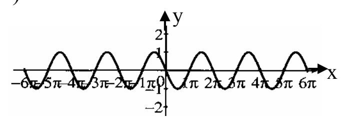
а)



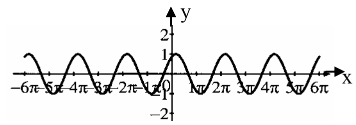
б)



в)

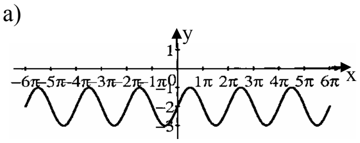


г)

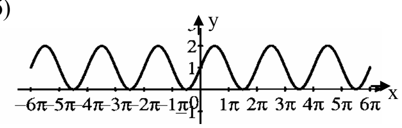


175.

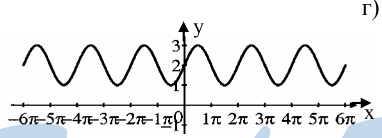
а)



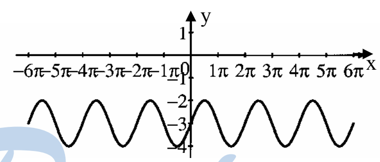
б)



в)

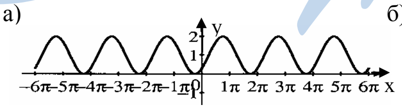


г)

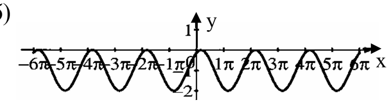


176.

а)

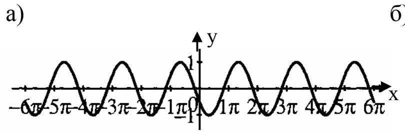


б)

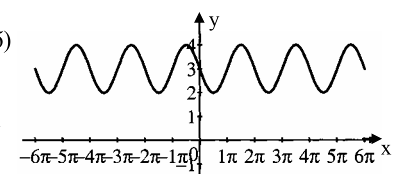


177.

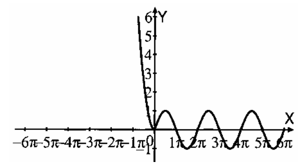
а)



б)



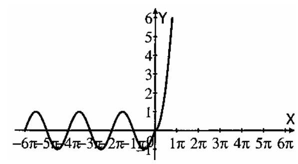
178. $f(x)=\begin{cases} x^2, x<0 \\ \sin x, \quad x \ge 0 \end{cases}$



а)
1) Область определения D(f)=R
2) Область значений E(f)=[-1;+∞)
3) При x>0 функция периодична, T=2π
4) Функция ни четная, ни нечетная
5) f(x)=0 при x=πn, n≥0, x=0 при y=0
6) Промежутки знакопостоянства:
   f(x)>0 при x<0, $x \in (2\pi n, \pi+2\pi n)$, n≥0
   f(x)<0 при $x \in (2\pi n-\pi, 2\pi n)$, n≥0
7) f_{min}=–1, f_{max}=+∞
8) Функция убывает при x≤0 и $x \in \left(\frac{\pi}{2}+2\pi n; \frac{3}{2}\pi+2\pi n\right)$, n≥0

   возрастает при $x \in \left[0; \frac{\pi}{2}\right] \cup \left[2\pi n-\frac{\pi}{2}; 2\pi n+\frac{\pi}{2}\right]$, n≥1

б)



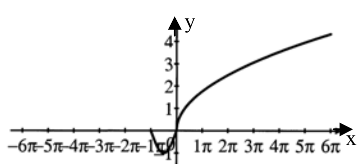
1) D(f)=R
2) E(f)=[-1;+∞)
3) При x<0 T=2π
4) ни четная, ни нечетная
5) f(x)=0 при x=πn, n≤0
   x=0 при y=0
6) f(x)>0 при x>0, $x \in (2\pi n, 2\pi n+\pi)$, n≤0
   f(x)<0 при $x \in (2\pi n-\pi, 2\pi n)$, n≤0
7) f_{min}=–1, f_{max}=+∞
8) убывает при $x \in \left(\frac{\pi}{2}+2\pi n; \frac{3}{2}\pi+2\pi n\right)$, n≤0

   возрастает при $x \in \left[2\pi n-\frac{\pi}{2}; 2\pi n+\frac{\pi}{2}\right] \cup \left[\frac{\pi}{2}; +\infty\right]$, n<0

179. $f(x) = \begin{cases} \sin x, & -\pi \le x \le 0 \\ x\sqrt{x}, & x \ge 0 \end{cases}$

а) $f\left(-\frac{\pi}{2}\right) = \sin\left(-\frac{\pi}{2}\right) = -1$, $f(0) = 0$, $f(1) = 1$, $f(\pi^2) = \pi$

б)



в) 1) D(f)=[- π;+ ∞); 2) E(f)=[-1;+∞); 3) непериодичная;
4) ни четная, ни нечетная; 5) f(x)=0 при x=0;

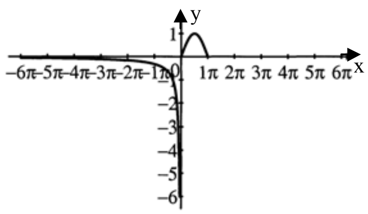
6) f(x)>0 при x>0, f(x)<0 при $x \in [-\pi;0)$;   7) $f_{min}=-1$, $f_{max}=+\infty$;

8) убывает при $x \in \left[-\pi;-\frac{\pi}{2}\right]$, возрастает при $x \geq -\frac{\pi}{2}$.

180. $f(x) = \begin{cases} \frac{1}{x}, & x < 0 \\ \sin x, & 0 \leq x \leq \pi \end{cases}$

а) $f(-2) = -\frac{1}{2}$, $f(0) = 0$, $f(1)=\sin 1$;

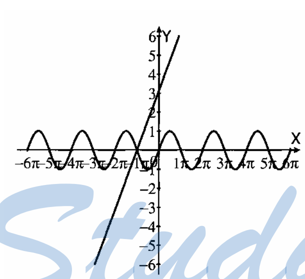
б)

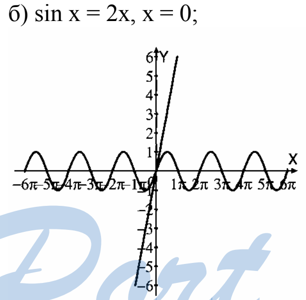


в) 1) $D(f)=(-\infty; \pi]$; 2) $E(f)=(-\infty;+1]$; 3) непериодичная;
4) ни четная, ни нечетная; 5) f(x)=0 при x=0, x= π;
6) f(x)<0 при x<0, f(x)>0 при $x \in (0; \pi)$;   7) $f_{min}=-\infty$, $f_{max}=1$;

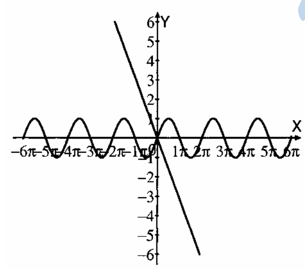
8) убывает при x<0, $x \in \left[-\frac{\pi}{2};\pi\right]$, возрастает при $x \in \left[0;\frac{\pi}{2}\right]$.

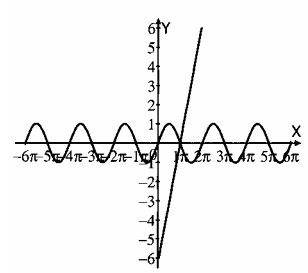
181. а) $\sin x = x + \pi$, $x = -\pi$;   б) $\sin x = 2x$, $x = 0$;





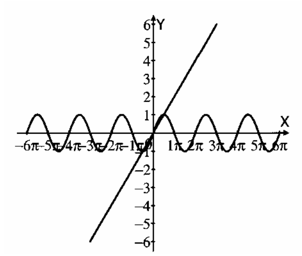
в) $\sin x = -x$, $x = 0$;   г) $\sin x = 2x - 2\pi$, $x = \pi$



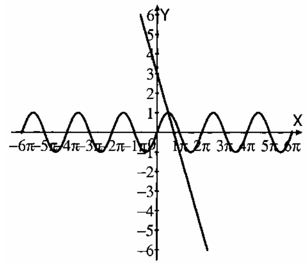


182.

а) $\sin x = \frac{2}{\pi}\ x,\ \ x = 0,\ \ x = \pm\ \frac{\pi}{2}$;

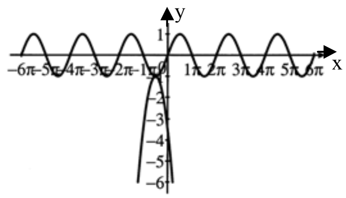


б) $\sin x = -\frac{4}{\pi}x + 3,\ \ \ x = \frac{\pi}{2}$.

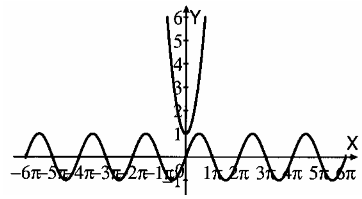


183.

а) $\sin x + 1 = -(x+\frac{\pi}{2})^2,\ \ x = -\ \frac{\pi}{2}$;



б) $\sin x = x^2 + 1$. решений нет.



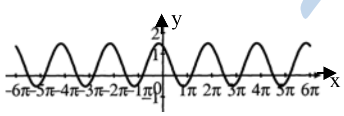
184. а) $\begin{cases} y = \sin x \\ y = x^2 + 4x - 1 \end{cases}$; $\begin{cases} y = \sin x \\ y = (x+2)^2 - 3 \end{cases}$ система имеет 2 решения.

б) $\begin{cases} y = \sin x \\ y = \frac{1}{x} \end{cases}$ система имеет бесконечное множество решений.

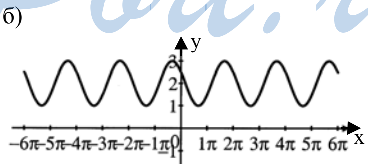
в) $\begin{cases} y = \sin x \\ y = -3x^2 - 2 \end{cases}$ система не имеет решений.

г) $\begin{cases} y = \sin x \\ |x| - y = 0. \end{cases}$ система имеет одно решение.

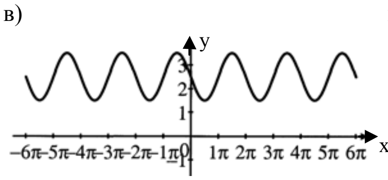
185. а)



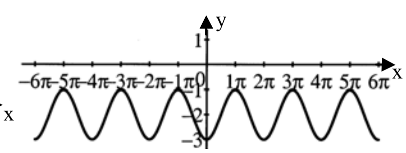
б)



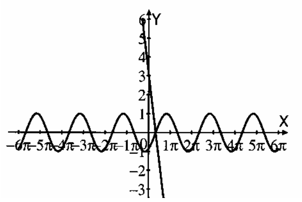
в)

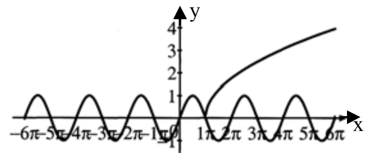


г)

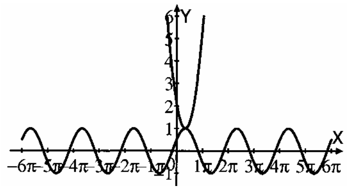


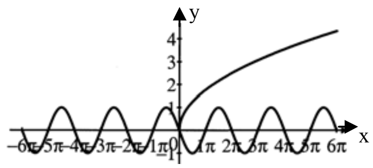
186. а) $\sin\left(x-\frac{\pi}{3}\right)=\pi-3x$, $x=\frac{\pi}{3}$; б) $\sin x=\sqrt{x-\pi}$, $x=\pi$.





в) $\sin\left(x+\frac{\pi}{6}\right)-1=\left(x-\frac{\pi}{3}\right)^2$, $x=\frac{\pi}{3}$. г) $-\sin x=\sqrt{x}$, $x=0$.





187. $y=\sin\left(x-\frac{\pi}{4}\right)+\frac{1}{2}$.

а) $x\in\left[\frac{\pi}{4};\frac{3\pi}{4}\right]$, $y_{max}=\frac{3}{2}$, $y_{min}=\frac{1}{2}$; б) $\left(\frac{3\pi}{4};\frac{9\pi}{4}\right)$, $y_{min}=-\frac{1}{2}$;

в) $[0;\pi)$, $y_{max}=\frac{3}{2}$, $y_{min}=\frac{1-\sqrt{2}}{2}$; г) $\left[\frac{\pi}{4};+\infty\right)$, $y_{max}=\frac{3}{2}$, $y_{min}=-\frac{1}{2}$.

188.

а) $f(x)=x^5\sin\frac{x}{2}$, $f(-x)=-x^5(-1)\sin\frac{x}{2}=f(x)$;

б) $f(x)=\frac{\sin^2 x}{x^2-1}$, $f(-x)=\frac{\sin^2(-x)}{(-x^2)-1}=\frac{\sin^2 x}{x^2-1}=f(x)$;

в) $f(x)=\frac{2\sin\frac{x}{2}}{x^3}$, $f(-x)=\frac{-2\sin\frac{x}{2}}{-x^3}=\frac{2\sin\frac{x}{2}}{x^3}=f(x)$;

г) $f(x)=\sin^2 x-x^4$, $f(-x)=\sin^2(-x)-(-x^4)=f(x)$.

189.

а) $f(x)=-x-\sin x$, $f(-x)=-(-x)-\sin(-x)=-(-x-\sin x)=-f(x)$;

б) $f(x)=x^3\sin x^2$, $f(-x)=-x^3\cdot\sin(-x)^2=-x^3\sin x^2=-f(x)$;

в) $f(x)=\frac{x^2\sin x}{x^2-9}$, $f(-x)=-\frac{x^2\sin x}{x^2-9}=-f(x)$;

г) $f(x)=x^3-\sin x$, $f(-x)=-x^3+\sin x=-f(x)$.

190. а) $f(x)=2x^2-x+1$.

$f(\sin x)=2\sin^2 x-\sin x+1=2-2\cos^2 x-\sin x+1=3-2\cos^2 x-\sin x$.

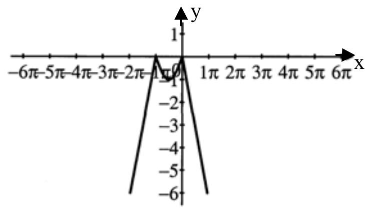
191. $f(x)=3x^2+2x-7$;

$f(\sin x)=3\sin^2 x+2\sin x-7=3-3\cos^2 x+2\sin x-7=-4-3\cos^2 x+2\sin x$.

192. $f(x)=\begin{cases}2x+2\pi, & x\le -\pi\\ \sin x, & -\pi<x\le 0\\ -2x, & x>0\end{cases}$

а) $f(-\pi-2)=-2\pi-4+2\pi=-4$, $f(-\frac{\pi}{6})=\sin(-\frac{\pi}{6})=-\frac{1}{2}$, $f(2)=-4$.

б)



в) 1) D(f)=R; 2) E(f)=(-∞;0]; 3) непериодичная; 4) ни четная, ни нечетная;
5) f(x)=0 при x=– π, x= 0; 6) f(x)<0 при x< – π ,x ∈ (– π;0), x>0;

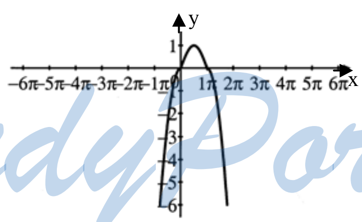
7) $f_{min}=-\infty$, $f_{max}=0$; 8) f(x) возрастает при $x\in(-\infty;-\pi]\cup\left[-\frac{\pi}{2};0\right]$,

убывает при $x\in\left[-\pi;\frac{\pi}{2}\right]\cup[0;+\infty)$.

193. $f(x)=\begin{cases}-x^2, & x<0\\ \sin x, & 0\le x\le \pi\\ -(x-\pi)^2, & x>\pi\end{cases}$

а) $f(-3)=-9$, $f(\frac{\pi}{2})=1$, $f(2\pi-3)=-(\pi-3)^2=-\pi^2+6\pi-9$.

б)

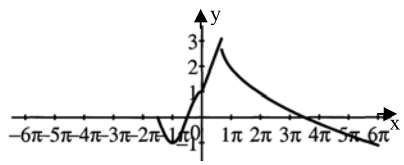


в)
1) D(f)=R
2) E(f)=(– ∞;1]
3) непериодичная,
4) ни четная, ни нечетная
5) f(x)=0 при x = 0, x = π
6) f(x)>0 при x ∈ (0; π), f(x)<0 при x<0; x> π
7) $f_{min}=-\infty$, $f_{max}=1$
8) f(x) возрастает при $x\in\left(-\infty;\frac{\pi}{2}\right]$, убывает при $x\in\left[\frac{\pi}{2};+\infty\right)$

194. $f(x) = \begin{cases} \sin(x + \frac{\pi}{2}), & -\frac{3\pi}{2} \le x \le 0 \\ x+1, & 0 < x < 2 \\ -\sqrt{x-2}+3, & x \ge 2 \end{cases}$

а) $f(0) = 1$, $f(6) = 1$, $f(-\pi - 2)$ = не определено, т.к. $(-\pi - 2) < -\frac{3\pi}{2}$.

б)



в) 1) $D(f)=\left[-\frac{3}{2}\pi;+\infty\right]$; 2) $E(f)=[-1;3]$; 3) непериодичная;

4) ни четная, ни нечетная; 5) $f(x)=0$ при $x=-\frac{\pi}{2}$, $x=11$;

6) $f(x)>0$ при $x\in\left(-\frac{\pi}{2};11\right)$, $f(x)<0$ при $x\in\left(-\frac{3}{2}\pi;-\frac{\pi}{2}\right)\cup(11;+\infty)$;

7) $f_{min}=-\infty$, $f_{max}=3$;
8) $f(x)$ возрастает при $x\in[-\pi;2]$, убывает при $x\in[2;+\infty)$.

§ 10. Функция y = cos x, ее свойства и график

195. а) $\cos\frac{\pi}{2}=0$; б) $\cos(-\pi)=-1$; в) $\cos\frac{5\pi}{6}=-\frac{\sqrt{3}}{2}$; г) $\cos(-\frac{2\pi}{3})=-\frac{1}{2}$.
196. а) $f(x) = \cos x$, $f(-x) = \cos x$; б) $f(x) = \cos x$, $f(3x) = \cos 3x$;
в) $f(x) = \cos x$, $f(x+2) = \cos(x+2)$; г) $f(x) = \cos x$, $f(x) - 6 = \cos x - 6$.

197. а) $y = 2\sin(-\frac{\pi}{2}) + \cos(-\frac{\pi}{2}) = -2$; б) $y = 2\sin\frac{\pi}{6} + \cos\frac{\pi}{6} = 1 + \frac{\sqrt{3}}{2}$.

198. а) $y = \cos(-\frac{\pi}{3}) - (-\frac{\pi}{3})^2 = \frac{1}{2} - \frac{\pi^2}{9}$; б) $y = \cos\pi - \pi^2 = -1 - \pi^2$.

199. а) $\frac{1}{\cos\frac{2\pi}{3}} = \frac{1}{-\frac{1}{2}} = -2$; б) $\frac{1}{\cos\frac{11\pi}{6}} = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}}$.

200. а) $y = 2\cos(-\frac{\pi}{2} - \frac{\pi}{4}) - 1 = -\sqrt{2} - 1$; б) $y = 2\cos(\frac{\pi}{4} - \frac{\pi}{4}) - 1 = 1$.

201. а) $y = \cos x$, $\frac{1}{2} = \cos\frac{\pi}{3}$, $(\frac{\pi}{3};\frac{1}{2})$ – принадлежит;

б) $y = \cos x$, $\frac{1}{2} \ne \cos\frac{\pi}{6}$, $(\frac{\pi}{6};\frac{1}{2})$ – не принадлежит;

в) y = cos x, $-\frac{1}{2} = \cos\frac{2\pi}{3}$, $(\frac{2\pi}{3}; -\frac{1}{2})$ – принадлежит;

г) y = cos x, $-\frac{\sqrt{3}}{2} = \cos\frac{5\pi}{6}$, $(\frac{5\pi}{6}; -\frac{\sqrt{3}}{2})$ – принадлежит.

202. $y = 2\cos(x - \frac{\pi}{6}) + 1$.

а) $\sqrt{3} + 1 = 2\cos(-\frac{\pi}{6}) + 1$, $(0; \sqrt{3} + 1)$ – принадлежит.

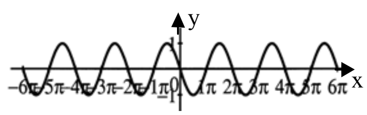
б) $1 \neq 2\cos(\frac{\pi}{6} - \frac{\pi}{6}) + 1$, $(\frac{\pi}{6}; 1)$ – не принадлежит.

в) $2 = 2\cos(\frac{\pi}{2} - \frac{\pi}{6}) + 1$, $(\frac{\pi}{2}; 2)$ – принадлежит.

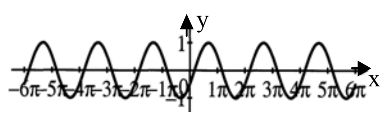
г) $3 = 2\cos(\frac{\pi}{6} - \frac{\pi}{6}) + 1$, $(\frac{\pi}{6}; 3)$ – принадлежит.

203.

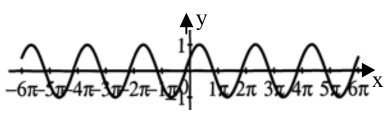
а)



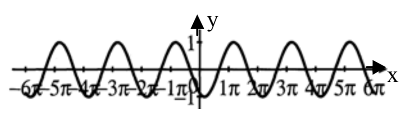
б)



в)

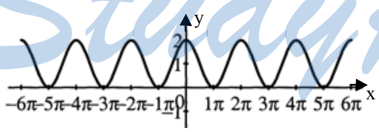


г)

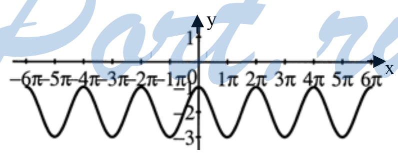


204.

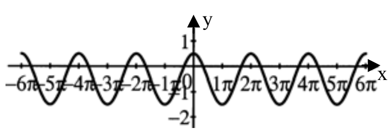
а)



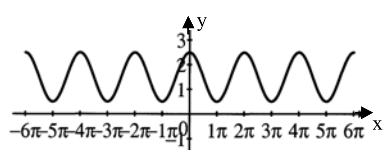
б)



в)

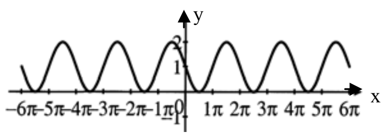


г)

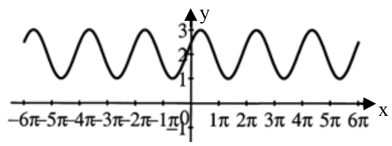


205.

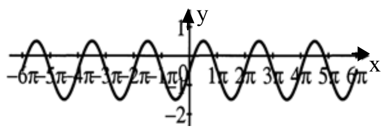
а)



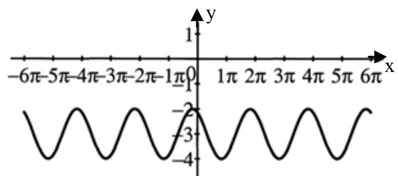
б)



в)



г)



206. y = cos x.

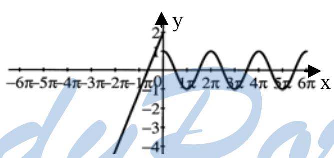
а) $x \in [\frac{\pi}{6}; \frac{2\pi}{3}]$ $y_{min} = -\frac{1}{2}$, $y_{max} = \frac{\sqrt{3}}{2}$.

б) $x \in (-\pi; \frac{\pi}{4})$ y_{min} = не существует, $y_{max} = 1$.

в) $x \in [-\frac{\pi}{4}; +\infty)$ $y_{min} = -1$, $y_{max} = 1$.

г) $x \in [-\frac{\pi}{3}; \frac{3\pi}{2})$ $y_{min} = -1$, $y_{max} = 1$.

207. а) $f(x)=\begin{cases} x+2, & x<0 \\ \cos x, & x \ge 0 \end{cases}$



1) D(f)=R; 2) E(f)=(– ∞;2); 3) при $x \ge 0$ T=2π; 4) ни четная, ни нечетная;

5) f(x)=0 при x = –2, $x = \frac{\pi}{2} + \pi n$, $n \ge 0$;

6) f(x)<0 при $x \in (-\infty; -2) \cup \left(\frac{\pi}{2} + 2\pi n; \frac{3}{2}\pi + 2\pi n\right)$, $n \ge 0$,

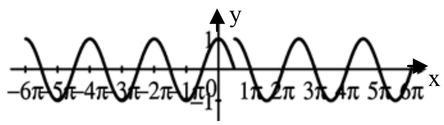
f(x)>0 при $x \in \left(-2; \frac{\pi}{2}\right) \cup \left(-\frac{\pi}{2} + 2\pi n; \frac{\pi}{2} + 2\pi n\right)$, $n \ge 1$;

7) $f_{min} = -\infty$, $f_{max} = 2$;

8) f(x) возрастает при $x \in (-\infty; 0) \cup (-\pi + 2\pi n; 2\pi n)$, $n \ge 0$,

убывает при $x \in (2\pi n; 2\pi n + \pi)$, $n \ge 0$.

б) f(x)= $\begin{cases} \cos x, & x \le \dfrac{\pi}{2} \\ \sin x, & x > \dfrac{\pi}{2} \end{cases}$



1) D(f)=R;   2) E(f)=[– 1;1];   3) на промежутках $\left(-\infty; \dfrac{\pi}{2}\right]$ и $\left(\dfrac{\pi}{2}; +\infty\right)$ T=2π;

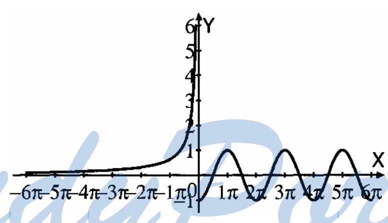
4) ни четная, ни нечетная;   5) f(x)=0 при x = $\dfrac{\pi}{2}$ -πn, n≥0, x= π(1+k), k≥0;

6) f(x)<0 при x ∈ $\left(\dfrac{\pi}{2} + 2\pi n; \dfrac{3}{2}\pi + 2\pi n\right)$, n≤–1,   x ∈ (– π+2 πk; 2πk), k≥1;

7) f_{min}=–1, f_{max}=1;   8) f(x) возрастает при x ∈ [–π+2πn; 2πn), n≤0,
x ∈ $\left[-\dfrac{\pi}{2} + 2\pi k; \dfrac{\pi}{2} + 2\pi k\right]$, k≥1,   убывает при x ∈ [2πn; π+2πn), n≤–1,

x ∈ $\left[0; \dfrac{\pi}{2}\right]$; x ∈ $\left[\dfrac{\pi}{2} + 2\pi k; \dfrac{3}{2}\pi + 2\pi k\right]$, k≥0;

в) f(x)= $\begin{cases} -\dfrac{2}{x}, & x < 0 \\ -\cos x, & x \ge 0 \end{cases}$



1) D(f)=R; 2) E(f)=[– 1;+∞); 3) при x≥0 T=2π; 4) ни четная, ни нечетная;

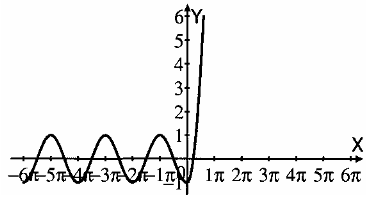
5) f(x)=0 при x = $\dfrac{\pi}{2}$ +πn, n≥–1;

6) f(x)<0 при x ∈ $\left[0; \dfrac{\pi}{2}\right) \cup \left(-\dfrac{\pi}{2} + 2\pi n; \dfrac{\pi}{2} + 2\pi n\right)$, n≥1,

f(x)>0 при x ∈ (– ∞; 0) $\cup \left(\dfrac{\pi}{2} + 2\pi n; \dfrac{3}{2}\pi + 2\pi n\right)$, n≥0;

7) f_{min}=–1, f_{max}=+∞; 8) f(x) возрастает при x ∈ (– ∞; 0) ∪ [2πn; π+2πn), n≥0,
убывает при x ∈ [2πk-π; 2πk), k≥1.

г) f(x)= $\begin{cases} -\cos x, & x < 0 \\ 2x^2 - 1, & x \geq 0 \end{cases}$



1) D(f)=R; 2) E(f)=[– 1;+∞); 3) при x<0 T=2π;   4) ни четная, ни нечетная;

5) f(x)=0 при x = $\frac{\pi}{2}$ –πn, n≥1, x= $\frac{\sqrt{2}}{2}$;

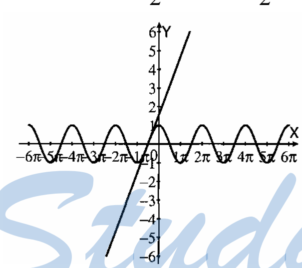
6) f(x)<0 при $x \in \left(-\frac{\pi}{2} - 2\pi n; \frac{\pi}{2} - 2\pi n\right) \cup \left(-\frac{\pi}{2}; \frac{\sqrt{2}}{2}\right)$, n≥1,

f(x)>0 при $x \in \left(\frac{\pi}{2} - 2\pi k; \frac{3}{2}\pi - 2\pi k\right) \cup \left(\frac{\sqrt{2}}{2}; +\infty\right)$, k≥1;

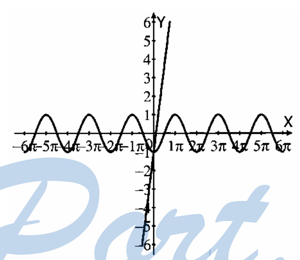
7) f_{min}=–1, f_{max}=+∞; 8) f(x) возрастает при x ∈ [–2πn; –2πn+π) ∪ [0;+∞), n≥1, убывает при x ∈ [–2πn-π; –2πn), n≥0.

208.

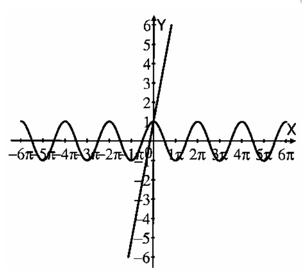
а) cos x = x + $\frac{\pi}{2}$;   x = – $\frac{\pi}{2}$.



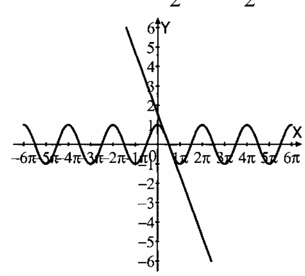
б) – cos x = 3x – 1; x = 0.



в) cos x = 2x + 1;   x = 0.

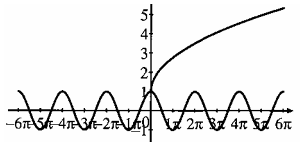


г) cos x = – x + $\frac{\pi}{2}$;x = $\frac{\pi}{2}$.

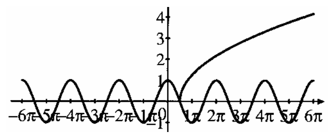


209.

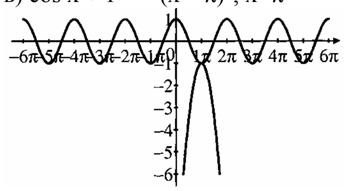
а) $\cos x = \sqrt{x} + 1$, $x = 0$;



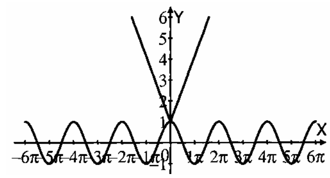
б) $\cos x = \sqrt{x - \frac{\pi}{2}}$, $x = \frac{\pi}{2}$;



в) $\cos x + 1 = -(x - \pi)^2$; $x = \pi$



г) $\cos x = |x| + 1$, $x = 0$.



210. а) $\begin{cases} y = \cos x \\ y = -x^2 + 2x - 3 \end{cases}$; $\begin{cases} y = \cos x \\ y = -(x-1)^2 - 2 \end{cases}$ решений нет.

б) $\begin{cases} y = \cos x \\ y = \frac{2}{x} \end{cases}$ бесконечное множество решений.

в) $\begin{cases} y = \cos x \\ y = x^2 - 3 \end{cases}$ 2 решения; г) $\begin{cases} y = \cos x \\ |x| - y = 0 \end{cases}$; $\begin{cases} y = \cos x \\ y = |x| \end{cases}$ 2 решения.

211. а) $f(x) = x^2 \cos x$, $f(-x) = (-x)^2 \cos(-x) = x^2 \cos x = f(x)$;

б) $f(x) = \frac{\cos x^3}{4 - x^2}$, $f(-x) = \frac{\cos(-x)^3}{4 - (-x)^2} = \frac{\cos x^3}{4 - x^2} = f(x)$;

в) $f(x) = \frac{\cos 5x + 1}{|x|}$, $f(-x) = \frac{\cos(-5x) + 1}{|-x|} = \frac{\cos 5x + 1}{|x|} = f(x)$;

г) $f(x) = (4 + \cos x)(\sin^6 x - 1)$,
$f(-x) = (4 + \cos(-x))(\sin^6(-x) - 1) = (4 + \cos x)(\sin^6 x - 1) = f(x)$.

212. а) $f(x) = \sin x \cos x$, $f(-x) = -\sin(-x)\cos(-x) = -\sin x \cos x = -f(x)$;
б) $f(x) = x^5 \cos 3x$, $f(-x) = (-x)^5 \cos(-3x) = -x^5 \cos 3x = -f(x)$;

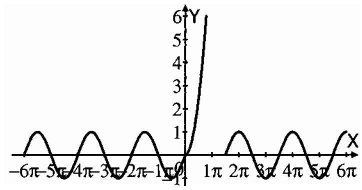
в) $f(x) = \frac{\cos x^3}{x(25 - x^2)}$, $f(-x) = \frac{\cos(-x)^3}{-x(25 - (-x)^2)} = -\frac{\cos x^3}{x(25 - x^2)} = -f(x)$;

г) $f(x) = x^{11} \cdot \cos x + \sin x$,
$f(-x) = (-x)^{11} \cdot \cos(-x) + \sin(-x) = (-x)^{11} \cdot \cos x - \sin x = f(-x)$.

213. $f(x) = 2x^2 - 3x - 2$,
$-f(\cos x) = -2\cos^2 x + 3\cos x + 2 = 2(1 - \cos^2 x) + 3\cos x = 2\sin^2 x + 3\cos x$

214. $f(x) = 5x^2 + x + 4$,
$f(\cos x) = 5\cos^2 x + \cos x + 4 = 5 - 5\sin^2 x + \cos x + 4 = -5\sin^2 x + \cos x + 9$.

215. см. рис. 70.



$$f(x)=\begin{cases} \sin x, & x \le 0 \\ x^2, & 0 < x < \frac{\pi}{2} \\ \cos x, & x \ge \frac{\pi}{2} \end{cases}$$

1) D(f)=R; 2) E(f)=[-1;+∞); 3) ни четная, ни нечетная;

4) при x≤0 и $x \ge \frac{\pi}{2}$ T=2π; 5) f(x)=0 при x=– πn, n≥0, $x=\frac{\pi}{2}+\pi k$, k≥0;

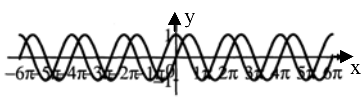
6) f(x)>0 при $x \in (-2\pi n; -2\pi+\pi) \cup (0; \frac{\pi}{2}) \cup (-\frac{\pi}{2}+2\pi k; \frac{\pi}{2}+2\pi k)$, n≥1, k≥1;

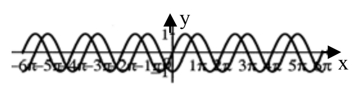
7) $f_{min}=-1$, $f_{max}=+\infty$;

8) f(x) возрастает при $x \in [-\frac{\pi}{2}-2\pi n; \frac{\pi}{2}-2\pi n]$, n≥1;

$x \in \cup \left[-\frac{\pi}{2}; \frac{\pi}{2}\right] \cup \left[2\pi; \frac{5}{2}\pi\right] \cup [\pi+2\pi n; 2\pi+2\pi n]$, n≥1.

216. а) sin x = cos x, $x = \frac{\pi}{4} + \pi k$. б) sin x = – cos x, $x = -\frac{\pi}{4} + \pi k$.





§ 11. Периодичность функций y = sin x, y = cos x

217. см. рис. 73. **218.** см. рис. 74.
219. см. рис. 75. **220.** см. рис. 76.
221. 32π является периодом функций y = sin x, y = cos x, но не основным.

222. а) $\sin 50{,}5\pi = \sin \frac{\pi}{2} = 1$; б) $\cos 51{,}75\pi = \cos \frac{\pi}{4} = \frac{\sqrt{2}}{2}$;

в) $\sin 25{,}25\pi = -\sin \frac{3\pi}{4} = -\frac{\sqrt{2}}{2}$; г) $\sin 30{,}5\pi = \sin \frac{\pi}{2} = 1$.

223. а) $\sin 390° = \sin 30° = \frac{1}{2}$; б) $\cos 750° = \cos 30° = \frac{\sqrt{3}}{2}$;

в) sin 540° = sin 180° = 0; г) $\cos 930° = \cos 150° = -\frac{\sqrt{3}}{2}$.

224. а) $\sin^2(x-8\pi) = 1-\cos^2(16\pi - x)$,
$\sin^2(x-8\pi) = \sin^2 x$; $1-\cos^2(16\pi-x)=1-\cos^2 x=\sin^2 x$;
б) $\cos^2(4\pi+x) = 1-\sin^2 x\,(22\pi-x)$; $\cos^2(4\pi+x)=\cos^2 x$,
$1-\sin^2 x(22\pi-x)=1-\sin^2 x=\cos^2 x$.

225. а) $y = \sin 2x$, $T = \pi$, $y(x + T) = \sin(2x + 2\pi) = \sin 2x = y(x)$;

б) $y = \cos 3x$, $T = \frac{2\pi}{3}$, $y(x + T) = \sin(3x + 2\pi) = \sin 3x = y(x)$;

в) $y = \sin\frac{x}{2}$, $T = 4\pi$, $y(x + T) = \sin(\frac{x}{2} + 2\pi) = \sin\frac{x}{2} = y(x)$;

г) $y = \cos\frac{3x}{4}$, $T = \frac{8\pi}{3}$, $y(x + T) = \cos(\frac{3x}{4} + 2\pi) = \cos\frac{3x}{4} = y(x)$.

226. а) $\sin 8 = \sin(8 - 2\pi)$; б) $\cos(-10) = \cos(-10 + 4\pi)$;
в) $\sin(-25) = \sin(-25 + 8\pi)$; г) $\cos 35 = \cos(35 - 10\pi)$.

227. а) $\cos(t + 4\pi) = ?$ $\cos(2\pi - t) = -\frac{3}{5}$,

$\cos t = -\frac{3}{5}$, $\cos(t + 4\pi) = \cos t = -\frac{3}{5}$;

б) $\sin(32\pi - t) = ?$ $\sin(2\pi - t) = \frac{5}{13}$, $\sin(32\pi - t) = \sin(2\pi - t) = \frac{5}{13}$.

228. а) $\sin(t + 2\pi) + \sin(t - 4\pi) = 1$, $\sin t + \sin t = 1$, $\sin t = ;$ $t=(-1)^k \frac{\pi}{6} + \pi k$;

б) $3\cos(2\pi + t) + \cos(t - 2\pi) + 2 = 0$, $4\cos t = -2$,

$\cos t = -\frac{1}{2}$, $t = \pm\frac{2\pi}{3} + 2\pi n$;

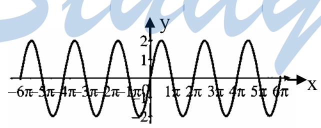
в) $\sin(t + 4\pi) + \sin(t - 6\pi) = \sqrt{3}$, $2\sin t = \sqrt{3}$, $\sin t = \frac{\sqrt{3}}{2}$, $t = (-1)^k \frac{\pi}{3} + \pi k$;

г) $\cos(t + 2\pi) + \cos(t - 8\pi) = \sqrt{2}$, $2\cos t = \sqrt{2}$, $\cos t = \frac{\sqrt{2}}{2}$, $t = \pm\frac{\pi}{4} + 2\pi k$.

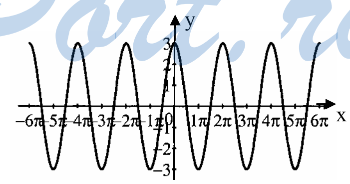
§ 12. Как построить график функции y = mf (x), если известен график функции y = f (x)

229.

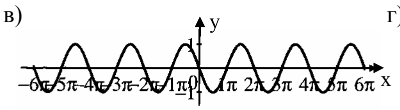
а)



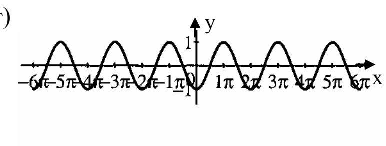
б)



в)

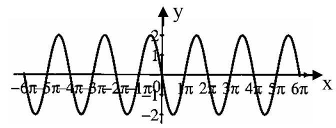


г)

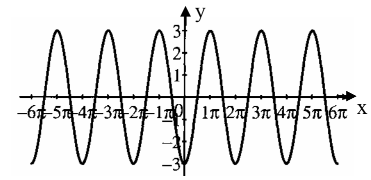


230.

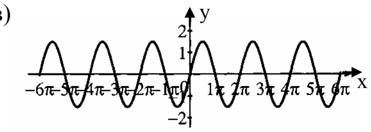
а)



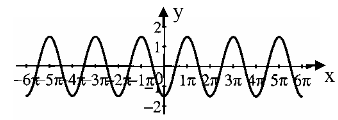
б)



в)



г)



231. $y = 2\cos x$

а) $x \in [-\frac{\pi}{2}; \frac{\pi}{2}]$, $y_{max} = 2$, $y_{min} = 0$;

б) $x \in (0; \frac{3\pi}{2})$, $y_{min} = -2$, $y_{max} = -0$;

в) $x \in [\frac{\pi}{3}; \frac{3\pi}{2}]$, $y_{max} = 1$, $y_{min} = -2$;

г) $x \in [-\frac{3\pi}{2}; -\frac{\pi}{4}]$, $y_{max} = \sqrt{2}$, $y_{min} = -2$.

232. $y = -3\sin x$.

а) $x \in [0; +\infty)$, $y_{max} = 3$, $y_{min} = -3$; б) $x \in (-\infty; \frac{\pi}{2})$, $y_{max} = 3$, $y_{min} = -3$;

в) $x \in [\frac{\pi}{4}; +\infty)$, $y_{max} = 3$, $y_{min} = -3$; г) $x \in (-\infty; 0)$, $y_{max} = 3$, $y_{min} = -3$.

233. $f(x) = 3\sin x$;

а) $f(-x) = -3\sin x$; б) $2f(x) = 6\sin x$;

в) $2f(x) + 1 = 6\sin x + 1$; г) $f(-x) + f(x) = -3\sin x + 3\sin x = 0$.

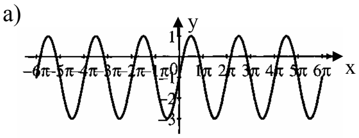
234. $f(x) = -\frac{1}{2}\cos x$;

а) $f(-x) = -\frac{1}{2}\cos$; б) $2f(x) = -\cos x$;

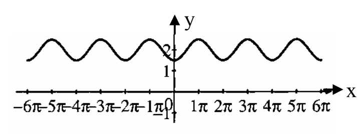
в) $f(x + 2\pi) = -\frac{1}{2}\cos x$; г) $f(-x) - f(x) = -\frac{1}{2}\cos x + \frac{1}{2}\cos x = 0$.

235.

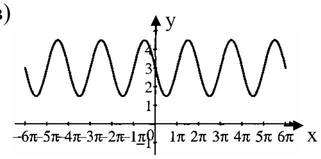
а)



б)



в)



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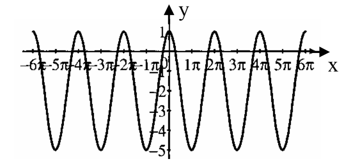
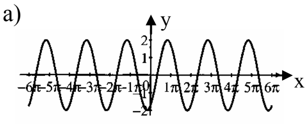
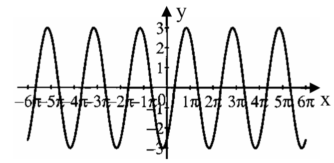


236.

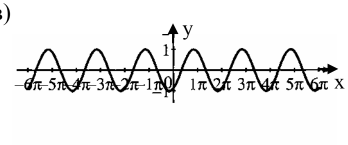
а)



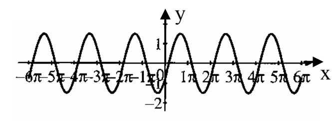
б)



в)



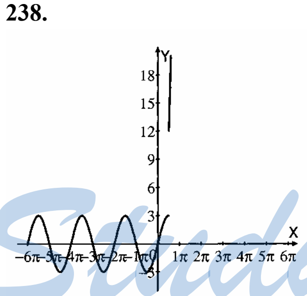
г)



237. а) $\begin{cases} x^2, & x<0 \\ \frac{1}{2}\sin x, & 0\le x\le\pi \end{cases}$

б) $\begin{cases} 1{,}5\cos x, & x\in[-\frac{\pi}{2};\frac{\pi}{2}] \\ x-\frac{\pi}{2}, & x>\frac{\pi}{2} \end{cases}$

238.



а) $f(x)=\begin{cases} 3\sin x, & x<\frac{\pi}{2} \\ 3x^3, & x\ge\frac{\pi}{2} \end{cases}$

1) D(f)=R

2) $E(f)=[-3;3]\cup\left[3\frac{\pi^3}{8};+\infty\right)$

3) при $x<\frac{\pi}{2}$ $T=2\pi$

4) ни четная, ни нечетная

5) f(x)=0 при $x=-\pi n$, $n\ge0$

6) $f_{min}=-3$, $f_{max}=+\infty$

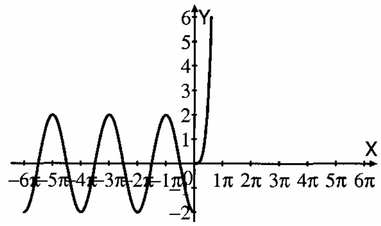
7) f(x)<0 при $x\in(-2\pi n-\pi;\ -2\pi n)$, $n\ge0$

f(x)>0 при $x\in(-2\pi n;\ -2\pi n+\pi)$, $n\ge0$

8) f(x) возрастает при $x\in[-2\pi n-\frac{\pi}{2};-2\pi n+\frac{\pi}{2}]$, $n\ge0$

f(x) убывает при $x\in[-2\pi n+\frac{\pi}{2};-2\pi n+\frac{3}{2}\pi]$, $n\ge0$

б) $f(x)=\begin{cases} -2\cos x, & x<0 \\ \frac{1}{2}x^4, & x\ge 0 \end{cases}$



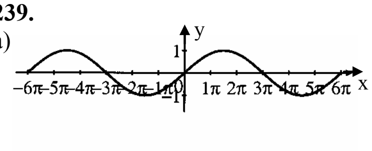
1) D(f)=R; 2) E(f)=[-2; +∞];
3) при x<0 T=2 π;
4) ни четная, ни нечетная;
5) f(x)=0 при $x=-\frac{\pi}{2}-\pi n$, n≥1, x=0;
6) $f_{min}=-3$, $f_{max}=+\infty$;
7) f(x)<0 при $x\in(-\frac{\pi}{2}-2\pi n;\ \frac{\pi}{2}-2\pi n)\cup(-\frac{\pi}{2};0)$, n≥1,

f(x)>0 при $x\in(\frac{\pi}{2}-2\pi n;\ \frac{3}{2}\pi-2\pi n)$, n≥1;
8) f(x) возрастает при $x\in[-2\pi n;-2\pi n+\pi]$, n≥1, x≥0,
f(x) убывает при $x\in[-2\pi n-\pi;-2\pi n]$, n≥0.

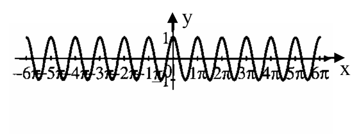
§ 13. Как построить график функции y = f (kx), если известен график функции y = f (x)

239.

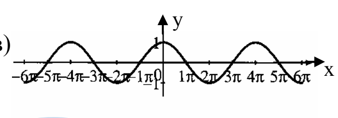
а)



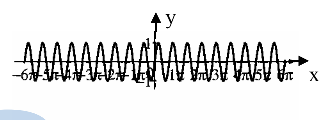
б)



в)

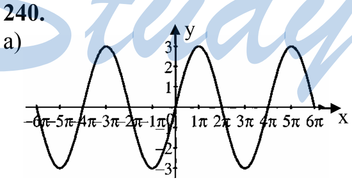


г)

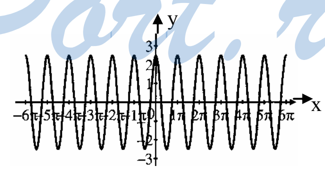


240.

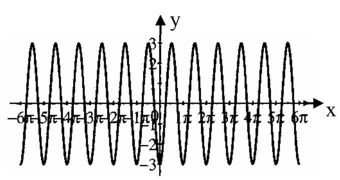
а)



б)



в)



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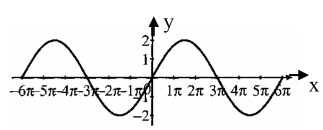
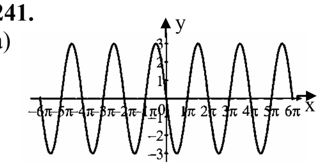
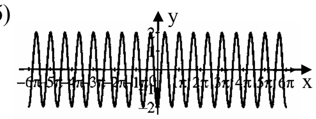


241.

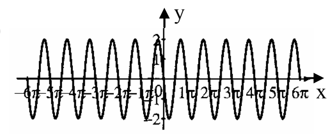
а)



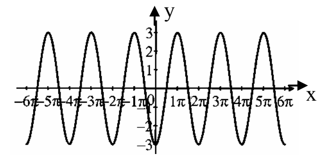
б)



в)



г)



242. y = sin 2x

а) $x \in [-\frac{\pi}{2}; 0]$, $y_{max} = 0$, $y_{min} = -1$; б) $x \in (-\frac{\pi}{4}; \frac{\pi}{2})$, $y_{max} = 1$;

в) $x \in [-\frac{\pi}{4}; \frac{\pi}{4}]$, $y_{min} = -1$, $y_{max} = 1$; г) $x \in (0; \pi]$, $y_{min} = -1$, $y_{max} = 1$.

243. $y = \cos\frac{x}{3}$

а) $x \in [0; +\infty)$, $y_{max} = 1$, $y_{min} = -1$; б) $x \in (-\infty; \pi)$, $y_{max} = 1$, $y_{min} = -1$;

в) $x \in [-\infty; \frac{\pi}{2}]$, $y_{max} = 1$, $y_{min} = -1$; г) $x \in (\frac{\pi}{3}, +\infty)$, $y_{max} = 1$, $y_{min} = -1$.

244. $f(x) = \cos\frac{x}{3}$

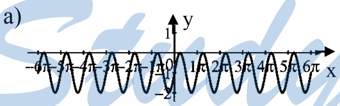
а) $f(-x) = \cos\frac{x}{3}$; б) $3f(x) = 3\cos\frac{x}{3}$; в) $f(-3x) = \cos x$; г) $f(-x) - f(x) = 0$.

245. $f(x) = \sin 2x$

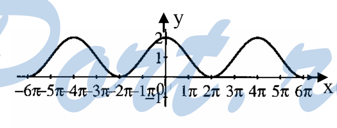
а) $f(-x) = -\sin 2x$; б) $2f(x) = 2\sin 2x$; в) $f(-3x) = -\sin 6x$; г) $f(-x) + f(x) = 0$.

246.

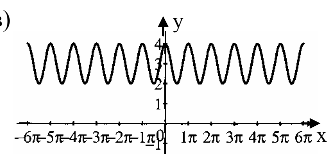
а)



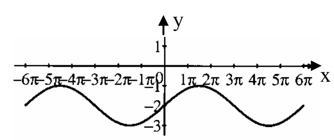
б)



в)

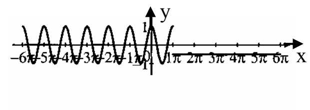


г)



247.

а) f(x)= $\begin{cases} \cos 2x, x \le \pi \\ -\frac{1}{2}, x > \pi \end{cases}$



1) D(f)=R; 2) E(f)=[-1;1]; 3) при x≤ π T= π; 4) ни четная, ни нечетная;

5) f(x)=0 при x=– $\frac{\pi}{4} - \frac{\pi n}{2}$ π, n≥0; x= $\frac{3}{4}$ π;

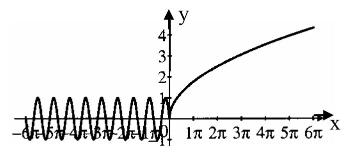
6) f_{min}=–1, f_{max}=1; 7) f(x)<0 при x ∈ $\left(\frac{\pi}{4} - \pi n; \frac{2}{4}\pi - \pi n\right) \cup (\pi;+\infty)$, n≥0,

f(x)>0 при x ∈ $\left(\frac{\pi}{4} - \pi n; \frac{\pi}{4} - \pi n\right) \cup \left(\frac{3}{4}\pi; \pi\right]$, n≥0;

8) f(x) возрастает при x ∈ $\left[\frac{\pi}{2} - \pi n; \pi - \pi n\right]$, n≥0,

f(x) убывает при x ∈ $\left[-\pi n; \frac{\pi}{2} - \pi n\right]$, n≥0.

б) f(x)= $\begin{cases} -\sin 3x, x < 0 \\ \sqrt{x}, x \ge 0 \end{cases}$



1) D(f)=R; 2) E(f)=[-1;+∞);

3) при x≤0 T= $\frac{2}{3}$ π; 4) ни четная, ни нечетная;

5) f(x)=0 при x=– $\frac{\pi n}{3}$, n≥0; 6) f_{min}=–1, f_{max}=+∞;

7) f(x)<0 при x ∈ $\left(-\frac{2}{3}\pi; \frac{\pi}{3} - \frac{2}{3}\pi n\right)$, n≥1,

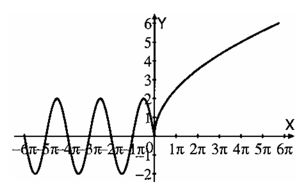
f(x)>0 при x ∈ $\left(-\frac{\pi}{3} - \frac{2}{3}\pi n; -\frac{2}{3}\pi n\right)$, n≥0, x≥0;

8) f(x) возрастает при x ∈ $\left[\frac{\pi}{6} - \frac{2}{3}\pi n; \frac{\pi}{3} - \frac{2}{3}\pi n\right]$, n≥1,

f(x) убывает при x ∈ $\left[-\frac{\pi}{6} - \frac{3}{2}\pi n; \frac{\pi}{6} - \frac{3}{2}\pi n\right]$, n≥1, x ∈ $\left[-\frac{\pi}{3}; 0\right)$.

248.

а) f(x)= $\begin{cases} -2\sin x, x < 0 \\ \sqrt{2x}, x \ge 0 \end{cases}$



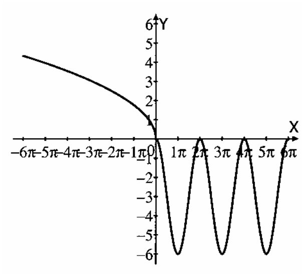
1) D(f)=R; 2) E(f)=[-2;+∞); 3) при x≤0 T= 2π;
4) ни четная, ни нечетная;
5) f(x)=0 при x=– πn, n≥0; 6) f_{min}=–2, f_{max}=+∞;

7) f(x)<0 при $x \in (-2\pi n; -2\pi n+\pi)$, $n \ge 1$, f(x)>0 при $x \in (-2\pi n-\pi; -2\pi n)$, $n \ge 0$;

8) f(x) возрастает при $x \in [-2\pi n+\frac{\pi}{2}; -2\pi n+\frac{3}{2}\pi] \cup [0; +\infty)$, $n \ge 1$,

f(x) убывает при $x \in [-2\pi n-\frac{\pi}{2}; -2\pi n+\frac{\pi}{2}] \cup [-\frac{\pi}{2}; 0)$, $n \ge 1$.

б) $f(x)=\begin{cases}\sqrt{-x}, & x \le 0\\ 3\cos x - 3, & x > 0\end{cases}$



1) D(f)=R
2) E(f)=[-6;+∞)
3) при $x \ge 0$ T= 2π
4) ни четная, ни нечетная
5) f(x)=0 при $x=2\pi n$, $n \ge 0$
6) $f_{min}=-6$, $f_{max}=+\infty$
7) f(x)<0 при $x \ne 2\pi n$, $n \ge 0$, f(x)>0 при x<0
8) f(x) возрастает при $x \in [-2\pi n-\pi; 2\pi n]$, $n \ge 1$,
f(x) убывает при $x \in [2\pi n; 2\pi n+\pi]$, $n \ge 0$, $x \le 0$

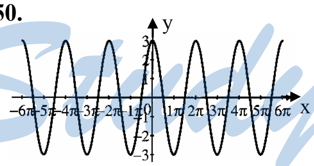
249. а) $y=\begin{cases}-x, & x<0\\ \sin 2x, & x \ge 0\end{cases}$; б) $y=\begin{cases}\cos 3x, & x \in [-\frac{\pi}{6}; \frac{\pi}{3}]\\ -1, & x > \frac{\pi}{3}\end{cases}$

в) $y=\begin{cases}\sin 2x, & x<0\\ 2\cos x, & x>0\end{cases}$; г) $y=\begin{cases}-2\sin x, & x \in [-2\pi; 0]\\ \cos\frac{x}{2}, & x \in (0; 3\pi]\end{cases}$

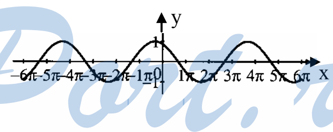
§ 14. График гармонического колебания

250.

а)

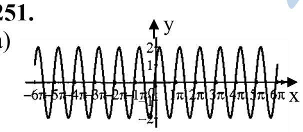


б)

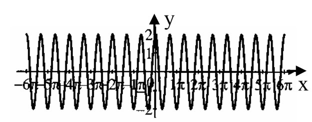


251.

а)

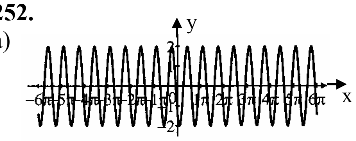


б)

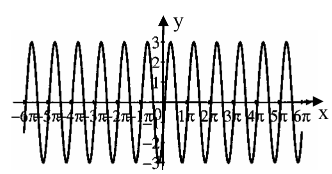


252.

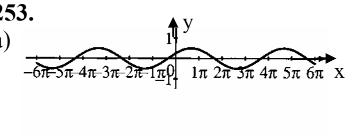
а)

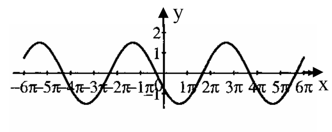


б)



253.

а) 

б) 

§ 15. Функции y = tg x, y = ctg x, их свойства и графики

254. y = tg x

а) $\text{tg}\,\frac{\pi}{4} = 1$; б) $\text{tg}\,\frac{2\pi}{3} = -\sqrt{3}$; в) $\text{tg}\,\frac{3\pi}{4} = -1$; г) $\text{tg}\,\pi = 0$.

255. t = tg x.

а) $x \in (\frac{\pi}{2}; \frac{3\pi}{2})$, $y_{min} = -$, $y_{max} = -$;

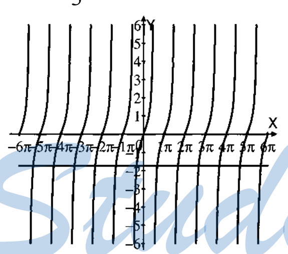
б) $x \in (\frac{3\pi}{4}; \pi]$, $y_{min} = -$, $y_{max} = 0$;

в) $x \in [-\frac{\pi}{4}; \frac{\pi}{6}]$, $y_{min} = -1$, $y_{max} = \frac{\sqrt{3}}{3}$

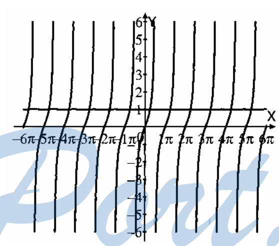
г) $x \in [\pi; -\frac{3\pi}{2})$, $y_{min} = 0$, $y_{max} = -$.

256.

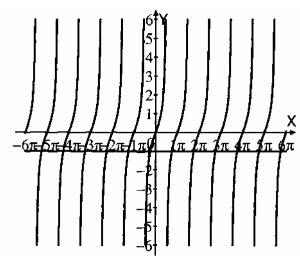
а) $x = \frac{\pi}{3} + \pi n$, $n \geq 0$;



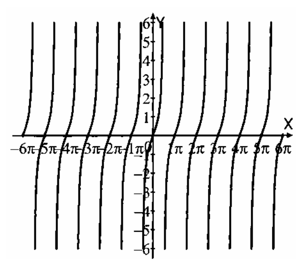
б) $x = \frac{\pi}{4} + \pi n$



в) $x = -\frac{\pi}{4} + \pi n$



г) $x = \pi n$.



257.

а) $\text{ctg}\,\frac{\pi}{4} = 1$; б) $x = \text{ctg}\,\frac{\pi}{3} = \frac{\sqrt{3}}{3}$; в) $\text{ctg}\,2\pi = –$; г) $\text{ctg}\,\frac{\pi}{2} = 0$.

258.

$y = \text{ctg}\,x$.

а) $x \in [\frac{\pi}{4}; \frac{\pi}{2}]$, $y_{max} = 1$, $y_{min} = 0$;

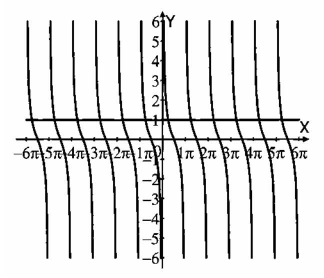
б) $x \in [\frac{\pi}{2}; \pi]$, $y_{max} = –$, $y_{min} = 0$;

в) $x \in [-\pi; 0]$, $y_{max} = –$, $y_{min} = –$;

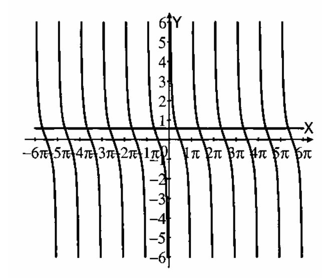
г) $x \in [\frac{\pi}{6}; \frac{3\pi}{4}]$, $y_{max} = \sqrt{3}$, $y_{min} = -1$.

259.

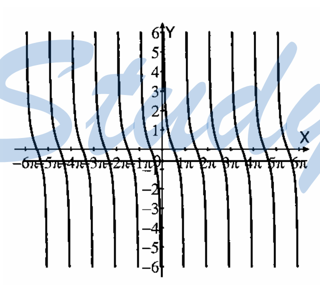
а) $x = \frac{\pi}{4} + \pi k$;



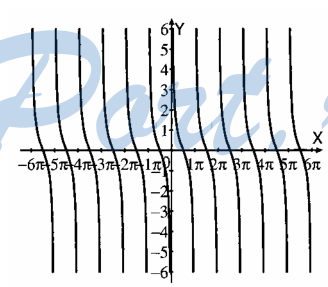
б) $x = \frac{\pi}{3} + \pi k$;



в) $x = -\frac{\pi}{3} + \pi k$;



г) $x = \frac{\pi}{2} + \pi k$.



260.

а) $f(x) = \text{tg}\,x - \cos x$, $f(-x) = -\text{tg}\,x - \cos x$, ни четная, ни нечетная;

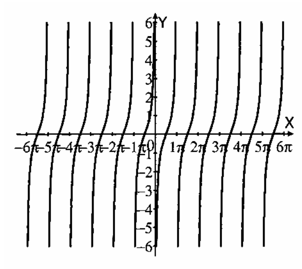
б) $f(x) = \text{tg}\,x + x$, $f(-x) = -\text{tg}x - x = -f(x)$, нечетная;

в) $f(x) = \text{ctg}^2\,x - x^4$, $f(-x) = \text{ctg}^2\,x - x^4$, четная;

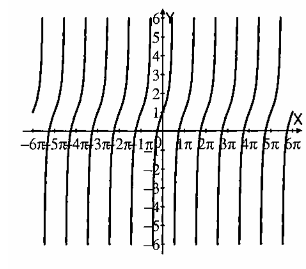
г) $f(x) = x^3 - \text{ctg}x$, $f(-x) = -x^3 + \text{ctg}x = -f(x)$, нечетная.

261.

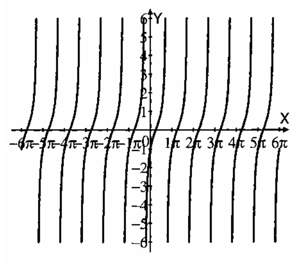
а)



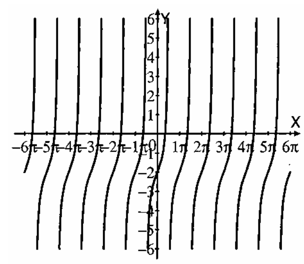
б)



в)

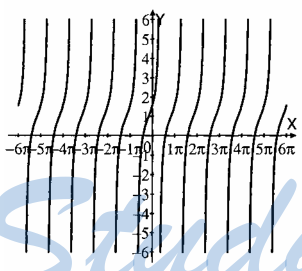


г)

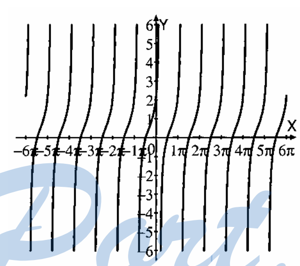


262.

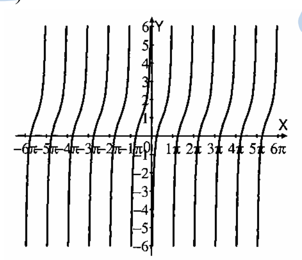
а)



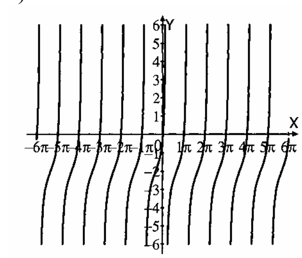
б)



в)

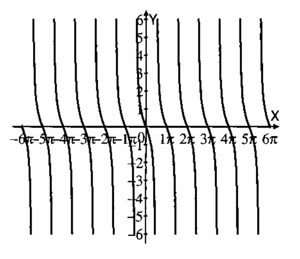


г)

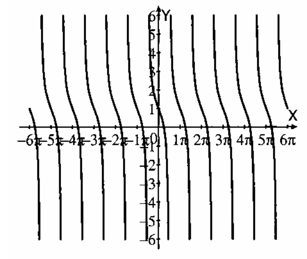


263.

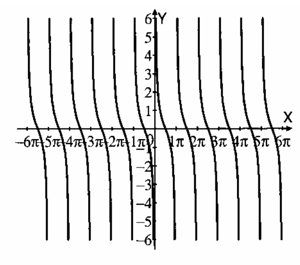
а)



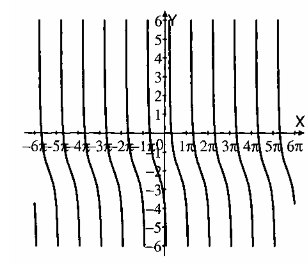
б)



в)

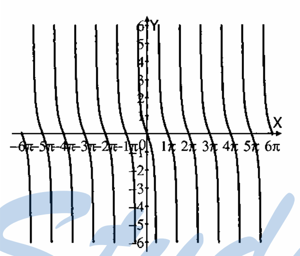


г)

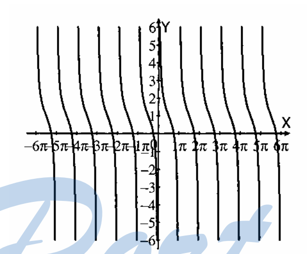


264.

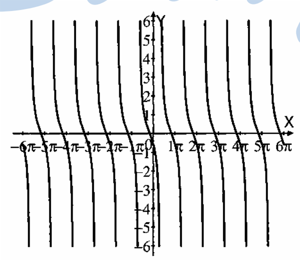
а)



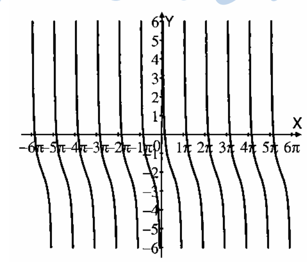
б)



в)



г)



265. а) $y = \text{tg } 2x$, $T = \frac{\pi}{2}$, $y(x + t) = \text{tg }(2x + \pi) = \text{tg } x$;

б) $y = \text{tg } \frac{x}{3}$, $T = 3\pi$, $y(x + T) = \text{tg }(\frac{x}{3} + \pi) = \text{tg } \frac{x}{3}$;

в) $y = \text{tg } 5x$, $T = \frac{\pi}{5}$, $y(x + T) = \text{tg }(5x + \pi) = \text{tg } 5x$;

.г) $y = \frac{2x}{5}$, $T = \frac{5\pi}{2}$, $y(x + T) = \text{tg }(\frac{2x}{5} + \pi) = \text{tg } \frac{2x}{5}$.

266. а) $y = \text{tg } x + \sin 2x - \text{tg } 3x - \cos 4x$, $T = \pi$,
$y(x + \pi) = \text{tg }(x + \pi) + \sin(2x + 2\pi) - \text{tg }(3x + 3\pi) - \cos(4x + 4\pi) = y(x)$;
б) $y = \sin 3x + \cos 5x + \text{ctg } x - 2\text{ tg } 2x$, $T=\pi$
$y(x + \pi) = \sin(3x + 3\pi) + \cos(5x + 5\pi) + \text{ctg }(\pi + x) - 2\text{tg }(2x + \pi) = -\sin 3x -$
$- \cos 5x + \text{ctg} x + \text{ctg} x - 2\text{tg} 2x \neq y(x)$, $\to \pi$ не есть период.

267. $\text{tg }(9\pi - x) = -\frac{3}{4}$; $\text{tg }(9\pi - x) = -\text{tg } x$; $\text{tg } x = \frac{3}{4}$, $\text{ctg } x = \frac{4}{3}$.

268. $\text{ctg }(7\pi - x) = \frac{5}{7}$; $\text{ctg } x = -\frac{5}{7}$, $\text{tg} = -\frac{7}{5}$

269. а) $\text{tg } 200^\circ - \text{tg } 201^\circ < 0$; б) $\text{tg } 1 - \text{tg } 1{,}01 < 0$;

в) $\text{tg } 2{,}2 - \text{tg } 2{,}1 > 0$; г) $\text{tg}\frac{3\pi}{5} - \text{tg}\frac{6\pi}{5} < 0$.

270. а) $f(x) = \text{tg}x \sin^2 x$, $f(-x) = -\text{tg } x \cdot \sin^2 x = -f(x)$, нечетная;

б) $f(x) = \frac{\text{tg}^2 x}{x^2 - 1}$, $f(-x) = \frac{\text{tg}^2 x}{x^2 - 1} = f(x)$, четная;

в) $f(x)=x^5\text{tg}x$, $f(-x)=x^5\text{tg}x=f(x)$, четная;
г)$f(x) = x^2 + \sin x + \text{tg}x$, $f(-x)=x^2-\sin x-\text{tg}x$, ни четная ,ни нечетная.

271. а) $f(x) = \sin x + \text{ctg}x$, $f(-x) = -\sin x - \text{ctg}x = -f(x)$, нечетная;

б) $f(x) = \frac{2\text{ctg}x}{x^3}$, $f(-x) = \frac{-2\text{ctg}x}{-x^3} = f(x)$, четная;

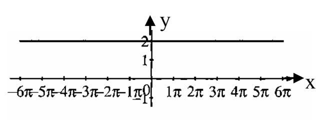
в) $f(x) = \frac{x^4 \text{ctg } x}{x^2 - 4}$, $f(-x) = -\frac{x^4 \text{ctg } x}{x^2 - 4} = -f(x)$, нечетная;

г) $f(x) = \text{ctg}x - x\cos x$, $f(-x) = -\text{ctg}x + x\cos x = -f(x)$, нечетная.

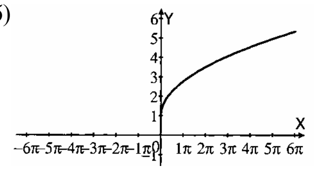
272. $f(x) = \text{tg } x$, $f(2x+2\pi)+f(7\pi-2x)=\text{tg}(2x+2\pi)+\text{tg }(7\pi-2x) = \text{tg } 2x - \text{tg}2x = 0$.

273. $f(x) = x^2 + 1$, $f(\text{tg } x) = \text{tg}^2 x + 1 = \frac{1}{\cos^2 x}$.

274. $f(x) = x^2 + 1$, $f(\text{ctg } x) = \text{ctg}^2 x + 1 = \frac{1}{\sin^2 x}$.

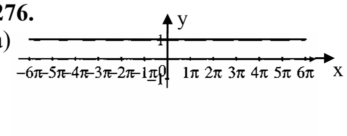
275.
а)


б)

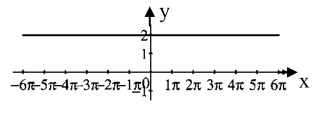


276.

а)

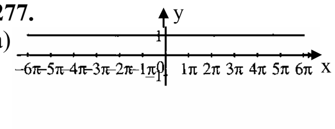


б)

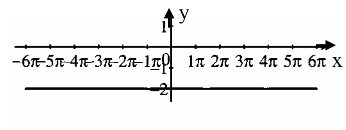


277.

а)



б)



Глава 2. Тригонометрические уравнения

§ 16. Первые представления о решении тригонометрических уравнений

278. а) $\cos t = \frac{\sqrt{2}}{2}$, $t = \pm\frac{\pi}{4} + 2\pi n$; б) $\sin t = -\frac{1}{2}$, $t = (-1)^{k+1}\frac{\pi}{6} + \pi k$;

в) $\cos t = -\frac{1}{2}$, $t = \pm\frac{2\pi}{3} + 2\pi n$; г) $\sin t = \frac{\sqrt{2}}{2}$, $t = (-1)^{k}\frac{\pi}{4} + \pi k$.

279. а) $\cos t = \frac{\sqrt{3}}{2}$, $t = \pm\frac{\pi}{6} + 2\pi k$; б) $\sin t = -\frac{\sqrt{3}}{2}$, $t = (-1)^{k+1}\frac{\pi}{6} + \pi k$;

в) $\cos t = -\frac{\sqrt{3}}{2}$, $t = \pm\frac{5\pi}{6} + 2\pi k$; г) $\sin t = \frac{\sqrt{3}}{2}$, $t = (-1)^{k}\frac{\pi}{3} + \pi k$.

280. а) $\sin t = 1$, $t = \frac{\pi}{2} + 2\pi k$; б) $\cos t = 2$, решений нет;

в) $\cos t = -1$, $t = \pi + 2\pi k$; г) $\sin t = -3$, решений нет.

281. а) $\operatorname{tg} t = \sqrt{3}$, $t = \frac{\pi}{3} + \pi n$; б) $\operatorname{ctg} t = -\frac{\sqrt{3}}{3}$, $t = -\frac{\pi}{3} + 2\pi n$;

в) $\operatorname{tg} t = -\frac{\sqrt{3}}{3}$, $t = -\frac{\pi}{6} + \pi n$; г) $\operatorname{ctg} t = \sqrt{3}$, $t = \frac{\pi}{6} + \pi n$.

282. а) $\sin t\,(2\cos t + 1) = 0$, $\left\{\begin{array}{l}\sin t = 0\\ \cos t = -\frac{1}{2}\end{array}\right.$, $t = \pi n$, $t = \pm\frac{2\pi}{3} + 2\pi n$;

б) $(\sin t - 1)(\cos t + 1) = 0$, $\left\{\begin{array}{l}\sin t = 1\\ \cos t = -1\end{array}\right.$, $t = \frac{\pi}{2} + 2\pi n$, $t = \pi + 2\pi n$;

в) $\cos t \cdot (2\sin t + 1) = 0$, $\left\{\begin{array}{l}\cos t = 0\\ \sin t = -\frac{1}{2}\end{array}\right.$, $t = \frac{\pi}{2} + \pi n$, $t = (-1)^{n+1}\frac{\pi}{6} + \pi n$;

г) $(2\sin t - \sqrt{2})(2\cos t + 1) = 0$,

$\left\{\begin{array}{l}\sin t = \frac{\sqrt{2}}{2}\\ \cos t = -\frac{1}{2}\end{array}\right.$, $t = (-1)^{n+1}\frac{\pi}{4} + \pi n$, $t = \pm\frac{2\pi}{3} + 2\pi n$.

283. а) $\cos\left(\frac{\pi}{2} - t\right) = 1$, $\sin t = 1$, $t = \frac{\pi}{2} + 2\pi n$;

б) $\cos(t - \pi) = 1$, $-\cos t = 1$, $t = \pi + 2\pi n$;

в) $\sin(\pi - t) = 1$, $\sin t = 1$, $t = \frac{\pi}{2} + 2\pi n$;

г) $\sin\left(t - \frac{\pi}{2}\right) = 1$, $\cos t = -1$, $t = \pi + 2\pi n$.

284. а) $3 - 4\sin^2 t = 0$, $\sin t = \pm\dfrac{\sqrt{3}}{2}$, $t = (-1)^k\dfrac{\pi}{3} + \pi k$, $t = (-1)^{k+1}\dfrac{\pi}{3} + \pi k$;

б) $\sin^2 t - \sin t = 0$, $\begin{cases}\sin t = 0\\ \sin t = 1\end{cases}$, $t = \pi n$, $t = \dfrac{\pi}{2} + 2\pi n$;

в) $4\sin^2 t - 1 = 0$, $\sin t = \pm\dfrac{1}{2}$, $t = (-1)^k\dfrac{\pi}{6} + \pi k$, $t = (-1)^{k+1}\dfrac{\pi}{6} + \pi k$;

г) $2\sin^2 t + \sin t = 0.$, $\begin{cases}\sin t = 0\\ \sin t = -\dfrac{1}{2}\end{cases}$, $t = \pi n$, $t = (-1)^{k+1}\dfrac{\pi}{6} + \pi k$.

285. а) $3 - 4\cos^2 t = 0$, $\cos t = \pm\dfrac{\sqrt{3}}{2}$, $t = \pm\dfrac{\pi}{6} + \pi n$;

б) $2\cos^2 t - \cos t = 0$, $\begin{cases}\cos t = 0\\ \cos t = \dfrac{1}{2}\end{cases}$, $t = \dfrac{\pi}{2} + \pi n$, $t = \pm\dfrac{\pi}{3} + 2\pi n$;

в) $4\cos^2 t - 1 = 0$, $\cos t = \pm\dfrac{1}{2}$, $t = \pm\dfrac{\pi}{3} + 2\pi n$, $t = \pm\dfrac{2\pi}{3} + 2\pi n$;

г) $2\cos^2 t + \cos t = 0$, $\begin{cases}\cos t = 0\\ \cos t = -\dfrac{1}{2}\end{cases}$, $t = \dfrac{\pi}{2} + \pi n$, $t = \pm\dfrac{2\pi}{3} + 2\pi n$.

286. а) $2\sin^2 t + 3\sin t - 2 = 0$, $\sin t = \dfrac{-3 + \sqrt{9 - 4\cdot 2(-2)}}{4} = \dfrac{1}{2}$,

$t = (-1)^k\dfrac{\pi}{6} + \pi k$, $\sin t = -2$ не подходит;

б) $2\cos^2 t - 5\cos t + 2 = 0$, $\cos t = \dfrac{5+3}{4} = 2$ – не подходит,

$\cos t = \dfrac{1}{2}$, $t = \pm\dfrac{\pi}{3} + 2\pi n$;

в) $2\sin^2 t + \sin t - 1 = 0$, $\sin t = \dfrac{-1+3}{4} = \dfrac{1}{2}$, $t = (-1)^k\dfrac{\pi}{6} + \pi k$,

$\sin t = -1$, $t = -\dfrac{\pi}{2} + 2\pi n$;

г) $4\cos^2 t + 9\cos t + 5 = 0$, $\cos t = \dfrac{-9+1}{8} = -1$, $t = \pi + 2\pi n$,

$\cos t = \dfrac{-9-1}{8}$ – не подходит.

287. а) $2\cos^2 t + \sin t + 1 = 0$, $2 - 2\sin^2 t + \sin t + 1 = 0$, $2\sin^2 t - \sin t - 3 = 0$,

$\sin t = \dfrac{1+5}{4}$ не подходит, $\sin t = \dfrac{1-5}{4} = -1$, $t = -\dfrac{\pi}{2} + 2\pi k$;

б) $\sin^2 t + 3\cos t - 3 = 0$, $\cos^2 t - 3\cos t + 2 = 0$, $\cos t = 2$ не подходит, $\cos t = 1$; $t = 2\pi n$.

288. а) $\sin(\frac{\pi}{2} + t) - \cos(\pi + t) = 1$, $\cos t + \cos t = 1$, $\cos t = \frac{1}{2}$, $t = \pm\frac{\pi}{3} + 2\pi n$.

б) $\sin(\pi + t) + \sin(2\pi - t) - \cos(\frac{3\pi}{2} + t) + 1{,}5 = 0$,

$-\sin t - \sin t - \sin t = -\frac{3}{2}$, $\sin t = \frac{1}{2}$; $t = (-1)^k \frac{\pi}{6} + \pi k$;

в) $\cos(\frac{\pi}{2} - t) - \sin(\pi + t) = \sqrt{2}$, $\sin t + \sin t = \sqrt{2}$,

$\sin t = \frac{\sqrt{2}}{2}$, $t = (-1)^k \frac{\pi}{4} + \pi k$;

г) $\sin(\pi + t) + \cos(\frac{\pi}{2} + t) = \sqrt{3}$, $-\sin t - \sin t = \sqrt{3}$,

$\sin t = -\frac{\sqrt{3}}{2}$, $t = (-1)^{k+1} \frac{\pi}{3} + \pi k$.

§ 17. Арккосинус и решение уравнения cos *t = a*

289. а) $\arccos 0 = \frac{\pi}{2}$; б) $\arccos 1 = 0$; в) $\arccos \frac{\sqrt{3}}{2} = \frac{\pi}{6}$; г) $\arccos \frac{1}{2} = \frac{\pi}{3}$.

290. а) $\arccos(-\frac{\sqrt{2}}{2}) = \frac{3\pi}{4}$; б) $\arccos(-\frac{\sqrt{3}}{2}) = \frac{5\pi}{6}$;

в) $\arccos(-1) = \pi$; г) $\arccos(-\frac{1}{2}) = \frac{2\pi}{3}$.

291. а) $\arccos(-1) + \arccos 0 = \pi + \frac{\pi}{2} = \frac{3\pi}{2}$;

б) $\arccos \frac{1}{2} - \arccos \frac{\sqrt{3}}{2} = \frac{\pi}{3} - \frac{\pi}{6} = \frac{\pi}{6}$;

в) $\arccos(-\frac{\sqrt{2}}{2}) + \arccos \frac{\sqrt{2}}{2} = \frac{3\pi}{4} + \frac{\pi}{4} = \pi$;

г) $\arccos(-\frac{1}{2}) - \arccos \frac{1}{2} = \frac{2\pi}{3} - \frac{\pi}{3} = \frac{\pi}{3}$.

292. а) $\sin(\arccos(-\frac{1}{2})) = \sin \frac{2\pi}{3} = \frac{\sqrt{3}}{2}$; б) $\mathrm{tg}(\arccos \frac{\sqrt{3}}{2}) = \mathrm{tg}\frac{\pi}{6} = \frac{\sqrt{3}}{3}$;

в) $\mathrm{ctg}(\arccos 0) = \mathrm{ctg}\frac{\pi}{2} = 0$; г) $\sin(\arccos \frac{\sqrt{2}}{2}) = \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$.

293. а) $\cos t = \frac{1}{2}$, $t = \pm\frac{\pi}{3} + 2\pi n$; б) $\cos t = \frac{\sqrt{2}}{2}$, $t = \pm\frac{\pi}{4} + 2\pi n$;

в) $\cos t = 1$, $t = 2\pi n$; г) $\cos t = \frac{\sqrt{3}}{2}$, $t = \pm\frac{\pi}{6} + 2\pi n$.

294. а) $\cos t = -1$, $t = \pi + 2\pi n$; б) $\cos t = -\frac{\sqrt{3}}{2}$, $t = \pm\frac{5\pi}{6} + 2\pi n$;

в) $\cos t = -\frac{1}{2}$, $t = \pm\frac{2\pi}{3} + 2\pi n$; г) $\cos t = -\frac{\sqrt{2}}{2}$, $t = \pm\frac{3\pi}{4} + 2\pi n$.

295. а) $\cos t = \frac{1}{3}$, $t = \pm \arccos\frac{1}{3} + 2\pi n$; б) $\cos t = -1{,}1$, решений нет;

в) $\cos t = -\frac{3}{7}$, $t = \pm\arccos\left(-\frac{3}{7}\right) + 2\pi n$; г) $\cos t = 2{,}04$, решений нет.

296. а) $\cos t\left(2\arccos\frac{1}{2} - 3\arccos 0 - \arccos\left(-\frac{1}{2}\right)\right) =$

$=\cos\left(\frac{2\pi}{3} - \frac{3\pi}{2} - \frac{2\pi}{3}\right) = \cos\frac{3\pi}{2} = 0$;

б) $\frac{1}{3}\left(\arccos\frac{1}{3} + \arccos\left(-\frac{1}{3}\right)\right) = \frac{1}{3}\pi = \frac{\pi}{3}$.

297. а) $x \in [-1; 1]$; б) $|x| \le \frac{1}{2}$; в) $x \in [0; 2]$; г) $x \in [1; 2]$.

298. а) $\arccos\sqrt{5}$, – нет; б) $\arccos\frac{\sqrt{2}}{3}$, – да;

в) $\arccos\frac{\pi}{5}$, – да; г) $\arccos(-\sqrt{3})$, – нет.

299. $\operatorname{tg}(\arccos 0{,}1 + \arccos(-0{,}1) + x) = \operatorname{tg} x$, $\operatorname{tg}(\pi + x) = \operatorname{tg} x$.

300. а) $\frac{8\cos t - 3}{3\cos t + 2} = 1$, $\frac{8\cos t - 3 - 3\cos t - 2}{3\cos t + 2} = 0$, $\begin{cases} 5\cos t - 5 = 0 \\ \cos t \ne -\frac{2}{3} \end{cases}$,

$\cos t = 1$, $t = 2\pi n$;

б) $\frac{3\cos t + 1}{2} + \frac{5\cos t - 1}{3} = 1\frac{3}{4}$, $9\cos t + 3 + 10\cos t - 2 = \frac{7}{4}\cdot 6 = \frac{21}{2}$,

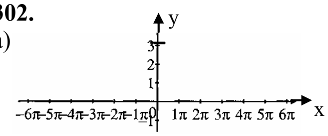
$19\cos t = \frac{19}{2}$, $\cos t = \frac{1}{2}$, $t = \pm\frac{\pi}{3} + 2\pi n$.

301. а) $6\cos^2 t + 5\cos t + 1 = 0$, $\cos t = \frac{-5+1}{12} = -\frac{1}{3}$, $t = \pm\arccos\left(-\frac{1}{3}\right) + 2\pi n$,

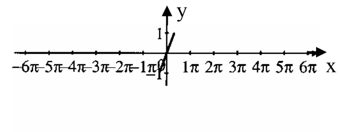
$\cos t = \frac{-5-1}{12} = -\frac{1}{2}$, $t = \pm\frac{2}{3}\pi + 2\pi n$.

302.

а)



б)



303. а) $\cos t > \frac{1}{2}$, $t \in (-\frac{\pi}{3} + 2\pi k;\ \frac{\pi}{3} + 2\pi k)$;

б) $\cos t \le -\frac{\sqrt{2}}{2}$, $t \in [\frac{3\pi}{4} + 2\pi k;\ \frac{5\pi}{4} + 2\pi k]$;

в) $\cos t \ge -\frac{\sqrt{2}}{2}$, $t \in [-\frac{3\pi}{4} + 2\pi k;\ -\frac{3\pi}{4} + 2\pi k]$;

г) $\cos t < \frac{1}{2}$, $t \in (\frac{\pi}{3} + 2\pi k;\ \frac{5\pi}{3} + 2\pi k)$.

304. а) $\cos t < \frac{2}{3}$, $t \in (\arccos\frac{2}{3} + 2\pi k;\ 2\pi - \arccos\frac{2}{3} + 2\pi k)$;

б) $\cos t > -\frac{1}{7}$, $t \in (-\arccos(-\frac{1}{7}) + 2\pi k;\ \arccos(-\frac{1}{7}) + 2\pi k)$;

в) $\cos t > \frac{2}{3}$, $t \in (-\arccos\frac{2}{3} + 2\pi k;\ \arccos\frac{2}{3} + 2\pi k)$;

г) $\cos t < -\frac{1}{7}$, $t \in (\arccos(-\frac{1}{7}) + 2\pi k;\ 2\pi - \arccos(-\frac{1}{7}) + 2\pi k)$.

305. а) $3\cos^2 t - 4\cos t \ge 4$, $3\cos^2 t - 4\cos t - 4 = 0$.

Найдем корни квадратного уравнения:

$\cos t = \frac{4 \pm \sqrt{16 + 4\cdot 3\cdot 4}}{6} = \frac{4 \pm 8}{6}$, $\cos t = -\frac{2}{3}$, $\cos t = 2$ не подходит,

$\cos t \le -\frac{2}{3}$, $t \in (\arccos(-\frac{2}{3}) + 2\pi k,\ 2\pi - \arccos\left(-\frac{2}{3}\right) + 2\pi k)$;

б) $6\cos^2 t + 1 > 5\cos t$.

Найдем корни квадратного уравнения:

$6\cos^2 t - 5\cos t + 1 = 0$, $\cos t = \frac{5+1}{12} = \frac{1}{2}$, $\cos t = \frac{1}{3}$,

$t \in (-\frac{\pi}{3} + 2\pi k;\ \frac{\pi}{3} + 2\pi k) \cup (\arccos\frac{1}{3} + 2\pi k;\ 2\pi - \arccos\frac{1}{3} + 2\pi k)$;

в) $3\cos^2 t - 4\cos t < 4$, $3\cos^2 t - 4\cos t - 4 < 0$.

Найдем корни квадратного уравнения:

$3\cos^2 t - 4\cos t - 4 = 0$, $\cos t = \frac{4 \pm \sqrt{16 + 4\cdot 3\cdot 4}}{6} = \frac{2 \pm 4}{3}$,

$\cos t = 2$ – не подходит, $\cos t = -\frac{2}{3} \to \cos t > -\frac{2}{3}$,

$t \in (-\arccos\left(-\frac{2}{3}\right) + 2\pi k; -\left(-\frac{2}{3}\right) + 2\pi k)$;

г) $6\cos^2 t + 1 \le 5\cos t$, $6\cos^2 t - 5\cos t + 1 \le 0$.
Найдем корни квадратного уравнения:

$6\cos^2 t - 5\cos t + 1 = 0$, $\cos t = \frac{5 \pm \sqrt{25 - 4 \cdot 6 \cdot 1}}{12} = \frac{5 \pm 1}{12}$, $\cos t = \frac{1}{2}$, $\cos t = \frac{1}{3}$,

$t \in (-\arccos\frac{1}{3} + 2\pi k; -\frac{\pi}{3} + 2\pi k)$, $(\frac{\pi}{3} + 2\pi k; \arccos\frac{1}{3} + 2\pi k)$.

306. а) $4\cos^2 t < 1$, $\cos^2 t < \frac{1}{4}$, $\cos t \in (-\frac{1}{2}; \frac{1}{2})$, $t \in (\frac{\pi}{3} + \pi k; \frac{2}{3}\pi + \pi k)$;

б) $3\cos^2 t < \cos t$, $\cos t\,(3\cos t - 1) < 0$, $\cos t \in (0; \frac{1}{3})$,

$t \in (-\frac{\pi}{2} + 2\pi n; -\arccos\frac{1}{3} + 2\pi n) \cup (\arccos\frac{1}{3} + 2\pi n; \frac{\pi}{2} + 2\pi n)$.

307. а) $\sin(\arccos\frac{3}{5}) = \sqrt{1 - \cos^2(\arccos\frac{3}{5})} = \sqrt{1 - \frac{9}{25}} = \frac{4}{5}$;

б) $\sin(\arccos(-\frac{4}{5})) = \sqrt{1 - \cos^2(\arccos(-\frac{4}{5}))} = \sqrt{1 - \frac{16}{25}} = \frac{3}{5}$.

308. а) $\mathrm{tg}(\arccos(-\frac{15}{3})) = \frac{\sqrt{1 - \cos^2(\arccos(-\frac{5}{13}))}}{\cos(\arccos(-\frac{5}{13}))} = \frac{\sqrt{1 - \frac{25}{169}}}{\frac{-5}{13}} = -\frac{12}{5}$;

б) $\mathrm{ctg}(\arccos(\frac{4}{5})) = \frac{\cos(\arccos(\frac{4}{5}))}{\sqrt{1 - \cos^2(\arccos\frac{4}{5})}} = \frac{4 \cdot 5}{5 \cdot 3} = \frac{4}{3}$.

§ 18. Арксинус и решение уравнения $\sin t = a$

309. а) $\arcsin\frac{\sqrt{3}}{2} = \frac{\pi}{3}$; б) $\arcsin 1 = \frac{\pi}{2}$; в) $\arcsin\frac{\sqrt{2}}{2} = \frac{\pi}{4}$; г) $\arcsin 0 = 0$.

310. а) $\arcsin(-\frac{\sqrt{3}}{2}) = -\frac{\pi}{3}$; б) $\arcsin(-\frac{1}{2}) = -\frac{\pi}{6}$;

в) $\arcsin(-1) = -\frac{\pi}{2}$; г) $\arcsin(-\frac{\sqrt{2}}{2}) = -\frac{\pi}{4}$.

311. а) $\arcsin 0 + \arccos 0 = \frac{\pi}{2}$; б) $\arcsin\frac{\sqrt{3}}{2} + \arccos\frac{\sqrt{3}}{2} = \frac{\pi}{2}$;

в) $\arcsin(-\frac{\sqrt{2}}{2}) + \arccos\frac{1}{2} = \frac{\pi}{12}$; г) $\arcsin(-1) + \arccos\frac{\sqrt{3}}{2} = -\frac{\pi}{3}$.

312. а) $\arccos(-\frac{1}{2}) + \arcsin(-\frac{1}{2}) = \frac{\pi}{2}$; б) $\arccos(-\frac{\sqrt{2}}{2}) - \arcsin(-1) = \frac{5\pi}{4}$;

в) $\arccos(-\frac{\sqrt{3}}{2}) + \arcsin(-\frac{\sqrt{3}}{2}) = \frac{\pi}{2}$; г) $\arccos\frac{\sqrt{2}}{2} - \arcsin(-\frac{\sqrt{3}}{2}) = \frac{7\pi}{12}$.

313. а) $\sin t = \frac{\sqrt{3}}{2}$, $t = (-1)^k \frac{\pi}{3} + \pi k$; б) $\sin t = \frac{\sqrt{2}}{2}$, $t = (-1)^k \frac{\pi}{4} + \pi k$;

в) $\sin t = 1$, $t = \frac{\pi}{2} + 2\pi n$; г) $\sin t = \frac{1}{2}$, $t = (-1)^k \frac{\pi}{6} + \pi k$.

314. а) $\sin t = -1$, $t = -\frac{\pi}{2} + 2\pi n$; б) $\sin t = -\frac{\sqrt{2}}{2}$, $t = (-1)^{k+1} \frac{\pi}{4} + \pi k$;

в) $\sin t = -\frac{1}{2}$, $t = (-1)^{k+1} \frac{\pi}{6} + \pi k$; г) $\sin t = -\frac{\sqrt{3}}{2}$, $t = (-1)^{k+1} \frac{\pi}{3} + \pi k$.

315. а) $\sin t = \frac{1}{4}$, $t = (-1)^k \arcsin\frac{1}{4} + \pi k$; б) $\sin t = 1{,}02$, решений нет;

в) $\sin t = -\frac{1}{7}$, $t = (-1)^k \arcsin(-\frac{1}{7}) + \pi k$; г) $\sin\frac{\pi}{3}$, решений нет.

316. а) $\sin(\arccos x + \arccos(-x)) = 0$, $\sin\pi = 0$;
б) $\cos(\arcsin x + \arcsin(-x)) = 1$, $\cos 0 = 1$.

317. а) $\sin(2\arcsin\frac{1}{2} - 3\arccos(-\frac{1}{2})) = \sin(\frac{\pi}{3} - 2\pi) = \frac{\sqrt{3}}{2}$;

б) $\cos(\frac{1}{2}\arcsin 1 + \arcsin(-\frac{\sqrt{2}}{2})) = \cos(\frac{\pi}{4} - \frac{\pi}{4}) = 1$.

318. а) $\operatorname{tg}(\arcsin\frac{\sqrt{3}}{2} + 2\arccos\frac{\sqrt{2}}{2}) = \operatorname{tg}(\frac{\pi}{3} + \frac{\pi}{2}) = -\frac{\sqrt{3}}{3}$;

б) $\operatorname{ctg}(3\arccos(-1) - \arcsin(-\frac{1}{2})) = \operatorname{ctg}(3\pi + \frac{\pi}{6}) = \sqrt{3}$.

319. а) $\arcsin x$, $x \in [-1; 1]$; б) $\arcsin(5 - 2x)$, $x \in [2; 3]$;
в) $\arcsin\frac{x}{2}$, $x \in [-2; 2]$; г) $\arcsin(x^2 - 3)$, $x \in [-2; -\sqrt{2}] \cup [\sqrt{2}; 2]$.

320. а) $\arcsin(-\frac{2}{3})$. Да; б) $\arcsin 1{,}5$. Нет;

в) $\arcsin(3 - \sqrt{20})$. Нет; г) $\arcsin(4 - \sqrt{20})$. Да.

321. а) $(2\cos x + 1)(2\sin x - \sqrt{3}) = 0$,

$$\left[\begin{array}{l}\cos x = -\frac{1}{2}\\ \sin x = \frac{\sqrt{3}}{2}\end{array}\right., \quad x = \pm\frac{2\pi}{3} + 2\pi k, \ x = (-1)^k \frac{\pi}{3} + 2\pi k;$$

б) $2\cos x - 3\sin x\cos x = 0$, $\cos x(2 - 3\sin x) = 0$,

$\begin{cases} \cos x = 0 \\ \sin x = \frac{2}{3} \end{cases}$, $x = \frac{\pi}{2} + \pi n$, $x = (-1)^k \arcsin\frac{2}{3} + \pi k$;

в) $4\sin^2 x - 3\sin x = 0$, $\sin x(4\sin x - 3) = 0$, $\sin x = 0$, $\sin x = \frac{3}{4}$,

$x = \pi n$, $x = (-1)^k \arcsin\frac{3}{4} + \pi k$;

г) $2\sin^2 x - 1 = 0$, $\sin x = \pm\frac{\sqrt{2}}{2}$, $x = \frac{\pi}{4} + \frac{\pi n}{2}$.

322. а) $6\sin^2 x + \sin x - 2 = 0$, $\sin x = \frac{-1+7}{12} = \frac{1}{2}$, $(-1)^n \arcsin\frac{\pi}{6} + \pi n$,

$\sin x = -\frac{2}{3}$, $(-1)^{k+1} \arcsin\frac{2}{3} + \pi k$;

б) $3\cos^2 x = 7(\sin x + 1)$, $3 - 3\sin^2 x = 7\sin x + 7$, $3\sin^2 x + 7\sin x + 4 = 0$,

$\sin x = \frac{-7+\sqrt{49-4\cdot 3\cdot 4}}{6} = \frac{-7\pm 1}{6}$,

$\sin x = \frac{-8}{6}$ — не подходит, $\sin x = -1$, $x = -\frac{\pi}{2} + 2\pi n$.

323. а) $\sin t > \frac{\sqrt{3}}{2}$, $t \in (\frac{\pi}{3} + 2\pi k; \frac{2\pi}{3} + 2\pi k)$;

б) $\sin t > -\frac{1}{2}$, $t \in (-\frac{\pi}{6} + 2\pi k; \frac{7\pi}{6} + 2\pi k)$;

в) $\sin t < \frac{\sqrt{3}}{2}$, $t \in (-\frac{4\pi}{3} + 2\pi k; \frac{\pi}{3} + 2\pi k)$;

г) $\sin t \le -\frac{1}{2}$, $t \in (\frac{7\pi}{6} + 2\pi k; \frac{11\pi}{6} + 2\pi k)$.

324. а) $\sin t < \frac{1}{3}$, $t \in (\pi - \arcsin\frac{1}{3} + 2\pi k; \arcsin\frac{1}{3} + 2\pi k)$;

б) $\sin t \ge -\frac{3}{5}$, $t \in (-\arcsin\frac{3}{5} + 2\pi k; \pi + \arcsin\frac{3}{5} + 2\pi k)$;

в) $\sin t \ge \frac{1}{3}$, $t \in (\arcsin\frac{1}{3} + 2\pi k; \pi - \arcsin\frac{1}{3} + 2\pi k)$;

г) $\sin t < -\frac{3}{5}$, $t \in (\pi + \arcsin\frac{3}{5} + 2\pi k; 2\pi - \arcsin\frac{3}{5} + 2\pi k)$.

325. а) $5\sin^2 t > 11\sin t + 12$, $5\sin^2 t - 11\sin t - 12 = 0$,

$\sin t = \frac{11+19}{10}$, не подходит. $\sin t = -\frac{8}{10}$,

$t \in (\pi + \arcsin\frac{4}{5} + 2\pi k; 2\pi - \arcsin\frac{4}{5} + 2\pi k)$

б) $5\sin^2 t \le 11t + 12$, $5\sin^2 t - 11t - 12 = 0$,

$\sin t = -\frac{4}{5}$, $t \in (-\arcsin\frac{4}{5} + 2\pi k; \pi + \arcsin\frac{4}{5} + 2\pi k)$.

326. а) $6\cos^2 t + \sin t > 4$, $6 - 6\sin^2 t + \sin t - 4 > 0$, $6\sin^2 t - \sin t - 2 < 0$,

$\sin t = \frac{1+7}{12} = \frac{3}{4}$, $\sin t = -\frac{1}{2}$,

$t \in (-\frac{\pi}{6} + 2\pi k; \arcsin\frac{2}{3} + 2\pi k) \cup (\pi - \arcsin\frac{2}{3} + 2\pi k; \frac{7\pi}{6} + 2\pi k)$;

б) $6\cos^2 t + \sin t \le 4$, $6\sin^2 t - \sin t - 2 = 0$, $\sin t = \frac{3}{4}$, $\sin t = -\frac{1}{2}$,

$t \in [\arcsin\frac{2}{3} + 2\pi k; \pi - \arcsin\frac{2}{3} + 2\pi k]$, $t \in [\frac{7\pi}{6} + 2\pi k; \frac{11\pi}{6} + 2\pi k]$.

327. а) $\cos(\arcsin(-\frac{5}{13})) = \sqrt{1 - \sin^2(\arcsin(-\frac{5}{13}))} = \sqrt{1 - \frac{25}{169}} = \frac{12}{13}$;

б) $\mathrm{tg}(\arcsin\frac{3}{5}) = \frac{3}{5}\cdot\frac{5}{4} = \frac{3}{4}$; в) $\cos(\arcsin\frac{8}{17}) = \sqrt{1 - \sin^2(\arcsin(-\frac{8}{17}))} = \frac{15}{17}$;

г) $\mathrm{ctg}(\arcsin(-\frac{4}{5})) = \frac{\sqrt{1 - \sin^2(\arcsin(-\frac{4}{5})}}{\sin(\arcsin(-\frac{4}{5}))} = -\frac{3}{5}\cdot\frac{5}{4} = -\frac{3}{4}$.

19. Арктангенс и решение уравнения tg *x* = *a*. *Арккотангенс и решение уравнения* ctg *x* = *a*

328. а) $\mathrm{arctg}\frac{\sqrt{3}}{3} = \frac{\pi}{6}$; б) $\mathrm{arctg}\,1 = \frac{\pi}{4}$; в) $\mathrm{arctg}\sqrt{3} = \frac{\pi}{3}$; г) $\mathrm{arctg}\,0 = 0$.

329. а) $\mathrm{arctg}(-1) = -\frac{\pi}{4}$; б) $\mathrm{arctg}(-\sqrt{3}) = -\frac{\pi}{3}$;

в) $\mathrm{arctg}(-\frac{\sqrt{3}}{3}) = -\frac{\pi}{6}$; г) $\mathrm{arctg}(-\frac{1}{\sqrt{3}}) = -\frac{\pi}{6}$.

330. а) $\mathrm{arctg}\,1 - \mathrm{arctg}\sqrt{3} = -\frac{\pi}{12}$; б) $\mathrm{arctg}(-\sqrt{3}) + \mathrm{arctg}\,0 = -\frac{\pi}{3}$

в) $\mathrm{arctg}\,1 - \mathrm{arctg}(-1) = \frac{\pi}{2}$; г) $\mathrm{arctg}\frac{\sqrt{3}}{3} + \mathrm{arctg}\sqrt{3} = \frac{\pi}{2}$.

331. а) $\mathrm{arcctg}\frac{\sqrt{3}}{3} = \frac{\pi}{3}$; б) $\mathrm{arcctg}\,1 = \frac{\pi}{4}$;

в) $\mathrm{arcctg}(-\frac{\sqrt{3}}{3}) = -\frac{\pi}{3}$; г) $\mathrm{arcctg}\,0 = \frac{\pi}{2}$.

332. а) arcctg (– 1) + arctg (– 1) = $\frac{\pi}{2}$; б) arcsin ($\frac{\sqrt{2}}{2}$) + arctg (– $\sqrt{3}$) = $\frac{7\pi}{12}$;

в) arctg (– $\frac{\sqrt{3}}{3}$) – arctg ($\frac{\sqrt{3}}{3}$) = $\frac{\pi}{2}$; г) arccos (– $\frac{1}{2}$)–arcctg (– $\sqrt{3}$) = – $\frac{\pi}{6}$.

333. а) tg x = 1, x = $\frac{\pi}{4}$ + πn; б) tg x = – $\frac{\sqrt{3}}{3}$, x = – $\frac{\pi}{6}$ + πn;

в) tg x = – 1, x = – $\frac{\pi}{4}$ + πn; г) tg x = $\frac{\sqrt{3}}{3}$, x = $\frac{\pi}{6}$ + πn.

334. а) tg x = 0, x =πn; б) tg x = – 2, x = – arctg 2 + πn;

в) tg x = – 3, x = – arctg 3 + πn; г) tg x = $\frac{1}{2}$, x = arctg $\frac{1}{2}$ + πn.

335. а) ctg x = 1, x = $\frac{\pi}{4}$ + πn; б) ctg x = $\sqrt{3}$, x = $\frac{\pi}{6}$ + πn;

в) ctg x = 0, x = $\frac{\pi}{2}$ + πn; г) ctg x = $\frac{\sqrt{3}}{2}$, x = $\frac{\pi}{3}$ + πn.

336. а) ctg x = – $\sqrt{3}$, x =– $\frac{\pi}{6}$ + πn; б) ctg x =– 1, x = – $\frac{\pi}{4}$ + πn;

в) ctg x = – $\frac{\sqrt{3}}{3}$, x =– $\frac{\pi}{3}$ + πn; г) ctg x =– 5, x = – arctg 5+ πn.

337. а) 2 arcsin (– $\frac{\sqrt{3}}{2}$)+arctg (– 1) + arccos $\frac{\sqrt{2}}{2}$ = – $\frac{2\pi}{3}$ – $\frac{\pi}{4}$ + $\frac{\pi}{4}$ = – $\frac{2\pi}{3}$;

б) 3 arcsin $\frac{1}{2}$ + 4 arccos (– $\frac{\sqrt{2}}{2}$)–arctg (– $\frac{\sqrt{3}}{3}$) = $\frac{\pi}{2}$ + 3π + $\frac{\pi}{6}$ = $\frac{11\pi}{3}$;

в) arctg (– $\sqrt{3}$) + arccos (– $\frac{\sqrt{3}}{2}$) + arcsin 1 = – $\frac{\pi}{3}$ + $\frac{5\pi}{6}$ + $\frac{\pi}{2}$ = π;

г) arcsin (– 1) – $\frac{3}{2}$ arccos $\frac{1}{2}$ + 3 arcctg (– $\frac{\sqrt{3}}{3}$)= – $\frac{\pi}{2}$ + $\frac{\pi}{2}$ + π = π.

338. а) sin (arctg (– $\sqrt{3}$)) = sin (– $\frac{\pi}{3}$)= – $\frac{\sqrt{3}}{2}$;

б) tg (arctg (– $\frac{\sqrt{3}}{3}$)) = tg (– $\frac{\pi}{3}$) = – $\frac{\sqrt{3}}{3}$;

в) cos (arctg 0) = cos 0 = 1; г) ctg (arctg (– 1) = ctg (– $\frac{\pi}{4}$) = –1.

339. а) tg (arcctg 1) = tg $\frac{\pi}{4}$ = 1; б) sin (arcctg $\sqrt{3}$) = sin $\frac{\pi}{6}$ = $\frac{1}{2}$;

в) cos (arcctg (–1))=cos $\frac{\pi}{4}$ = – $\frac{\sqrt{2}}{2}$; г) ctg (2arcctg (– $\frac{\sqrt{3}}{3}$) = ctg ($\frac{2\pi}{3}$) = $\sqrt{3}$.

340. а) $tg^2 x - 3 = 0$, $tg\, x = \pm\sqrt{3}$, $x = \pm\frac{\pi}{3} + \pi n$;

б) $2\, tg^2 x + 3\, tg\, x = 0$, $tg\, x = 0$, $tg\, x = -\frac{3}{2}$, $x = \pi n$, $x = -\, arcctg\, \frac{3}{2} + \pi n$;

в) $4\, tg^2 x - 9 = 0$, $tg\, x = \pm\frac{3}{2}$, $x = \pm\, arcctg\, \frac{3}{2} + \pi n$;

г) $3\, tg^2 x - 2\, tg\, x = 0$, $tg\, x = 0$, $tg\, x = \frac{2}{3}$, $x = \pi n$, $x = arcctg\, \frac{2}{3} + \pi n$.

341. а) $tg^2 x - 6 tg\, x + 5 = 0$, $tg\, x = 5$, $tg\, x = 1$, $x = arctg\, 5 + \pi n$, $x = \frac{\pi}{4} + \pi n$;

б) $tg^2 x - 2\, tg\, x - 3 = 0$, $tg\, x = 3$, $tg\, x = -1$, $x = arctg\, 3 + \pi n$, $x = -\frac{\pi}{4} + \pi n$.

342. а) $tg\,(\pi + x) = \sqrt{3}$, $tg\, x = \sqrt{3}$, $x = \frac{\pi}{3} + \pi n$;

б) $2\, ctg\,(2\pi + x) - tg(\frac{\pi}{2} + x) = \sqrt{3}$, $2ctg\, x + ctg\, x = \sqrt{3}$, $ctg\, x = \frac{\sqrt{3}}{3}$, $x = \frac{\pi}{3} + \pi n$;

в) $-\sqrt{3}\; tg\,(\pi - x) = 1$, $tg\, x = \frac{\sqrt{3}}{3}$, $x = \frac{\pi}{6} + \pi n$;

г) $ctg\,(2\pi - x) + ctg\,(\pi - x) = 2$, $ctg\, x = -1$, $x = \frac{\pi}{4} + \pi n$.

343. а) $tg\, x < \sqrt{3}$, $x \in (-\frac{\pi}{2} + \pi k;\ \frac{\pi}{3} + \pi k)$; б) $ctg\, x > 0$, $x \in (\pi k;\ \frac{\pi}{2} + \pi k)$;

в) $tg\, x < 0$, $x \in (-\frac{\pi}{2} + \pi k;\ \pi k)$; г) $ctg\, x > -1$, $x \in (\pi k;\ \frac{3\pi}{4} + \pi k)$.

344. а) $tg\, x < 3$, $x \in (-\frac{\pi}{2} + \pi k;\ arctg\, 3 + \pi k)$;

б) $3\, ctg\, x - 1 > 0$, $ctg\, x > \frac{1}{3}$, $x \in (\pi k;\ arctg\, \frac{1}{3} + \pi k)$;

в) $ctg\, x \le 2$, $x \in (arctg\, 2 + \pi k;\ \pi + 2\pi k)$;

г) $2\, tg\, x + 1 \ge 0$, $tg\, x \ge -\frac{1}{2}$, $x \in (-arctg\frac{1}{2} + \pi k;\ \frac{\pi}{2} + \pi k)$.

345. а) $tg^2 x > 9$, $\left[\begin{array}{l} tg\, x < -3 \\ tg\, x > 3 \end{array}\right.$, $x \in (-\frac{\pi}{2} + \pi k;\ arctg\, 3 + \pi k) \cup (arctg\, 3 + \pi k;\ \frac{\pi}{2} + \pi k)$

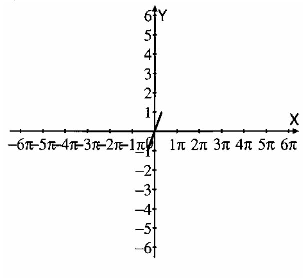
б) $tg^2 x > tg x$, $tgx(tgx - 1) > 0$, $\left[\begin{array}{l} tg\, x < 0 \\ tg\, x > 1 \end{array}\right.$, $x \in (-\frac{\pi}{2} + \pi k;\ \pi k) \cup (\frac{\pi}{4} + \pi k;\ \frac{\pi}{2} + \pi k)$;

в) $tg^2 x < 9$, $tg\, x \in (-3;\ 3)$, $x \in (-arctg\, 3 + \pi k;\ arctg\, 3 + \pi k)$;

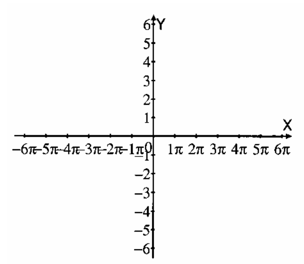
г) $tg^2 x < 2\, tg\, x$, $tg\, x\,(tg\, x - 2) < 0$, $tg\, x \in (0;\ 2)$, $x \in (\pi k;\ arctg\, x + \pi k)$.

346.

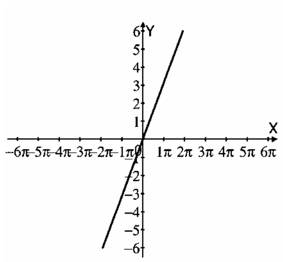
а)



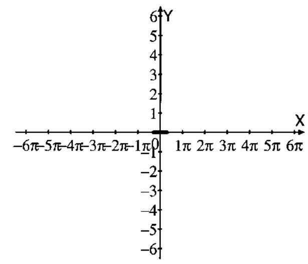
б)



в)

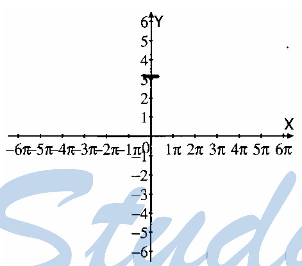


г)

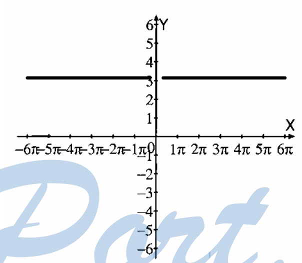


347.

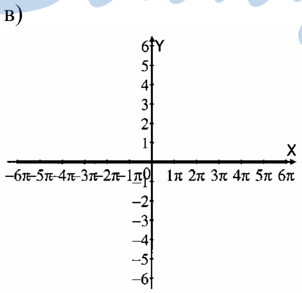
а)



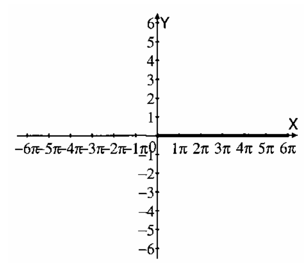
б)



в)



г)



348. а) $\sin(\text{arctg}\ \frac{3}{4})$, $\text{arctg}\ \frac{3}{4} = x$, $x \in (-\frac{\pi}{2}; \frac{\pi}{2})$, $\text{tg}\ x = \frac{3}{4}$,

$\sin x = \frac{3}{4}\sqrt{1-\sin^2 x}$, $16\sin^2 x = 9 - 9\sin^2 x$, $\sin^2 x = \frac{9}{25}$,

$\sin x = \pm\frac{3}{5} \Rightarrow \sin(\text{arctg}\ \frac{3}{4}) = \frac{3}{5}$;

б) $\cos(\text{arcctg}\ \frac{12}{5})$, $\text{arcctg}\ \frac{12}{5} = x$, $x \in (0; \pi)$, $\frac{12}{5} = \text{ctg}\ x$,

$\frac{12}{5} = \frac{\cos x}{\sqrt{1-\cos^2 x}}$, $144 - 144\cos^2 x = 25\cos^2 x$, $\cos^2 x = \frac{144}{169}$,

$\cos x = \pm\frac{12}{13} \Rightarrow \cos(\text{arcctg}\ \frac{12}{5}) = \frac{12}{13}$;

в) $\sin(\text{arcctg}\ (-\frac{4}{3}))$, $\text{arcctg}\ (-\frac{4}{3}) = x$, $x \in (0, \pi)$, $-\frac{4}{3} = \text{ctg}\ x$, $-\frac{4}{3} = \frac{\cos x}{\sin x}$,

$16\sin^2 x = 9 - 9\sin^2 x$, $\sin^2 x = \frac{9}{25}$, $\sin x = \pm\frac{3}{5} \Rightarrow \sin(\text{arctg}\ (-\frac{4}{3})) = \frac{3}{5}$;

г) $\cos(\text{arctg}\ (-\frac{5}{12}))$, $\text{arctg}\ (-\frac{5}{12}) = x$, $x \in (-\frac{\pi}{2}; \frac{\pi}{2})$, $-\frac{5}{12} = \text{tg}\ x$,

$25\cos^2 x = \frac{144}{160}$, $\cos x = \pm\frac{12}{13} \Rightarrow \cos(\text{arctg}\ (-\frac{5}{12})) = \frac{12}{13}$.

§ 20. Тригонометрические уравнения

349. а) $2\cos x + \sqrt{3} = 0$, $\cos x = -\frac{\sqrt{3}}{2}$, $x = \pm\frac{5\pi}{6} + 2\pi n$;

б) $2\sin x - 1 = 0$, $\sin x = \frac{1}{2}$, $x = (-1)^n\frac{\pi}{6} + \pi k$;

в) $2\cos x - 1 = 0$, $\cos x = \frac{1}{2}$, $x = \pm\frac{\pi}{3} + 2\pi n$;

г) $2\sin x + \sqrt{2} = 0$, $\sin x = -\frac{\sqrt{2}}{2}$, $x = (-1)^{k+1} + \frac{\pi}{4} + \pi k$.

350. а) $\text{tg}\ x + \sqrt{3} = 0$, $\text{tg}\ x = -\sqrt{3}$, $x = -\frac{\pi}{3} + \pi n$;

б) $\sqrt{3}\ \text{tg}\ x - 1 = 0$, $\text{tg}\ x = \frac{\sqrt{2}}{2}$, $x = \frac{\pi}{6} + \pi n$;

в) $\text{ctg}\ x + 1 = 0$, $\text{ctg}\ x = -1$, $x = -\frac{\pi}{4} + \pi n$;

г) $\sqrt{3}\ \text{ctg}\ x - 1 = 0$, $\text{ctg}\ x = \frac{\sqrt{3}}{3}$, $x = \frac{\pi}{3} + \pi n$.

351. а) $\sin 2x = \frac{\sqrt{2}}{2}$, $2x = (-1)^k \frac{\pi}{4} + \pi k$, $x = (-1)^k \frac{\pi}{8} + \frac{\pi k}{2}$;

б) $\cos\left(\frac{x}{3}\right) = -\frac{1}{2}$, $\frac{x}{3} = \pm\frac{2\pi}{3} + 2\pi n$, $x = \pm 2\pi + 6\pi n$;

в) $\sin\frac{x}{4} = \frac{1}{2}$, $\frac{x}{4} = (-1)^k \frac{\pi}{6} + \pi k$, $x = (-1)^k \frac{2\pi}{3} + 4\pi k$;

г) $\cos 4x = 0$, $4x = \frac{\pi}{2} + \pi n$, $x = \frac{\pi}{8} + \frac{\pi n}{4}$.

352. а) $\sin\left(-\frac{x}{3}\right) = \frac{\sqrt{2}}{2}$, $\sin\frac{x}{3} = -\frac{\sqrt{2}}{2}$, $\frac{x}{3} = (-1)^{k+1}\frac{\pi}{4} + \pi k$,

$x = (-1)^{k+1}\frac{3\pi}{4} + 3\pi k$;

б) $\cos(-2x) = -\frac{\sqrt{3}}{2}$, $2x = \pm\frac{5\pi}{6} + 2\pi n$, $x = \pm\frac{5\pi}{12} + \pi n$;

в) $\mathrm{tg}(-4x) = \frac{\sqrt{3}}{3}$, $\mathrm{tg}\, 4x = -\frac{\sqrt{3}}{3}$, $4x = -\frac{\pi}{6} + \pi n$, $x = -\frac{\pi}{24} + \frac{\pi n}{4}$;

г) $\mathrm{ctg}\left(-\frac{x}{2}\right) = 1$, $\mathrm{ctg}\frac{x}{2} = -1$, $\frac{x}{2} = -\frac{\pi}{4} + \pi n$, $x = -\frac{\pi}{2} + 2\pi n$.

353. а) $2\cos\left(\frac{x}{2} - \frac{\pi}{6}\right) = \sqrt{3}$, $\frac{x}{2} - \frac{\pi}{6} = \pm\frac{\pi}{6} + 2\pi n$, $x = \pm\frac{\pi}{3} + \frac{\pi}{3} + 4\pi n$;

б) $\sqrt{3}\,\mathrm{tg}\left(\frac{x}{3} + \frac{\pi}{6}\right) = 3$, $\mathrm{tg}\left(\frac{x}{3} + \frac{\pi}{6}\right) = \sqrt{3}$, $\frac{x}{3} + \frac{\pi}{6} = \frac{\pi}{3} + \pi n$, $x = \frac{\pi}{2} + 3\pi n$;

в) $2\sin\left(3x - \frac{\pi}{4}\right) = -\sqrt{2}$, $3x - \frac{\pi}{4} = (-1)^{k+1}\frac{\pi}{4} + \pi k$,

$x = (-1)^{k+1}\frac{\pi}{12} + \frac{\pi}{12} + \frac{\pi k}{3}$;

г) $\sin\left(\frac{x}{2} - \frac{\pi}{6}\right) + 1 = 0$, $\frac{x}{2} - \frac{\pi}{6} = -\frac{\pi}{2} + 2\pi n$, $x = -\frac{2\pi}{3} + 4\pi n$.

354. а) $\cos\left(\frac{\pi}{6} - 2x\right) = -1$, $2x - \frac{\pi}{6} = \pi + 2\pi n$, $x = \frac{\pi}{12} + \pi n$;

б) $\mathrm{tg}\left(\frac{\pi}{4} - \frac{x}{2}\right) = -1$, $\frac{x}{2} - \frac{\pi}{4} = \frac{\pi}{4} + \pi n$, $x = \pi + 2\pi n$;

в) $2\sin\left(\frac{\pi}{3} - \frac{x}{4}\right) = \sqrt{3}$, $\frac{x}{4} - \frac{\pi}{3} = (-1)^{k+1}\frac{\pi}{3} + \pi n$,

$x = (-1)^{k+1}\frac{4\pi}{3} + \frac{4\pi}{3} + 4\pi k$;

г) $2\cos\left(\frac{\pi}{4} - 3x\right) = \sqrt{2}$, $3x - \frac{\pi}{4} = \pm\frac{\pi}{4} + 2\pi n$, $x = \pm\frac{\pi}{12} + \frac{\pi}{12} + \frac{2\pi n}{3}$.

355. а) $3\sin^2 x - 5\sin x - 2 = 0$, $\sin x = \frac{5+7}{6}$ не подходит.

$\sin x = -\frac{1}{3}$, $x = (-1)^{k+1}\arcsin\frac{1}{3} + \pi n$;

б) $3\sin^2 2x + 10\sin 2x + 3 = 0$, $\sin 2x = \frac{-10 \pm \sqrt{100 - 4\cdot 3\cdot 3}}{6} = \frac{-5 \pm 4}{3}$,

$\sin 2x = \frac{-5-4}{3}$ не подходит; $\sin 2x = -\frac{1}{3}$, $x = (-1)^{k+1}\frac{\arcsin\frac{1}{3}}{2} + \frac{\pi k}{2}$;

в) $4\sin^2 x + 11\sin x - 3 = 0$, $\sin x = \frac{-11 \pm \sqrt{121 + 4\cdot 4\cdot 3}}{8} = \frac{-11 \pm 13}{8}$,

$\sin x = \frac{-11-13}{8}$ не подходит; $\sin x = \frac{1}{4}$, $x = (-1)^{k+1}\arcsin\frac{1}{4} + \pi n$;

г) $2\sin^2\frac{x}{2} - 3\sin\frac{x}{2} + 1 = 0$, $\sin\frac{x}{2} = \frac{-3 \pm \sqrt{9 - 4\cdot 2\cdot 1}}{4} = \frac{3 \pm 1}{4}$,

$\sin\frac{x}{2} = \frac{3+1}{4} = 1$, $\frac{x}{2} = \frac{\pi}{2} + 2\pi n$, $x = \pi + 4\pi n$, $\sin\frac{x}{2} = \frac{1}{2}$,

$\frac{x}{2} = (-1)^k\frac{\pi}{6} + \pi k$, $x = (-1)^k\frac{\pi}{3} + 2\pi k$.

356. а) $6\cos^2 x + \cos x - 1 = 0$, $\cos x = \frac{-1 \pm \sqrt{1 + 4\cdot 6\cdot 1}}{12} = \frac{-1 \pm 5}{12}$,

$\cos x = -\frac{1}{2}$; $x = \pm\frac{2}{3}\pi + 2\pi n$, $\cos x = \frac{1}{3}$, $x = \pm\arccos\frac{1}{3} + 2\pi n$;

б) $2\cos^2 3x - 5\cos 3x - 3 = 0$, $\cos 3x = \frac{5+7}{4}$ не подходит,

$\cos 3x = -\frac{1}{2}$, $3x = \pm\frac{2\pi}{3} + 2\pi n$, $x = \pm\frac{2\pi}{9} + \frac{2\pi n}{3}$.

в) $2\cos^2 x - \cos x - 3 = 0$, $\cos x = \frac{1+5}{4}$ не подходит, $\cos x = -1$, $x = \pi + 2\pi n$;

г) $2\cos^2\frac{x}{3} + 3\cos\frac{x}{3} - 2 = 0$, $\cos\frac{x}{3} = \frac{-3-5}{4}$ не подходит,

$\cos\frac{x}{3} = \frac{1}{2}$, $\frac{x}{3} = \pm\frac{\pi}{3} + 2\pi n$, $x = \pm\pi + 6\pi n$.

357. а) $2\sin^2 x + 3\cos x = 0$, $2 - 2\cos^2 x - 3\cos x - 2 = 0$,

$\cos x = \frac{3+5}{4}$ не подходит, $\cos x = -\frac{1}{2}$, $x = \pm\frac{2\pi}{3} + 2\pi n$;

б) $8\sin^2 2x + \cos 2x + 1 = 0$, $8 - 8\cos^2 x + \cos 2x + 1 = 0$,

$8\cos^2 x - \cos 2x - 9 = 0$;

$\cos 2x = \frac{1+17}{16}$ не подходит, $\cos 2x = -1$, $2x=\pi+2\pi n$, $x = \frac{\pi}{2} + \pi n$;

в) $5\cos^2 x + 6\sin x - 6 = 0$, $5 - 5\sin^2 x + 6\sin x - 6 = 0$,

$5\sin^2 x - 6\sin x + 1 = 0$, $\sin x\ \frac{6 \pm \sqrt{36 - 4\cdot 5\cdot 1}}{10} = \frac{3 \pm 2}{5}$,

$\sin x = 1$, $x = \frac{\pi}{2} + 2\pi n$, $\sin x = \frac{1}{5}$, $x = (-1)^n \arcsin\frac{1}{5} + \pi n$;

г) $4\sin 3x + \cos^2 3x = 4$, $\sin^2 3x - 4\sin 3x + 3 = 0$, $\sin 3x = 3$ не подходит.

$\sin 3x = 1$, $3x = \frac{\pi}{2} + 2\pi n$, $x = \frac{\pi}{6} + \frac{2\pi n}{3}$.

358. а) $3\,\mathrm{tg}^2 x + 2\,\mathrm{tg}\, x - 1 = 0$, $\mathrm{tg}\, x = \frac{-1+2}{3} = \frac{1}{3}$, $x = \mathrm{arctg}\frac{1}{3} + \pi n$,

$\mathrm{tg}\, x = -1$, $x = -\frac{\pi}{4} + \pi n$;

б) $\mathrm{ctg}^2\, 2x - 6\,\mathrm{ctg}\, 2x + 5 = 0$, $\mathrm{ctg}\, 2x = 5$, $2x = \mathrm{arcctg}\, 5 + \pi n$,

$x = \frac{\mathrm{arctg}\ 5}{2} + \frac{\pi n}{2}$, $\mathrm{ctg}\, 2x = 1$, $2x = \frac{\pi}{4} + \pi n$, $x = \frac{\pi}{8} + \frac{\pi n}{2}$;

в) $2\,\mathrm{tg}^2 x + 3\,\mathrm{tg}\, x - 2 = 0$, $\mathrm{tg}\, x = \frac{-3+5}{4} = \frac{1}{2}$, $x = -\mathrm{arctg}\frac{1}{2} + \pi n$,

$\mathrm{tg}\, x = -2$, $x = -\mathrm{arctg}\, 2 + \pi n$;

г) $7\,\mathrm{ctg}^2\frac{x}{2} + 2\,\mathrm{ctg}\frac{x}{2} = 5$, $7\,\mathrm{ctg}^2\frac{x}{2} + 2\,\mathrm{ctg}\frac{x}{2} - 5 = 0$,

$\mathrm{ctg}\frac{x}{2} = \frac{-1-6}{7} = -1$, $\frac{x}{2} = -\frac{\pi}{4} + \pi n$, $x = -\frac{\pi}{2} + 2\pi n$,

$\mathrm{ctg}\frac{x}{2} = \frac{-1+6}{7} = \frac{5}{7}$, $\frac{x}{2} = \mathrm{arcctg}\frac{5}{7} + \pi n$, $x = 2\,\mathrm{arcctg}\frac{5}{7} + 2\pi n$.

359. а) $(\sin x - \frac{1}{2})(\sin x + 1) = 0$, $\sin x = \frac{1}{2}$, $x = (-1)^k\frac{\pi}{6} + \pi k$,

$\sin x = -1$, $x = -\frac{\pi}{2} + 2\pi n$;

б) $(\cos x + \frac{1}{2})(\cos x - 1) = 0$, $\cos x = -\frac{1}{2}$, $x = \pm\frac{2\pi}{3} + 2\pi n$, $\cos x = 1$, $x = 2\pi n$;

в) $(\cos x - \frac{\sqrt{2}}{2})(\sin x + \frac{\sqrt{2}}{2}) = 0$, $\cos x = \frac{\sqrt{2}}{2}$, $x = \pm\frac{\pi}{4} + 2\pi n$,

$\sin x = -\frac{\sqrt{2}}{2}$, $x = (-1)^{n+1}\frac{\pi}{4} + \pi n$;

г) $(1 + \cos x)\cdot(\sqrt{2}\sin x - 1) = 0$, $\cos x = -1$, $x = \pi + 2\pi n$,

$\sin x = \frac{\sqrt{2}}{2}$, $x = (-1)^n\frac{\pi}{4} + \pi n$.

360. а) $\sin x + \sqrt{3}\cos x = 0$, $\operatorname{tg} x = -\sqrt{3}$, $\cos x \neq 0$, $x = -\frac{\pi}{3} + \pi n$;

б) $\sin x + \cos x = 0$, $\operatorname{tg} x = -1$; $\cos x \neq 0$, $x = -\frac{\pi}{4} + \pi n$;

в) $\sin x - 3\cos x = 0$, $\operatorname{tg} x = 3$, $\cos x \neq 0$, $x = \operatorname{arctg} 3 + \pi n$;

г) $\sqrt{3}\sin x + \cos x = 0$, $\operatorname{tg} x = -\frac{\sqrt{3}}{3}$, $\cos x \neq 0$, $x = -\frac{\pi}{6} + \pi n$.

361. а) $\sin^2 x + \sin x \cos x = 0$, $\sin x(\sin x + \cos x) = 0$, $\sin x = 0$, $x = \pi n$,

$\sin x + \cos x = 0$, $x = -\frac{\pi}{4} + \pi n$;

б) $\sqrt{3}\sin x \cos x + \cos^2 x = 0$, $\cos x(\sqrt{3}\sin x + \cos x) = 0$, $\cos x = 0$,

$x = \frac{\pi}{2} + \pi n$, $\sqrt{3}\sin x + \cos x = 0$, $x = -\frac{\pi}{6} + \pi n$;

в) $\sin^2 x = 3\sin x \cos x$, $\sin x(\sin x - 3\cos x) = 0$, $\sin x = 0$, $x = \pi n$,
$\sin x - 3\cos x = 0$, $x = \operatorname{arctg} 3 + \pi n$;

г) $\sqrt{3}\cos^2 x = \sin x \cos x$, $\cos x(\sqrt{3}\cos x - \sin x) = 0$, $\cos x = 0$,

$x = \frac{\pi}{2} + \pi n$, $\sqrt{3}\cos x - \sin x = 0$, $\operatorname{tg} x = \sqrt{3}$, $x = \frac{\pi}{3} + \pi n$.

362. а) $\sin^2 x + 2\sin x \cos x - 3\cos^2 x = 0$, $\operatorname{tg}^2 x + 2\operatorname{tg} x - 3 = 0$, $\operatorname{tg} x = -3$,

$x = -\operatorname{arctg} 3 + \pi n$, $\operatorname{tg} x = 1$, $x = \frac{\pi}{4} + \pi n$;

б) $\sin^2 x - 4\sin x \cos x + 3\cos^2 x = 0$, $\operatorname{tg}^2 x - 4\operatorname{tg} x + 3 = 0$, $\operatorname{tg} x = 3$,

$x = \operatorname{arctg} 3 + \pi n$, $\operatorname{tg} x = 1$, $x = \frac{\pi}{4} + \pi n$;

в) $\sin^2 x + \sin x \cos x - 2\cos^2 x = 0$, $\operatorname{tg}^2 x + \operatorname{tg} x - 2 = 0$,

$\operatorname{tg} x = \frac{-1 \pm 3}{2}$, $\operatorname{tg} x = 1$; $x = \frac{\pi}{4} + \pi n$, $\operatorname{tg} x = -2$, $x = \operatorname{arctg} 2 + \pi n$;

г) $3\sin^2 x + \sin x\cos x - 2\cos^2 x = 0$, $3\operatorname{tg}^2 x + \operatorname{tg} x - 2 = 0$, $\cos x \neq 0$,

$\operatorname{tg} x = \frac{-1 \pm \sqrt{1 + 4\cdot 3\cdot 2}}{6} = \frac{-1 \pm 5}{6}$, $\operatorname{tg} x = -1$, $x = -\frac{\pi}{4} + \pi n$,

$\operatorname{tg} x = \frac{2}{3}$, $x = \operatorname{arctg}\frac{2}{3} + \pi n$.

363. а) $\sin^2\frac{3x}{4} - \frac{\sqrt{2}}{2} = \sin x - \cos^2\frac{3x}{4} + 1$, $1 - \frac{\sqrt{2}}{2} - 1 = \sin x$,

$\sin x = -\frac{\sqrt{2}}{2}$, $x = (-1)^{k+1}\frac{\pi}{4} + \pi k$;

б) $\cos^2 2x - 1 - \cos x = \frac{\sqrt{3}}{2} - \sin^2 2x$, $\cos x = -\frac{\sqrt{3}}{2}$, $x = \pm\frac{5\pi}{6} + 2\pi n$.

364. а) $\sin x = \frac{1}{2}$, $x \in [0 ; \pi]$; $x = \frac{\pi}{6}$, $\frac{5\pi}{6}$; $\frac{13\pi}{6}$; $\frac{17\pi}{6}$.

б) $\cos x = -\frac{1}{2}$, $x \in [-2\pi; 3\pi]$; $x = -\frac{4\pi}{3}$; $-\frac{2\pi}{3}$; $\frac{2\pi}{3}$; $\frac{4\pi}{3}$; $\frac{8\pi}{3}$.

365. а) $\sin 3x = \frac{\sqrt{2}}{2}$, $x \in [0 ; 2\pi]$; $3x = (-1)^k \frac{\pi}{4} + \pi k$;

$3x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{9\pi}{4}, \frac{11\pi}{4}, \frac{17\pi}{4}, \frac{19\pi}{4}$; $x = \frac{\pi}{12}, \frac{\pi}{4}, \frac{3\pi}{4}, \frac{11\pi}{12}, \frac{17\pi}{12}, \frac{19\pi}{12}$.

б) $\cos 3x = \frac{\sqrt{3}}{2}$, $x \in [-\pi ; \pi]$; $3x = \pm \frac{\pi}{6} + 2\pi n$;

$3x = -\frac{13\pi}{6}; -\frac{11\pi}{6}; -\frac{\pi}{6}; \frac{11\pi}{6}; \frac{13\pi}{6}$; $x = -\frac{13\pi}{18}; -\frac{11\pi}{18}; -\frac{\pi}{18}; \frac{11\pi}{18}; \frac{13\pi}{18}$.

в) $\operatorname{tg} \frac{x}{2} = \frac{\sqrt{3}}{3}$, $x \in [-3\pi; 3\pi]$; $\frac{x}{2} = \frac{\pi}{6} + \pi n$;

$x = \frac{\pi}{3} + 2\pi n$, $x = -\frac{5\pi}{3}\ \frac{\pi}{3}\ \frac{7\pi}{3}$;

г) $\operatorname{ctg} 4x = -1$, $x \in [0; \pi]$; $4x = -\frac{\pi}{4} + \pi n$,

$4x = \frac{3\pi}{4}; \frac{7\pi}{4}; \frac{11\pi}{4}; \frac{15\pi}{4}$; $x = \frac{3\pi}{16}; \frac{7\pi}{16}; \frac{11\pi}{16}; \frac{15\pi}{16}$.

366. а) $\sin 3x = -\frac{1}{2}$, $x \in [-4; 4]$; $3x = (-1)^{k+1} \frac{\pi}{6} + \pi k$;

$x = (-1)^{k+1} \frac{\pi}{18} + \frac{\pi k}{3}$; $x = -\frac{\pi}{6}; -\frac{5\pi}{6}; \frac{7\pi}{6}$.

б) $\cos x = 1$, $x \in [-6; 16]$; $x = 2\pi n$, $x = 0; 2\pi; 4\pi$.

367. а) $\sin \frac{x}{2} = 0$, $x \in [-12; 18]$, $\frac{x}{2} = \pi n$, $x = -2\pi; 0; 2\pi; 4\pi$;

б) $\cos 3x = \frac{-\sqrt{2}}{2}$, $x \in [1; 7]$, $3x = \pm \frac{3\pi}{4} + 2\pi n$,

$x = \pm \frac{\pi}{4} + \frac{2\pi n}{3}$, $x = \frac{11\pi}{12}; \frac{19\pi}{12}; \frac{5}{12}\pi; \frac{13}{12}\pi; \frac{7}{4}\pi$.

368. $\sin (2x - \frac{\pi}{4}) = -1$, $2x - \frac{\pi}{4} = -\frac{\pi}{2} + 2\pi n$, $x = -\frac{\pi}{8} + \pi n$.

а) $x = \frac{7\pi}{8}$; б) $-\frac{\pi}{8}$; $\frac{7\pi}{8}$; в) $-\frac{\pi}{8}$; г) $-\frac{\pi}{8}$.

369. $\cos (\frac{\pi}{3} - 2x) = \frac{1}{2}$, $2x - \frac{\pi}{3} = \pm \frac{\pi}{3} + 2\pi n$, $x = \frac{\pi}{3} + \pi n$, $x = \pi n$.

а) $\frac{\pi}{3}$; б) 0; $\frac{\pi}{3}$; π; $\frac{4\pi}{3}$; в) $-\frac{2\pi}{3}$; г) $-\frac{2\pi}{3}$; 0; $\frac{\pi}{3}$.

370. а) $\sqrt{16-x^2}\sin x = 0$, $|x| \le 4$, x = 4, x = – 4,
sin x = 0, x = πn, n =0, ± 1.
Ответ: x = ± 4; x = πn, n =0, ± 1…

б) $\sqrt{7x-x^2}(2\cos x - 1) = 0$, 0≤ x ≤7, $7x - x^2 = 0$, x = 0, x = 7,

2cos x – 1 = 0, cos x = $\frac{1}{2}$, x = $\frac{\pi}{3}$, x = $\frac{5\pi}{3}$.

Ответ: $x = 0; \frac{\pi}{3}; \frac{5\pi}{3}; 7$.

371. а) $(\sqrt{2}\cos x - 1)\cdot\sqrt{4x^2 - 7x + 3} = 0$, $4x^2 - 7x + 3 \ge 0$,

$x \ge \frac{7+1}{8} = 1$, $x \le \frac{3}{4}$, $\sqrt{2}\cos x - 1 = 0$, $x = \pm\frac{\pi}{4} + 2\pi n$.

Ответ: x = 1, x = $\frac{3}{4}$, x = $-\frac{\pi}{4}$; x = ± $\frac{\pi}{4}$ + 2πn. n = ± 1; ± 2; ± 3…

б) $(2\sin x - \sqrt{3})\sqrt{3x^2 - 7x + 4} = 0$; $3x^2 - 7x + 4 \ge 0$; x≤1; x≥$\frac{4}{3}$

2sinx- $\sqrt{3} = 0$, sinx = $\frac{\sqrt{3}}{2}$, $x = \frac{2\pi}{3} + 2\pi k; x = \frac{\pi}{3} + 2\pi k; k = \pm1, \pm2, \ldots$;

$3x^2 - 7x + 4 = 0$ x = 1; x = $\frac{4}{3}$.

Ответ: 1; $\frac{4}{3}; \frac{2\pi}{3} + 2\pi k; \frac{\pi}{3} + 2\pi k; k = \pm1; \pm2; \ldots$

372. а) tg x – 2 ctg x + 1 = 0, $tg^2x + tg\,x - 2 = 0$,

tg x = – 2, x = – arctg 2 + πn, tg x = 1, x = $\frac{\pi}{4}$ + πn;

б) $\frac{tg\,x + 5}{2} = \frac{1}{\cos^2 x}$, $2\,tg^2x - tg\,x - 3 = 0$,

tg x = $\frac{1+5}{4} = \frac{3}{2}$, x = arctg $\frac{3}{2}$ + πk, tg x = – 1; x = – $\frac{\pi}{4}$ + πk;

в) 2 ctg x –3 tg x + 5 = 0, $2ctg^2x + 5\,ctg\,x - 3 = 0$, ctg x = $\frac{-5+7}{4} = \frac{1}{2}$,

x = arcctg $\frac{1}{2}$ + πn, ctg x = – 3, x = – arcctg 3 + πn;

г) $\frac{7 - ctg\,x}{4} = \frac{1}{\sin^2 x}$, $7 - ctg\,x = 4\,ctg^2 x + 4$, $4\,ctg^2 x + ctg\,x - 3 = 0$,

ctg x = $\frac{-1+7}{8} = \frac{3}{4}$, x = arctg $\frac{3}{4}$ + πn, ctg x = – 1, x = – $\frac{\pi}{4}$ + πn.

373. а) $2\cos^2\frac{x}{2}+\sqrt{3}\cos\frac{x}{2}=0$, $\cos\frac{x}{2}\left(2\cos\frac{x}{2}+\sqrt{3}\right)=0$,

$\cos\frac{x}{2}=0$, $\frac{x}{2}=\frac{\pi}{2}+\pi k$, $x=\pi+2\pi k$,

$\cos=\frac{\sqrt{3}}{2}$, $\frac{x}{2}=\pm\frac{5\pi}{6}+2\pi n$, $x=\pm\frac{5}{3}\pi+4\pi n$;

б) $4\cos^2\left(x-\frac{\pi}{6}\right)-3=0$, $\cos\left(x-\frac{\pi}{6}\right)=\pm\frac{\sqrt{3}}{2}$,

$x-\frac{\pi}{6}=\pm\frac{\pi}{6}+2\pi n$, $x=\frac{\pi}{3}+2\pi n$, $x=2\pi n$,

$x-\frac{\pi}{6}=\pm\frac{5\pi}{6}+2\pi n$, $x=\pi+2\pi n$, $x=-\frac{2}{3}\pi+2\pi n$, $x=\frac{\pi}{3}+\pi n$, $x=\pi n$;

в) $\sqrt{3}\,\mathrm{tg}^2\,3x-3\,\mathrm{tg}\,3x=0$, $\mathrm{tg}\,3x\left(\sqrt{3}\,\mathrm{tg}\,3x-3\right)=0$, $\mathrm{tg}\,3x=0$,

$3x=\pi n$, $x=\frac{\pi n}{3}$, $\mathrm{tg}\,3x=\sqrt{3}$, $3x=\frac{\pi}{3}+\pi n$, $x=\frac{\pi}{9}+\frac{\pi n}{3}$;

г) $4\sin^2\left(2x+\frac{\pi}{3}\right)=-1=0$, $\sin\left(2x+\frac{\pi}{3}\right)=\pm\frac{1}{2}$,

$\left(2x+\frac{\pi}{3}\right)=(-1)^n\frac{\pi}{6}+\pi n$, $x=(-1)^n\frac{\pi}{12}-\frac{\pi}{6}+\frac{\pi n}{2}$,

$2x+\frac{\pi}{3}=(-1)^{n+1}\frac{\pi}{6}+\pi n$, $x=(-1)^n\frac{\pi}{12}-\frac{\pi}{6}+\frac{\pi n}{2}$.

374. а) $\sin^2 x-\frac{12-\sqrt{2}}{2}\cdot\sin x-3\sqrt{2}=0$, $\sin x=6$, решений нет.

$\sin x=-\frac{\sqrt{2}}{2}$, $x=(-1)^{k+1}\frac{\pi}{4}+\pi k$;

б) $\cos^2 x-\frac{8-\sqrt{3}}{2}\cos x-2\sqrt{3}=0$, $\cos x=4$, не подходит.

$\cos x=-\frac{\sqrt{2}}{2}$, $x=\pm\frac{5\pi}{6}+2\pi n$.

375. а) $\sin 2x=\cos 2x$, $\mathrm{tg}\,2x=1$, $\cos 2x\neq 0$, $2x=\frac{\pi}{4}+\pi n$, $x=\frac{\pi}{8}+\frac{\pi n}{2}$;

б) $\sqrt{3}\sin 3x=\cos 3x$, $\mathrm{ctg}\,3x=\sqrt{3}$, $\sin 3x\neq 0$, $3x=\frac{\pi}{6}+\pi n$, $x=\frac{\pi}{18}+\frac{\pi n}{3}$;

в) $\sin\frac{x}{2}=\sqrt{3}\cos\frac{x}{2}$, $\mathrm{tg}\frac{x}{2}=\sqrt{3}$, $\cos\frac{x}{2}\neq 0$, $\frac{x}{2}=\frac{\pi}{3}+\pi n$, $x=\frac{2}{3}\pi+2\pi n$;

г) $\sqrt{3}\sin 17x=\sqrt{6}\cos 17x$, $\mathrm{tg}\,17x=\sqrt{3}$, $\cos 17x\neq 0$, $17x=\frac{\pi}{3}+\pi n$, $x=\frac{\pi}{51}+\frac{\pi n}{17}$.

376. а) $2\sin^2 2x - 5\sin 2x\cos 2x + \cos^2 2x = 0$,
$2\,tg^2 2x - 5\,tg\,2x + 1 = 0$, $\cos 2x \neq 0$,

$tg\,2x = \frac{5-\sqrt{17}}{4}$, $x = \frac{1}{2}\,arctg\,\frac{5-\sqrt{17}}{4} + \frac{\pi n}{2}$, $x = \frac{1}{2}\,arctg\,\frac{5+\sqrt{17}}{4} + \frac{\pi n}{2}$;

б) $3\sin^2 3x + 10\sin 3x\cos 3x + 3\cos^2 3x = 0$, $2\sin x\cos 3x = -\frac{3}{5}$,

$3tg^2 3x + 10tg\,3x + 3 = 0$, $tg\,3x = \frac{-10\pm\sqrt{100-4\cdot3\cdot3}}{6} = \frac{-10\pm8}{6}$,

$tg\,3x = -3$; $x = \frac{1}{3}\,arctg(-3) + \frac{1}{3}\pi n$, $tg\,3x = -\frac{1}{3}$; $x = \frac{1}{3}\,arctg\,(-\frac{1}{3}) + \frac{1}{3}\pi n$,

$6x = (-1)^{k+1}\frac{1}{6}\arcsin\frac{3}{5} + \frac{\pi k}{6}$.

377. а) $\sin^2\frac{x}{2} = 3\cos^2\frac{x}{2}$, $\cos^2\frac{x}{2} = \frac{1}{4}$, $\cos\frac{x}{2} = \pm\frac{1}{2}$,

$x = \pm\frac{2\pi}{3} + 4\pi n$, $x = \pm\frac{4\pi}{3} + 4\pi n$;

б) $\sin^2 4x = \cos^2 4x$, $tg^2 4x = 1$, $\cos 4x \neq 0$, $tg\,4x = \pm 1$,

$4x = \frac{\pi}{4} + \pi n$, $x = \frac{\pi}{16} + \frac{\pi n}{4}$, $4x = -\frac{\pi}{4} + \pi n$, $x = -\frac{\pi}{16} + \pi n$.

378. а) $5\sin^2 x - 14\sin x\cos x - 3\cos^2 x = 2$,
$3\sin^2 x - 14\sin x\cos x - 5\cos^2 x = 0$, $3tg^2 x - 14tg\,x - 5 = 0$, $\cos x \neq 0$,

$tg^2 x = \frac{7+8}{3} = 5$, $x = arctg\,5 + \pi k$, $tg\,x = -\frac{1}{3}$, $x = -\,arctg\,\frac{1}{3} + \pi k$;

б) $3\sin^2 x - \sin x\cos x = 2$, $\sin^2 x - \sin x\cos x - 2\cos^2 x = 0$, $tg^2 x - tg\,x - 2 = 0$,

$\cos x \neq 0$, $tg\,x = 2$, $x = arctg\,2 + \pi n$, $tg\,x = -1$; $x = -\frac{\pi}{4} + \pi n$.

в) $2\cos^2 x - \sin x\cos x + 5\sin^2 x = 3$, $2\sin^2 x - \sin x\cos x - \cos^2 x = 0$,

$2\,tg^2 x - tgx - 1 = 0$, $\cos x \neq 0$, $tg\,x = 1$, $x = \frac{\pi}{4} + \pi k$, $tg\,x = -\frac{1}{2}$, $x = -\,arctg\frac{1}{2} + \pi k$;

г) $4\sin^2 x - 2\sin x\cos x = 3$, $\sin^2 x - 2\sin x\cos x - 3\cos^2 x = 0$,

$tg^2 x - 2\,tgx - 3 = 0$, $\cos x \neq 0$, $tg\,x = -1$, $x = -\frac{\pi}{4} + \pi k$, $tg\,x = 3$, $x = arctg 3 + \pi k$.

379. а) $\sqrt{3}\sin x\cos x + \cos^2 x = 0$, $\cos x(\sqrt{3}\sin x + \cos x) = 0$, $\cos x = 0$,

$x = \frac{\pi}{2} + \pi n$, $\sqrt{3}\sin x + \cos x = 0$, $\sqrt{3}\,tg\,x = -1$, $tg\,x = \frac{\sqrt{3}}{3}$, $x = -\frac{\pi}{6} + \pi n$;

б) $2\sin^2 x - 3\sin x\cos x + 4\cos^2 x = 4$, $3\sin x\cos x = 2 - 2\cos^2 x$,
$3\sin x\cos x = 2\sin^2 x$, $\sin x(3\cos x - 2\sin x) = 0$, $\sin x = 0$, $x = \pi n$,

$3\cos x - 2\sin x = 0$, $tg\,x = \frac{3}{2}$, $\cos x \neq 0$, $x = arctg\,\frac{3}{2} + \pi n$.

380. а) $3\sin^2 2x - 2 = \sin 2x\cos 2x$, $\sin^2 2x - \sin 2x\cos 2x - 2\cos^2 2x = 0$,
$tg^2 2x - tg\, 2x - 2 = 0$, $\cos^2 2x \neq 0$, $tg\, 2x = 2$, $2x = arctg\, 2 + \pi n$,

$x = \frac{1}{2} arctg\, 2 + \frac{\pi n}{2}$, $tg\, 2x = -1$, $2x = -\frac{\pi}{4} + \pi n$, $x = -\frac{\pi}{2} + \frac{\pi n}{2}$;

б) $2\sin^2 4x - 4 = 3\sin 4x\cos 4x - 4\cos^2 4x = 0$, $2+2\cos^2 4x - 4 = 3\sin 4x\cos 4x$,
$2\sin^2 4x + 3\sin 4x\cos 4x = 0$, $\sin 4x(2\sin 4x + 3\cos 4x) = 0$, $\sin 4x = 0$,

$4x = \pi n$, $x = \frac{\pi n}{4}$, $2\sin 4x + 3\cos 4x = 0$, $2tg\, 4x = -3$, $\cos 4x \neq 0$,

$x = -\frac{1}{4} arctg\frac{3}{2} + \frac{\pi n}{4}$.

381. а) $\sin^2\frac{x}{2} - 3 = 2\sin\frac{x}{2}\cos\frac{x}{2}$, $2\sin^2\frac{x}{2} + 2\sin\frac{x}{2}\cos\frac{x}{2} + 3\cos^2\frac{x}{2} = 0$,

$2\, tg^2\frac{x}{2} + 2\, tg\frac{x}{2} + 3 = 0$, $\cos\frac{x}{2} \neq 0$, решений нет;

б) $3\sin^2\frac{x}{3} + 4\cos^2\frac{x}{3} = 3 + \sqrt{3}\sin\frac{x}{3}\cos\frac{x}{3}$, $\cos^2\frac{x}{3} - 3\sqrt{3}\sin\frac{x}{3}\cos\frac{x}{3} = 0$,

$\cos\frac{x}{3}(\cos\frac{x}{3} - \sqrt{3}\sin\frac{x}{3}) = 0$, $\cos\frac{x}{3} = 0$, $\frac{x}{3} = \frac{\pi}{2} + \pi n$, $x = \frac{3\pi}{2} + 3\pi n$,

$\cos\frac{x}{3} - \sqrt{3}\sin\frac{x}{3} = 0$, $ctg\frac{x}{3} = \sqrt{3}$, $\sin\frac{x}{3} \neq 0$,

$\frac{x}{3} = \frac{\pi}{6} + \pi n$, $x = \frac{\pi}{2} + 3\pi n$.

382. а) $\sin(\frac{\pi}{2} + 2x) + \cos(\frac{\pi}{2} - 2x) = 0$, $\cos 2x + \sin 2x = 0$,

$tg\, 2x = -1$, $\cos 2x \neq 0$, $2x = -\frac{\pi}{4} + \pi n$, $x = -\frac{\pi}{8} + \frac{\pi n}{2}$;

б) $2\sin(\pi - 3x) + \cos(2\pi - 3x) = 0$, $2\sin 3x + \cos 3x = 0$,

$tg\, 3x = -\frac{1}{2}$, $\cos 3x \neq 0$, $3x = -arctg\frac{1}{2} + \pi n$, $x = -\frac{1}{3} arctg\frac{1}{2} + \frac{\pi n}{3}$.

383. а) $\cos(\frac{\pi}{2} - \frac{x}{2}) - 3\cos(\pi - \frac{x}{2}) = 0$, $\sin\frac{x}{2} + 3\cos\frac{x}{2} = 0$,

$tg\frac{x}{2} = -3$, $\cos\frac{x}{2} \neq 0$, $\frac{x}{2} = -arctg\, 3x + \pi n$, $x = -2\, arctg\, 3 + 2\pi n$;

б) $\sqrt{3}\sin(\pi - \frac{x}{3}) + 3\sin(\frac{\pi}{2} - \frac{x}{3}) = 0$, $\sqrt{3}\sin\frac{x}{3} + 3\cos\frac{x}{3} = 0$,

$tg\frac{x}{3} = -\sqrt{3}$, $\cos\frac{x}{3} \neq 0$, $x = -\pi + 3\pi n$.

384. а) $|\sin x| = |\cos x|$, $\sin x = \pm\cos x$, $tgx = \pm 1$, $\cos x \neq 0$, $x = \pm\frac{\pi}{4} + \pi n$.

б) $|\sin 2x| = |\sqrt{3}\cos 2x|$, $\sin 2x = \pm\sqrt{3}\cos 2x$, $\text{tg}\, 2x = \pm\sqrt{3}$, $\cos 2x \neq 0$,

$2x = \pm\frac{\pi}{3} + \pi n$, $x = \pm\frac{\pi}{6} + \frac{\pi n}{2}$.

385. а) $\sin(2x - \frac{\pi}{6}) + \cos(\frac{13\pi}{6} - 2x) = 0$, $\sin(2x - \frac{\pi}{6}) + \cos(2x - \frac{\pi}{6}) = 0$,

$\text{tg}(2x - \frac{\pi}{6}) = -1$, $\cos(2x - \frac{\pi}{6}) \neq 0$, $2x - \frac{\pi}{6} = -\frac{\pi}{4} + \pi n$, $x = -\frac{\pi}{24} + \frac{\pi n}{2}$;

б) $\sin(\frac{x}{2} + \frac{\pi}{3}) = \sqrt{3}\cos(\frac{47\pi}{3} - \frac{x}{2})$, $\sin(\frac{x}{2} + \frac{\pi}{3}) = \sqrt{3}\cos(\frac{x}{2} + \frac{\pi}{3})$,

$\text{tg}(\frac{x}{2} + \frac{\pi}{3}) = \sqrt{3}$, $\cos(\frac{x}{2} + \frac{\pi}{3}) \neq 0$, $\frac{x}{2} + \frac{\pi}{3} = \frac{\pi}{3} + \pi n$; $x = 2\pi n$, $x = 2\pi n$.

386. а) $\sin^2 x - 5\cos x = \sin x\cos x - 5\sin x$, $\sin x(\sin x + 5) - \cos x(\sin x + 5) = 0$,

$(\sin x + 5)(\sin x - \cos x) = 0$, $\sin x - \cos x = 0$, $x = \frac{\pi}{4} + \pi n$;

б) $\cos^2 x - 7\sin x + \sin x\cos x = 7\cos x$, $\cos x(\cos x - 7) + \sin x(\cos x - 7) = 0$,

$(\cos x - 7)(\cos x + \sin x) = 0$, $\cos x + \sin x = 0$, $x = -\frac{\pi}{4} + \pi n$.

387. а) $\sin^2 x + \cos(\frac{\pi}{2} - x)\sin(\frac{\pi}{2} - x) - 2\cos^2 x$, $\text{tg}^2 x + \text{tg}\, x - 2 = 0$, $\cos x \neq 0$,

$\text{tg}\, x = -2$, $x = -\text{arctg}\, 2 + \pi n$, $\text{tg}\, x = 1$, $x = \frac{\pi}{4} + \pi n$;

б) $\sin^2 3x + 3\cos^2 3x - 4\sin\left(\frac{\pi}{2} + 3x\right)\cos\left(\frac{\pi}{2} + 3x\right) = 0$,

$\sin^2 3x + 3\cos^2 3x + 4\sin 3x\cos 3x = 0$, $\text{tg}^2 3x + 4\,\text{tg}\, 3x + 3 = 0$, $\cos x \neq 0$,

$\text{tg}\, 3x = -1$, $x = -\frac{\pi}{12} + \frac{\pi k}{3}$, $\text{tg}\, 3x = -3$, $x = -\frac{1}{3}arctg 3 + \frac{\pi k}{3}$.

Ответ: $-\frac{\pi}{12} + \frac{\pi k}{3}; -\frac{1}{3}arctg 3 + \frac{\pi k}{3}$.

в) $\sin^2 x + 2\sin(\pi - x)\cos x - 3\cos^2(2\pi - x) = 0$, $\text{tg}^2 x + 2\,\text{tg}\, x - 3 = 0$,

$\cos x \neq 0$, $\text{tg}\, x = -3$, $x = -\text{arctg}\, 3 + \pi n$, $\text{tg}\, x = 1$, $x = \frac{\pi}{4} + \pi n$;

г) $\sin^2(2\pi - 3x) + 5\sin(\pi - 3x)\cos 3x + 4\sin^2(\frac{3\pi}{2} - 3x) = 0$,

$\text{tg}^2 3x + 5\,\text{tg}\, 3x + 4 = 0$, $\text{tg}\, 3x = -4$, $3x = -\text{arctg}\, 4 + \pi n$, $x = -\frac{1}{3}\text{arctg} 4 + \frac{\pi n}{3}$,

$\text{tg}\, x = -1$, $3x = -\frac{\pi}{4} + \pi n$, $x = -\frac{\pi}{12} + \frac{\pi n}{3}$.

388. а) $3\sin^2\frac{x}{2} + \sin\frac{x}{2}\sin(\frac{\pi}{2} - \frac{x}{2}) = 2$,

$\sin^2 \frac{x}{2} + \sin \frac{x}{2} \cos \frac{x}{2} - 2\cos^2 x = 0$, $\operatorname{tg}^2 \frac{x}{2} + \operatorname{tg} \frac{x}{2} - 2 = 0$, $\cos \frac{x}{2} \neq 0$,

$\operatorname{tg} \frac{x}{2} = -2$, $\frac{x}{2} = -\operatorname{arctg} 2 + \pi n$, $x = -2 \operatorname{arctg} 2 + 2\pi n$, $\operatorname{tg} \frac{x}{2} = 1$,

$\frac{x}{2} = \frac{\pi}{4} + \pi n$, $x = \frac{\pi}{2} + 2\pi n$;

б) $2\cos^2 \frac{x}{2} - 3 \sin (\pi - \frac{x}{2}) \cos (2\pi - \frac{x}{2}) + 7 \sin^2 \frac{x}{2} = 3$,

$4 \sin^2 \frac{x}{2} - 3 \sin \frac{x}{2} \cos \frac{x}{2} - \cos^2 \frac{x}{2} = 0$, $4 \operatorname{tg}^2 \frac{x}{2} - 3 \operatorname{tg} \frac{x}{2} - 1 = 0$,

$\cos x \neq 0$, $\operatorname{tg} \frac{x}{2} = \frac{3+5}{8} = 1$, $\frac{x}{2} = \frac{\pi}{4} + \pi n$, $x = \frac{\pi}{2} + 2\pi n$, $\operatorname{tg} \frac{x}{2} = -\frac{1}{4}$,

$\frac{x}{2} = -\operatorname{arctg} \frac{1}{4} + \pi n$, $x = -2 \operatorname{arctg} \frac{1}{4} + 2\pi n$;

в) $4\cos^2 (\frac{\pi}{2} + x) + \sqrt{3} \sin (\frac{3\pi}{2} - x) \sin (\pi + x) + 3\cos^2 (\pi + x) = 3$,

$\sin^2 x + \sqrt{3} \sin x \cos x = 0$; $\sin x(\sin x + \sqrt{3} \cos x) = 0$, $\sin x = 0$, $x = \pi n$,

$\sin x + \sqrt{3} \cos x = 0$, $\operatorname{tg} x = -\sqrt{3}$, $\cos x \neq 0$, $x = -\frac{\pi}{3} + \pi n$;

г) $3 \sin^2 (x - \frac{3\pi}{2}) - 2 \cos (\frac{3\pi}{2} + x) \cos (\pi + x) + 2 \sin^2 (x - \pi) = 2$,

$\cos^2 x + 2 \sin x \cos x = 0$, $\cos x (\cos x + 2 \sin x) = 0$, $\cos x = 0$,

$x = \frac{\pi}{2} + \pi n$, $\cos x + 2 \sin x = 0$, $\operatorname{tg} x = -\frac{1}{2}$, $\cos x \neq 0$, $x = -\operatorname{arctg} \frac{1}{2} + \pi n$.

389. а) $2 \sin^2 (\pi + x) - 5 \cos (\frac{\pi}{2} + x) + 2 = 0$, $2 \sin^2 x + 5 \sin x + 2 = 0$,

$\sin x = \frac{-5-3}{4}$ не подходит. $\sin x = -\frac{1}{2}$. $x = (-1)^{k+1} \frac{\pi}{6} + \pi k$;

б) $2\cos^2 x + 5 \cos (\frac{\pi}{2} - x) - 4 = 0$, $2 \sin^2 x - 5 \sin x + 2 = 0$,

$\sin x = \frac{5+3}{4}$ не подходит. $\sin x = \frac{1}{2}$, $x = (-1)^k \frac{\pi}{6} + \pi k$;

в) $2 \cos^2 x + \sin (\frac{\pi}{2} - x) - 1 = 0$, $2 \cos^2 x + \cos x - 1 = 0$,

$\cos x = \frac{-1-3}{4} = -1$, $x = \pi + 2\pi k$, $\cos x = \frac{1}{2}$, $x = \pm \frac{\pi}{3} + 2\pi n$;

г) $5 - 5 \sin (3 (\pi - x)) = \cos^2 (\pi - 3x)$, $5 - 5 \sin (3\pi - 3x) = \cos^2 3x$,
$\sin^2 3x - 5 \sin 3x + 4 = 0$. $\sin 3x = 4$ не подходит. $\sin 3x = 1$.

$3x = \frac{\pi}{2} + 2\pi n$, $x = \frac{\pi}{6} + \frac{2\pi n}{3}$.

390. а) $2\,\text{tg}^2\,2x + 3\,\text{tg}\,(\pi + 2x) = 0$, $2\,\text{tg}^2\,2x + 3\,\text{tg}\,2x = 0$,

$\text{tg}\,2x\,(2\,\text{tg}\,2x + 3) = 0$, $\text{tg}\,2x = 0$, $2x = \pi n$, $x = \frac{\pi n}{2}$, $\text{tg}\,2x = -\frac{3}{2}$,

$2x = -\text{arctg}\,\frac{3}{2} + \pi n$, $x = -\frac{1}{2}\,\text{arctg}\,\frac{3}{2} + \frac{\pi n}{2}$;

б) $\text{tg}^2\,3x - 6\,\text{ctg}\,(\frac{\pi}{2} - 3x) = 0$, $\text{tg}^2\,3x - 6\,\text{tg}\,3x = 0$, $\text{tg}\,3x\,(\text{tg}\,3x - 6) = 0$,

$\text{tg}\,3x = 0$, $x = \frac{\pi n}{3}$, $\text{tg}\,3x = 6$, $x = \frac{1}{3}\,\text{arctg}\,6 + \frac{\pi n}{3}$.

391. а) $3\,\text{tg}^2\,\frac{x}{2} - 2\,\text{ctg}\,(\frac{3\pi}{2} + \frac{x}{2}) - 1 = 0$, $3\,\text{tg}^2\,\frac{x}{2} + 2\,\text{tg}\,\frac{x}{2} - 1 = 0$,

$\text{tg}\,\frac{x}{2} = \frac{-1-2}{3} = -1$, $\frac{x}{2} = -\frac{\pi}{4} + \pi n$, $x = -\frac{\pi}{2} + 2\pi n$, $\text{tg}\,\frac{x}{2} = \frac{1}{3}$, $x = 2\text{arctg}\,\frac{1}{3} + 2\pi n$;

б) $3\,\text{tg}^2\,4x - 2\,\text{ctg}\,(\frac{\pi}{2} - 4x) = 1$, $3\,\text{tg}^2\,4x - 2\,\text{tg}\,4x - 1 = 0$,

$\text{tg}\,4x = 1$, $x = \frac{\pi}{16} + \frac{\pi n}{4}$, $\text{tg}\,4x = -\frac{1}{3}$, $x = -\frac{1}{4}\,\text{arctg}\,\frac{1}{3} + \frac{\pi n}{4}$.

392. а) $\text{tg}\,(\pi + x) + 2\,\text{tg}\,(\frac{\pi}{2} + x) + 1 = 0$, $\text{tg}\,x - 2\,\text{ctg}\,x + 1 = 0$,

$\text{tg}^2x + \text{tg}\,x - 2 = 0$, $\text{tg}\,x = -2$, $x = -\text{arctg}\,2 + \pi n$, $\text{tg}\,x = 1$, $x = \frac{\pi}{4} + \pi n$;

б) $2\,\text{ctg}\,x - 3\,\text{ctg}\,(\frac{\pi}{2} - x) + 5 = 0$, $2\,\text{ctg}\,x - 3\,\text{tg}\,x + 5 = 0$, $3\text{tg}^2\,x - 5\,\text{tg}\,x - 2 = 0$,

$\text{tg}x = \frac{5 \pm \sqrt{25 + 4\cdot 3\cdot 2}}{6} = \frac{5 \pm 7}{6}$, $\text{tg}\,x = 2$, $x = \text{arctg}2 + \pi n$, $\text{tg}\,x = -\frac{1}{3}$, $x = -\text{arctg}\frac{1}{3} + \pi n$.

393. а) $\sin^2 x + \cos^2 2x + \cos^2(\frac{3\pi}{2} + 2x) + 2\cos x\,\text{tg}\,x = 1$,

$\sin^2 x + \cos^2 2x + \sin^2 2x + 2\sin x - 1 = 0$, $\sin^2 x + 2\sin x = 0$, $\sin x\,(\sin x + 2) = 0$,
$\sin x = 0$, $x = \pi n$;

б) $2\cos^2 x - \sin(x - \frac{\pi}{2}) + \text{tg}x\,\text{tg}\,(x + \frac{\pi}{2}) = 0$, $2\cos^2 x + \cos x - 1 = 0$,

$\cos x = \frac{-1-3}{4} = -1$, $x = \pi + 2\pi n$, $\cos x = \frac{1}{2}$, $x = \pm\frac{\pi}{3} + 2\pi n$.

394. а) $\sin 2x < \frac{1}{2}$, $2x \in (-\frac{7\pi}{6} + 2\pi n; \frac{\pi}{6} + 2\pi n)$, $x \in (-\frac{7\pi}{12} + \pi n; \frac{\pi}{12} + \pi n)$.

б) $3\cos 4x < 1$, $\cos 4x < \frac{1}{3}$, $4x \in (\arccos\frac{1}{3} + 2\pi n.; 2\pi - \arccos\frac{1}{3} + 2\pi n)$,

$x \in (\frac{1}{4}\arccos\frac{1}{3} + \frac{\pi n}{2}; \frac{\pi}{2} - \frac{1}{4}\arccos\frac{1}{3} + \frac{\pi n}{2})$;

в) $\cos 3x > \frac{\sqrt{3}}{2}$, $3x \in (-\frac{\pi}{6} + 2\pi n; \frac{\pi}{6} + 2\pi n)$, $x \in (-\frac{\pi}{18} + \frac{2\pi n}{3}; \frac{\pi}{18} + \frac{2\pi n}{3})$;

г) $7 \sin \frac{x}{2} > -1$, $\sin \frac{x}{2} > \frac{-1}{7}$, $\frac{x}{2} \in (-\arcsin \frac{1}{7} + 2\pi n; \arcsin \frac{1}{7} + 2\pi n + \pi)$,

$x \in (-2 \arcsin \frac{1}{7} + 4\pi n; 2 \arcsin \frac{1}{7} + 2\pi + 4\pi n)$.

395. а) $\sin (2x - \frac{\pi}{3}) > \frac{1}{3}$, $2x - \frac{\pi}{3} \in (\arcsin \frac{1}{3} + 2\pi n; \pi - \arcsin \frac{1}{3} + 2\pi n)$,

$x \in (\frac{1}{2}\arcsin \frac{1}{3} + \frac{\pi}{6} + \pi n; \frac{2\pi}{3} - \frac{1}{2}\arcsin \frac{1}{3} + \pi n)$;

б) $\cos (\frac{\pi}{4} - x) < \frac{\sqrt{2}}{2}$, $x - \frac{\pi}{4} \in (\frac{\pi}{4} + 2\pi n; \frac{7\pi}{4} + 2\pi n)$, $x \in (\frac{\pi}{2} + 2\pi n; 2\pi + 2\pi n)$;

в) $\cos (3x - \frac{\pi}{6}) > -\frac{1}{4}$, $3x - \frac{\pi}{6} \in (-\arccos (-\frac{1}{4}) + 2\pi n; \arccos (-\frac{1}{4}) + 2\pi n)$,

$x \in (\frac{\pi}{18} - \frac{1}{3}\arccos (-\frac{1}{4}) + \frac{2\pi n}{3}; \frac{\pi}{18} + \frac{1}{3}\arccos (-\frac{1}{4}) + \frac{2\pi n}{3})$.

г) $\sin (\frac{3\pi}{4} - x) < \frac{\sqrt{3}}{2}$, $\sin (x - \frac{3\pi}{4}) > -\frac{\sqrt{3}}{2}$,

$x - \frac{3\pi}{4} \in (-\frac{\pi}{3} + 2\pi n.; \frac{4\pi}{3} + 2\pi n)$, $x \in (-\frac{\pi}{3} + \frac{3\pi}{4} + 2\pi n; \frac{4\pi}{3} + \frac{3\pi}{4} + 2\pi n)$.

396. а) $\sin^2 x - 6 \sin x \cos x + 5 \cos^2 x > 0$, $\operatorname{tg}^2 x - 6 \operatorname{tg} x + 5 > 0$, $\cos x \neq 0$,

$\operatorname{tg} x < 1$, $\operatorname{tg} x > 5$, $x \in (-\frac{\pi}{2} + \pi n; \frac{\pi}{4} + \pi n)$, $(\operatorname{arctg} 5 + \pi n; \frac{\pi}{2} + \pi n)$;

б) $\sin^2 x - 6 \sin x \cos x + 5 \cos^2 x < 0$, $\operatorname{tg}^2 x - 6 \operatorname{tg} x + 5 < 0$, $\operatorname{tg} x \in (1; 5)$,

$x \in (\frac{\pi}{4} + \pi n; \operatorname{arctg} 5 + \pi n)$.

397. а) $y = \sin x + \sqrt{-\cos^2 x}$, $\cos^2 x \geq 0$, $\cos x = 0$, $x = \frac{\pi}{2} + \pi n$.

Область значений функции: $\{-1, 1\}$.

б) $y = \cos x + \sqrt{-\sin^2 x}$, $\sin^2 x \geq 0$, $\sin x = 0$, $x = \pi n$.

Область значений функции: $\{-1, 1\}$.

398. а) $y = \cos 3x + \sqrt{\cos^2 3x - 1} = \cos 3x + \sqrt{-\sin^2 x}$, $\sin^2 x \geq 0$, sin–3x=0,

$x = \frac{\pi n}{3}$. Область значений функции: $\{-1; 1\}$.

б) $y = \sin 2x + \sqrt{\sin^2 4x - 1} = \sin 2x + \sqrt{-\cos^2 4x}$, $\cos^2 4x \geq 0$, $\cos 4x = 0$,

$x = \frac{\pi}{8} + \frac{\pi n}{4}$, $2x = \frac{\pi}{4} + \frac{\pi n}{2}$. Область значений функции: $\{\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\}$.

Глава 3. Преобразование тригонометрических выражений

§ 21. Синус и косинус суммы аргументов

399. а) sin 105° = sin (60° + 45°) = sin 60° cos 45° + sin 45° cos 60° =

$= \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2}\ \frac{1}{2} = \frac{\sqrt{6}-\sqrt{2}}{4}$;

б) $\cos 105° = \cos (60° + 45°) = \frac{1}{2}\ \frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2}\ \frac{\sqrt{2}}{2} = \frac{\sqrt{2}-\sqrt{6}}{4}$.

400. а) sin (α+β)–sin αcos β=sin αcos β + sin βcos α – sin αcos β=sin β cos α.

б) $\sin (\frac{\pi}{3} + \alpha) - \frac{1}{2}\sin\alpha = \sin\frac{\pi}{3}\cos\alpha + \cos\frac{\pi}{3}\sin\alpha - \frac{1}{2}\sin\alpha = \frac{\sqrt{3}}{2}\cos\alpha$.

в) sin α sin β + cos (α + β) = sin α sin β + cos α cos β–sin α sin β = cos α cos β.

г) $\cos (\alpha + \frac{\pi}{4}) + \frac{\sqrt{2}}{2}\sin\alpha = \frac{\sqrt{2}}{2}\cos\alpha - \frac{\sqrt{2}}{2}\sin\alpha + \frac{\sqrt{2}}{2}\sin\alpha = \frac{\sqrt{2}}{2}\cos\alpha$.

401. а) sin (α + β) + sin (– α) cos (– β) = sin α cos β.
sin α cos β + cos α sin β = sin β cosα. тождество неверно.
б) cos (α + β) + sin (– α) sin (– β) = cos α cos β.
cos (α+β)+sin (–α) sin (–β)=cos α cos β – sin α sin β+ sin α sin β = cos α cos β

402. а) sin 74° cos 16° + cos 74° sin 16° = sin (74° + 16°) = sin 90° = 1.

б) $\cos 23° \cos 22° - \sin 23° \sin 22° = \cos 45° = \frac{\sqrt{2}}{2}$.

в) sin 89° cos 1° + cos 89° sin 1° = sin (89° + 1°) = sin 90° = 1.
г) cos 178° cos 2° – sin 176° sin 2° = cos (178° + 2°) = – 1.

403. а) $\sin\frac{\pi}{5}\cos\frac{\pi}{20} + \cos\frac{\pi}{5}\sin\frac{\pi}{20} = \sin(\frac{\pi}{5} + \frac{\pi}{20}) = \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$.

б) $\cos\frac{2\pi}{7}\cos\frac{5\pi}{7} - \sin\frac{2\pi}{7}\sin\frac{5\pi}{7} = \cos(\frac{2\pi}{7} + \frac{5\pi}{7}) = \cos\pi = -1$.

в) $\sin\frac{\pi}{12}\cos\frac{11\pi}{12} + \cos\frac{\pi}{12}\sin\frac{11\pi}{12} = \sin(\frac{\pi}{12} + \frac{11\pi}{12}) = \sin(\frac{\pi}{12} + \frac{11\pi}{12}) = 0$.

г) $\cos\frac{2\pi}{15}\cos\frac{\pi}{5} - \sin\frac{2\pi}{15}\sin\frac{\pi}{5} = \cos(\frac{2\pi}{15} + \frac{\pi}{5}) = \cos\frac{\pi}{3} = \frac{1}{2}$.

404. а) $\frac{\sqrt{3}}{2}\cos x + \frac{1}{2}\sin x = \sin(\frac{\pi}{3} + x)$. $\sin\frac{\pi}{3}\cos x + \cos\frac{\pi}{3}\sin x = \sin(\frac{\pi}{3} + x)$.

б) $\frac{1}{2}\cos x - \frac{\sqrt{3}}{2}\sin x = \cos(x + \frac{\pi}{3})$.

405. а) sin 5x cos 3x + cos 5x sin 3x = sin 8x, sin (5x + 3x) = sin 8x;
б) cos 5x cos 3x – sin 5x sin 3x = cos 8x, cos (5x + 3x) = cos 8x;

в) $\sin 7x \cos 4x + \cos 7x \sin 4x = \sin 11x$, $\sin(7x + 4x) = \sin 11x$;
г) $\cos 2x \cos 12x - \sin 2x \sin 12x = \cos 14x$.

406. а) $\sin 2x \cos x + \cos 2x \sin x = 1$, $\sin 3x = 1$,

$3x = \frac{\pi}{2} + 2\pi n$, $x = \frac{\pi}{6} + \frac{2\pi n}{3}$;

б) $\cos 3x \cos 5x = \sin 3x \sin 5x$, $\cos 3x \cos 5x - \sin 3x \sin 5x = 0$,

$\cos 8x = 0$, $8x = \frac{\pi}{2} + \pi n$, $x = \frac{\pi}{16} + \frac{\pi n}{8}$.

407. а) $\sin 6x \cos x + \cos 6x \sin x = \frac{1}{2}$, $\sin 7x = \frac{1}{2}$,

$7x = (-1)^k \frac{\pi}{6} + \pi k$, $x = (-1)^k \frac{\pi}{42} + \frac{\pi k}{7}$;

б) $\cos 5x \cos 7x - \sin 5x \sin 7x = -\frac{\sqrt{3}}{2}$, $\cos 12x = -\frac{\sqrt{3}}{2}$,

$12x = \pm \frac{5\pi}{6} + 2\pi n$, $x = \pm \frac{5\pi}{72} + \frac{\pi n}{6}$.

408. $\sin t = \frac{3}{5}$, $0 < t < \frac{\pi}{2}$, $\cos t = \frac{4}{5}$.

а) $\sin(\frac{\pi}{3} + t) = \sin\frac{\pi}{3}\cos t + \sin t \cos\frac{\pi}{3} = \frac{\sqrt{3}}{2} \cdot \frac{4}{5} + \frac{1}{2} \cdot \frac{3}{5} = \frac{2\sqrt{3}}{5} + \frac{3}{10} = \frac{4\sqrt{3}+3}{10}$.

б) $\cos(\frac{\pi}{2} + t) = -\sin t = -\frac{3}{5}$; в) $\sin(\frac{\pi}{2} + t) = \cos t = \frac{4}{5}$;

г) $\cos(\frac{\pi}{3} + t) = \cos\frac{\pi}{3}\cos t - \sin\frac{\pi}{3}\sin t = \frac{1}{2} \cdot \frac{4}{5} - \frac{\sqrt{3}}{2} \cdot \frac{3}{5} = \frac{4-3\sqrt{3}}{10}$.

409. $\cos t = -\frac{5}{13}$, $\frac{\pi}{2} < t < \pi$ $\sin t = \frac{12}{13}$.

а) $\sin(t + \frac{\pi}{6}) = \sin t \frac{\sqrt{3}}{2} + \cos t \frac{1}{2} = \frac{\sqrt{3}}{2} \frac{12}{13} - \frac{5}{13} \frac{1}{2} =$

$= \frac{\sqrt{3}}{2} \frac{12}{13} - \frac{5}{13} \frac{1}{2} = \frac{12\sqrt{3}-5}{26}$;

б) $\cos(t + \frac{3\pi}{2}) = \sin t = \frac{12}{13}$;

в) $\cos(t + \frac{\pi}{6}) = \cos t \cos\frac{\pi}{6} - \sin t \sin\frac{\pi}{6} = -\frac{5}{13} \frac{\sqrt{3}}{2} - \frac{12}{13} \frac{1}{2} = \frac{-5\sqrt{3}-12}{26}$;

г) $\sin(t + \frac{3\pi}{2}) = -\cos t = \frac{5}{13}$.

410. $\cos\alpha = \frac{15}{17}$, $\cos\beta = \frac{4}{5}$, $0 < \alpha < \frac{\pi}{2}$;

$0 < \beta < \frac{\pi}{2}$ $\quad \sin\alpha = \frac{8}{17}$ $\qquad \sin\beta = \frac{3}{5}$.

а) $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \sin\beta\cos\alpha = \frac{8}{17}\cdot\frac{4}{5} + \frac{3}{5}\cdot\frac{15}{17} = \frac{77}{85}$.

б) $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta = \frac{5}{17}\cdot\frac{4}{5} - \frac{8}{17}\cdot\frac{3}{5} = \frac{60-24}{85} = \frac{36}{85}$.

411. $\sin\alpha = \frac{4}{5}$ $\cos\beta = -\frac{15}{17}$ $\qquad \frac{\pi}{2} < \alpha < \pi,\ \frac{\pi}{2} < \beta < \pi$

$\cos\alpha = -\frac{3}{5}$, $\qquad \sin\beta = \frac{8}{17}$.

а) $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \sin\beta\cos\alpha = \frac{4}{5}\cdot(-\frac{15}{17}) + \frac{8}{17}\cdot(-\frac{3}{5}) = -\frac{84}{85}$;

б) $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta = \frac{13}{85}$.

412. $\sin\alpha = \frac{9}{41}$, $\quad \sin = -\frac{40}{41}$, $0 < \alpha < \frac{\pi}{2}$, $\frac{3\pi}{2} < \beta < 2\pi$

$\cos\alpha = \frac{40}{41}$, $\qquad \cos\beta = \frac{9}{41}$.

$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \sin\beta\cos\alpha = \left(\frac{9}{41}\right)^2 - \left(\frac{40}{41}\right)^2 = \frac{9-40}{41}\cdot\frac{9+40}{41} = -\frac{1519}{1681}$;

б) $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta = \frac{40}{41}\ \frac{9}{41} + \frac{9}{41}\ \frac{40}{41} = \frac{720}{1681}$.

413. а) $\sin 75^\circ \cos 75^\circ = \frac{1}{4}$.

$\sin 75^\circ \cos 75^\circ = \frac{1}{2}(\sin 75^\circ\cos 75^\circ + \sin 75^\circ \cos 75^\circ) = \frac{1}{2}\sin 150^\circ = \frac{1}{2}\sin 30^\circ = \frac{1}{4}$;

б) $\cos^2 75^\circ - \sin^2 75^\circ - \sin 75^\circ \sin 75^\circ = \cos(75^\circ + 75^\circ) = \cos 150^\circ = -\cos 30^\circ = -\frac{\sqrt{3}}{2}$;

в) $\sin 105^\circ \cos 105^\circ = -\frac{1}{4}$, $\sin 105^\circ \cos 105^\circ = \frac{1}{2}(\sin 105^\circ \cos 105^\circ +$

$+ \sin 105^\circ \cos 105^\circ) = \sin 210^\circ = -\frac{1}{2}\sin 30^\circ = -\frac{1}{4}$;

г) $\cos^2 75^\circ + \sin^2 75^\circ = 1$.

414. а) $\sin 2x = 2\sin x\cos x$,
$\sin 2x = \sin(x+x) = \sin x\cos x + \sin x\cos x = 2\sin x\cos x$;
б) $\cos 2x = \cos^2 x - \sin^2 x$,
$\cos 2x = \cos(x+x) = \cos x\cos x - \sin x\sin x = \cos^2 x - \sin^2 x$.

415. а) $\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$,
$\sin(\alpha-\beta) = \sin\alpha\cos(-\beta) + \cos\alpha\sin(-\beta) = \sin\alpha\cos\beta - \sin\beta\cos\alpha$;

б) $\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$,
$\cos(\alpha + (-\beta)) = \cos\alpha\cos(-\beta) - \sin\alpha\sin(-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$.

416. а) $\frac{\sqrt{2}}{2}\sin x + \frac{\sqrt{2}}{2}\cos x = 1$, $\sin\frac{\pi}{4}\sin x + \frac{\sqrt{2}}{2}\cos x = 1$,

$\cos(x - \frac{\pi}{4}) = 1$, $x - \frac{\pi}{4} = 2\pi n$, $x = \frac{\pi}{4} + 2\pi n$;

б) $\sin x + \cos x = 1$, $\sin(x + \frac{\pi}{4}) = \frac{1}{\sqrt{2}}$,

$x + \frac{\pi}{4} = (-1)^k\frac{\pi}{4} + \pi n$, $x = (-1)^k\frac{\pi}{4} - \frac{\pi}{4} + \pi n$;

в) $\frac{\sqrt{3}}{2}\cos x - \frac{1}{2}\sin x = 1$, $\cos\frac{\pi}{6}\cos x - \sin\frac{\pi}{6}\sin x = 1$,

$x + \frac{\pi}{6} = 2\pi n$, $x = -\frac{\pi}{6} + 2\pi n$;

г) $\sqrt{3}\cos x - \sin x = 1$, $\frac{\sqrt{3}}{2}\cos x - \frac{1}{2}\sin x = \frac{1}{2}$,

$x + \frac{\pi}{6} = \pm\frac{\pi}{3} + 2\pi n$, $x = \pm\frac{\pi}{3} - \frac{\pi}{6} + 2\pi n$.

417. а) $\sin x\cos 3x + \cos x\sin 3x > \frac{1}{2}$, $\sin 4x > \frac{1}{2}$,

$4x \in (\frac{\pi}{6} + 2\pi n; \frac{5\pi}{6} + 2\pi n)$, $x \in (\frac{\pi}{24} + \frac{\pi n}{2}; \frac{5\pi}{24} + \frac{\pi n}{2})$;

б) $\cos 2x\cos 5x - \sin 2x\sin 5x < -\frac{1}{3}$, $\cos 7x < -\frac{1}{3}$,

$7x \in (\pi - \arccos\frac{1}{3} + 2\pi n; \pi + \arccos\frac{1}{3} + 2\pi n)$,

$x \in (\frac{\pi}{7} - \frac{1}{7}\arccos\frac{1}{3} + \frac{2\pi n}{7}; \frac{\pi}{7} + \frac{1}{7}\arccos\frac{1}{3} + \frac{2\pi n}{7})$;

в) $\sin x\cos\frac{x}{2} + \cos x\sin\frac{x}{2} \le -\frac{2}{7}$, $\sin\frac{3x}{2} \le -\frac{2}{7}$,

$\frac{3x}{2} \in [-\pi + \arcsin\frac{2}{7} + 2\pi n; -\arcsin\frac{2}{7} + \frac{4\pi n}{3}]$,

$x \in [-\frac{2}{3}\pi + \frac{2}{3}\arcsin\frac{2}{7} + \frac{4}{3}\pi n; -\frac{2}{3}\arcsin\frac{2}{7} + \frac{4\pi n}{3}]$;

г) $\cos\frac{x}{2}\cos\frac{x}{4} - \sin\frac{x}{2}\sin\frac{x}{4} > \frac{\sqrt{2}}{2}$, $\cos\frac{3x}{4} > \frac{\sqrt{2}}{2}$,

$\frac{3x}{4} \in (-\frac{\pi}{4} + 2\pi n; \frac{\pi}{4} + 2\pi n)$, $x \in (-\frac{\pi}{3} + \frac{8\pi n}{3}; \frac{\pi}{3} + \frac{8\pi n}{3})$.

418. а) $\sin(60° - \beta) = \sin 60° \cos\beta - \cos 60° \sin\beta = \frac{\sqrt{3}}{2}\cos\beta - \frac{1}{2}\sin\beta$.

б) $\cos(\beta - 30°) = \cos\beta\cos 30° + \sin\beta \sin 30° = \frac{\sqrt{3}}{2}\cos\beta + \frac{1}{2}\sin\beta$.

в) $\sin(\alpha - 30°) = \sin\alpha\cos 30° - \sin 30°\cos\alpha = -\frac{\sqrt{3}}{2}\sin\alpha + \frac{1}{2}\cos\alpha$.

г) $\cos(60° - \alpha) = \cos 60°\cos\alpha + \sin 60°\sin\alpha = \frac{1}{2}\cos\alpha + \frac{\sqrt{3}}{2}\sin\alpha$.

419. а) $\sin\left(\frac{5\pi}{6} - \alpha\right) - \frac{1}{2}\cos\alpha = \sin\frac{5\pi}{6}\cos\alpha - \sin\alpha\cos\frac{5\pi}{6} -$

$- \frac{1}{2}\cos\alpha = \frac{1}{2}\cos\alpha + \frac{\sqrt{3}}{2}\sin\alpha - \frac{1}{2}\cos\alpha = \frac{\sqrt{3}}{2}\sin\alpha$.

б) $\sqrt{3}\cos\alpha - 2\cos\left(\alpha - \frac{\pi}{6}\right) = \sqrt{3}\cos\alpha - 2\cos\alpha\cos\frac{\pi}{6} - 2\sin\alpha\sin\frac{\pi}{6} = -\sin\alpha$.

в) $\frac{\sqrt{3}}{2}\sin\alpha + \cos\left(\alpha - \frac{5\pi}{3}\right) = \frac{\sqrt{3}}{2}\sin\alpha + \cos\alpha\cos\frac{5\pi}{3} + \sin\alpha\sin\frac{5\pi}{3} = \frac{1}{2}\cos\alpha$.

г) $\sqrt{2}\sin\left(\alpha - \frac{\pi}{4}\right) - \sin\alpha = \sqrt{2}\sin\alpha\cos\frac{\pi}{4} - \sqrt{2}\cos\alpha\sin\frac{\pi}{4} - \sin\alpha = -\cos\alpha$.

420. а) $\cos(\alpha - \beta) - \cos\alpha\cos\beta = \cos\alpha\cos\beta + \sin\alpha\sin\beta - \cos\alpha\cos\beta = \sin\alpha\sin\beta$
б) $\sin(\alpha+\beta) + \sin(\alpha-\beta) = \sin\alpha\cos\beta + \sin\beta\cos\alpha + \sin\alpha\cos\beta - \sin\beta\cos\alpha = 2\sin\alpha\cos\beta$
в) $\sin\alpha\cos\beta - \sin(\alpha - \beta) = \sin\alpha\cos\beta - \sin\alpha\cos\beta + \sin\beta\cos\alpha = \sin\beta\cos\alpha$
г) $\cos(\alpha-\beta) - \cos(\alpha+\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta - \cos\alpha\cos\beta + \sin\alpha\sin\beta = 2\sin\alpha\sin\beta$.
421. а) $\cos 107°\cos 17° + \sin 107°\sin 17° = \cos(107° - 17°) = \cos 90° = 0$.

б) $\cos 36°\cos 24° - \sin 36°\sin 24° = \cos(36° + 24°) = \cos 60° = \frac{1}{2}$.

в) $\sin 63°\cos 27° + \cos 63°\sin 27° = \sin(63° + 27°) = \sin 90° = 1$.

г) $\sin 51°\cos 21° - \cos 51°\sin 21° = \sin(51° - 21°) = \frac{1}{2}$.

422. а) $\cos\frac{5\pi}{8}\cos\frac{3\pi}{8} + \sin\frac{5\pi}{8}\sin\frac{3\pi}{8} = \cos\left(\frac{5\pi}{8} - \frac{3\pi}{8}\right) = \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$.

б) $\sin\frac{2\pi}{15}\cos\frac{\pi}{5} + \cos\frac{2\pi}{15}\sin\frac{\pi}{5} = \sin\left(\frac{2\pi}{15} + \frac{\pi}{5}\right) = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$.

в) $\cos\frac{\pi}{12}\cos\frac{\pi}{4} - \sin\frac{\pi}{12}\sin\frac{\pi}{4} = \sin\left(\frac{\pi}{12} - \frac{\pi}{4}\right) = \cos\frac{\pi}{3} = \frac{1}{2}$.

г) $\sin\frac{\pi}{12}\cos\frac{\pi}{4} - \cos\frac{\pi}{12}\sin\frac{\pi}{4} = \sin\left(\frac{\pi}{12} - \frac{\pi}{4}\right) = -\sin\frac{\pi}{6} = -\frac{1}{2}$.

423. а) $\frac{\sqrt{3}}{2}$ cosx– $\frac{1}{2}$ sinx=sin ($\frac{\pi}{3}$ – x), sin $\frac{\pi}{3}$ cosx–cos $\frac{\pi}{3}$ sin x = sin ($\frac{\pi}{3}$ – x).

б) $\frac{1}{2}$ cos x + $\frac{\sqrt{3}}{2}$ sin x = cos ($\frac{\pi}{3}$ – x), cos $\frac{\pi}{3}$ cos x+sin $\frac{\pi}{3}$ sin x=cos ($\frac{\pi}{3}$ – x).

424. а) cos (α – β) + sin (– α) sin β = cos α cos β,
cos α cos β + sin α sin β – sin α sin β = cos α cos β;
б) sin (30° – α) – cos (60° – α) = – $\sqrt{3}$ sin α,
sin 30° cos α – sin α cos 30° – cos 60° cos α – sin 60° sin α =
= $\frac{1}{2}$ cos α – $\frac{\sqrt{3}}{2}$ sin α – $\frac{1}{2}$ cos α – $\frac{\sqrt{3}}{2}$ sin α = – $\sqrt{3}$ sin α;
в) sin (α – β) – cos α sin (– β) = sin α cos β,
sin α cos β – sin β cos α + sin β cos α = sin α cos β;
г) sin (30° – α) + sin (30° + α) = cos α,
$\frac{1}{2}$ cos α – $\frac{\sqrt{3}}{2}$ sin α + $\frac{1}{2}$ cos α + $\frac{\sqrt{3}}{2}$ sin α = cos α.

425. а) sin (α – β) – sin (α + β) = – 2 cos α sin β,
sin α cos β – sin β cos α – sin α cos β – sin β cos α = –2cos α sin β;
б) cos (α – β) + cos (α + β) = 2 cos α cos β,
cos α cos β + sin α sinβ + cos α cos β – sin α sin β = 2 cos α cos β.

426. а) cos 6x cos 5x + sin 6x sin 5x = – 1, cos (6x – 5x) = – 1,
cos x = – 1, x = π + 2πn;
б) sin 3x cos 5x – sin 5x cos 3x = $\frac{1}{2}$, sin (3x – 5x) = $\frac{1}{2}$, sin 2x = – $\frac{1}{2}$,

2x = (– 1)$^{k+1}$ $\frac{\pi}{6}$ + πk, x = (– 1)$^{k+1}$ $\frac{\pi}{12}$ + $\frac{\pi k}{2}$.

427. а) sin 15° = sin (45° – 30°) = $\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2} = \frac{\sqrt{6}-\sqrt{2}}{2}$;

б) cos 15° = cos (45° – 30°) = $\frac{\sqrt{2}}{2} \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \frac{1}{2} = \frac{\sqrt{6}+\sqrt{2}}{2}$;

в) sin 15° cos 15° = $\frac{1}{2}$ (sin (15° + 15°)) = $\frac{1}{2}$ sin 30° = $\frac{1}{4}$;

г) cos^2 15° – sin^2 15° = cos (15° + 15°) = $\frac{\sqrt{3}}{2}$.

428. а) Опечатка в условии:
sin 77° cos 17° – sin13° cos 73° = sin 77° cos 17° – cos 77 sin 17°=
= sin (77° – 17°) = sin 60° = $\frac{\sqrt{3}}{2}$;

б) cos 125°cos 5°+sin 55°cos 85°=–cos 55°cos 5°+sin 55°cos 5°= –cos 60°=– $\frac{1}{2}$.

429. а) $\sin\left(\frac{\pi}{6}+t\right)\cos\left(\frac{\pi}{3}-t\right)+\sin\left(\frac{2\pi}{3}+t\right)\sin\left(\frac{\pi}{3}-t\right)=$

$\sin\left(\frac{\pi}{6}+t\right)\cos\left(\frac{\pi}{3}-t\right)+\sin\left(\frac{\pi}{3}-t\right)\sin\left(\frac{\pi}{3}-t\right)=\cos^2 t\left(\frac{\pi}{3}-t\right)+\sin^2\left(\frac{\pi}{3}-t\right)=1.$

б) $\cos\left(\frac{\pi}{4}+t\right)\cos\left(\frac{\pi}{12}-t\right)-\cos\left(\frac{\pi}{4}-t\right)\cos\left(\frac{5\pi}{12}+t\right)=$

$=\cos\left(\frac{\pi}{12}-t\right)\sin\left(\frac{\pi}{4}-t\right)-\sin\left(\frac{\pi}{12}-t\right)\cos\left(\frac{\pi}{4}-t\right)=\sin\left(\frac{\pi}{4}-t-\frac{\pi}{12}+t\right)=\sin\frac{\pi}{6}=\frac{1}{2}.$

430. а) $\frac{\cos 105^\circ\cos 55^\circ+\sin 105^\circ\cos 85^\circ}{\sin 95^\circ\cos 5^\circ-\cos 95^\circ\sin 185^\circ}=\frac{\sin 15^\circ\cos 5^\circ+\cos 15^\circ\sin 15^\circ}{\cos^2 5^\circ-\sin^2 5^\circ}=$

$=\frac{-\sin 10^\circ}{\cos 10^\circ}=-\operatorname{tg} 10^\circ.$

б) $\frac{\sin 25^\circ\cos 5^\circ-\cos 25^\circ\cos 85^\circ}{\cos 375^\circ\cos 5^\circ-\sin 15^\circ\sin 365^\circ}=\frac{\sin(25^\circ-5^\circ)}{\cos 15^\circ\cos 5^\circ-\sin 15^\circ\sin 5^\circ}=\frac{\sin 20^\circ}{\cos 20^\circ}=\operatorname{tg} 20^\circ.$

431. а) $\frac{\sin(\alpha+\beta)-\cos\alpha\sin\beta}{\sin(\alpha-\beta)+\cos\alpha\sin\beta}=\frac{\sin\alpha\cos\beta}{\sin\alpha\cos\beta}=1.$

б) $\frac{\sin(\alpha-\beta)+2\cos\alpha\sin\beta}{2\cos\alpha\cos\beta-\cos(\alpha-\beta)}=\frac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)}=\operatorname{tg}(\alpha+\beta).$

в) $\frac{\cos(\alpha+\beta)+\sin\alpha\sin\beta}{\cos(\alpha-\beta)-\sin\alpha\sin\beta}=\frac{\cos\alpha\ \cos\beta}{\cos\alpha\ \cos\beta}=1.$

г) $\frac{\cos(\alpha-\beta)+2\sin\alpha\sin\beta}{2\sin\alpha\cos\beta-\sin(\alpha-\beta)}=\operatorname{ctg}(\alpha+\beta).$

432. $\sin t=\frac{5}{13}$, $\frac{\pi}{2}<t<\pi$, $\cos t=-\frac{12}{13}$.

а) $\sin\left(\frac{\pi}{3}-t\right)=\sin\frac{\pi}{3}\cos t-\cos\frac{\pi}{3}\sin t=\frac{\sqrt{3}}{2}\cdot\frac{12}{13}-\frac{1}{2}\cdot\frac{5}{13}=\frac{-12\sqrt{3}-5}{26};$

б) $\cos\left(t-\frac{\pi}{2}\right)=\sin t=\frac{5}{13}$; в) $\sin\left(\frac{\pi}{2}-t\right)=\sin\frac{\pi}{2}\cos t-\cos\frac{\pi}{2}\sin t=\cos t=-\frac{12}{13};$

г) $\cos\left(\frac{\pi}{3}-t\right)=\cos\frac{\pi}{3}\cos t+\sin\frac{\pi}{3}\sin t=-\frac{1}{2}\cdot\frac{12}{13}+\frac{\sqrt{3}}{2}\cdot\frac{5}{13}=\frac{5\sqrt{3}-12}{26}.$

433. $\cos t=\frac{3}{5}$, $\frac{3\pi}{2}<t<2\pi$

а) $\sin\left(t-\frac{\pi}{6}\right)=\sin t\cos\frac{\pi}{6}-\sin\frac{\pi}{6}\cos t=-\frac{4}{5}\frac{\sqrt{3}}{2}-\frac{1}{2}\frac{3}{5}=-\frac{4\sqrt{3}+3}{10};$

б) $\sin\left(t-\frac{3\pi}{2}\right)=\sin t\cos\frac{3\pi}{2}-\sin\frac{3\pi}{2}\cos t=\cos t=\frac{3}{5};$

в) $\cos\left(t-\frac{3\pi}{2}\right)=\cos t\cos\frac{3\pi}{2}+\sin t\sin\frac{3\pi}{2}=-\sin t=\frac{4}{5}$;

г) $\cos(t-\frac{\pi}{6})=\cos t\cos\frac{\pi}{6}+\sin t\sin\frac{\pi}{6}=\cos t\frac{\sqrt{3}}{2}+\sin t\frac{1}{2}=$

$=\frac{3}{5}\cdot\frac{\sqrt{3}}{2}-\frac{4}{5}\cdot\frac{1}{2}=\frac{3\sqrt{3}-4}{10}$.

434. $\sin\alpha=\frac{4}{5}$, $\cos\beta=-\frac{15}{17}$, $\frac{\pi}{2}<\alpha<\pi$, $\frac{\pi}{2}<\beta<\pi$

а) $\sin(\alpha-\beta)=\sin\alpha\cos\beta-\sin\beta\cos\alpha=-\frac{4}{5}\cdot\frac{15}{17}+\frac{8}{17}\cdot\frac{3}{5}=\frac{-60+24}{85}=\frac{-36}{85}$.

б) $\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta=\frac{3}{5}\cdot\frac{15}{17}+\frac{4}{5}\cdot\frac{8}{17}=\frac{77}{85}$.

435. $\sin\beta=-\frac{12}{13}$, $\cos\alpha=-\frac{4}{5}$, $\pi<\beta<\frac{3\pi}{2}$

$\cos\beta=-\frac{5}{13}$, $\sin=\frac{3}{5}$, $\frac{\pi}{2}<\alpha<\pi$;

а) $\sin(\alpha-\beta)=\sin\alpha\cos\beta-\sin\beta\cos\alpha=-\frac{3}{5}\cdot\frac{5}{13}-\frac{12}{13}\cdot\frac{4}{5}=\frac{-15-48}{65}=-\frac{63}{65}$;

б) $\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta=\frac{20}{65}-\frac{36}{65}=-\frac{16}{65}$.

436. а) $\dfrac{\sqrt{2}\cos\alpha-2\cos(\frac{\pi}{4}-\alpha)}{2\sin(\frac{\pi}{6}+\alpha)-\sqrt{3}\sin\alpha}=-\sqrt{2}\ \text{tg}\,\alpha.$

$\dfrac{\sqrt{2}\cos\alpha-\sqrt{2}\cos\alpha-\sqrt{2}\sin\alpha}{\cos\alpha+\sqrt{3}\sin\alpha-\sqrt{3}\sin\alpha}=\dfrac{-\sqrt{2}\sin\alpha}{\cos\alpha}=-\sqrt{2}\ \text{tg}\,\alpha.$

б) $\dfrac{\cos\alpha-2\cos(\frac{\pi}{3}+\alpha)}{2\sin(-\frac{\pi}{6}+\alpha)-\sqrt{3}\sin\alpha}=-\sqrt{3}\ \text{tg}\,\alpha.$

$\dfrac{\cos\alpha-\cos\alpha+\sqrt{3}\sin\alpha}{2\sin\cos\frac{\pi}{6}-2\sin\frac{\pi}{6}\cos\alpha-\sqrt{3}\sin\alpha}=\dfrac{\sqrt{3}\sin\alpha}{-\cos\alpha}=-\sqrt{3}\ \text{tg}\,\alpha.$

437. а) $\sqrt{2}\cos(\frac{\pi}{4}-x)-\cos x=\frac{1}{2}$, $\cos x+\sin x-\cos x=\frac{1}{2}$,

$\sin x=\frac{1}{2}$, $x=(-1)^k\frac{\pi}{6}+\pi k$;

б) $\sqrt{2}\sin(\frac{\pi}{4}-\frac{x}{2})+\sin\frac{x}{2}=\frac{\sqrt{3}}{2}$, $\cos\frac{x}{2}-\sin\frac{x}{2}+\sin\frac{x}{2}=\frac{\sqrt{3}}{2}$,

$\cos\frac{x}{2}=\frac{\sqrt{3}}{2}$, $\frac{x}{2}=\pm\frac{\pi}{6}+2\pi n$, $x=\pm\frac{\pi}{3}+4\pi n$.

438. а) $\frac{\sqrt{2}}{2}\sin x-\frac{\sqrt{2}}{2}\cos x=1$, $\sin(x-\frac{\pi}{4})=1$,

$x-\frac{\pi}{4}=\frac{\pi}{2}+2\pi n$, $x=\frac{3\pi}{4}+2\pi n$;

б) $\sin x-\cos x=1$, $\sin(x-\frac{\pi}{4})=\frac{\sqrt{2}}{2}$,

$x-\frac{\pi}{4}=(-1)^n\frac{\pi}{4}+2\pi n$, $x=(-1)^n\frac{\pi}{4}+\frac{\pi}{4}+2\pi n$;

в) $\frac{\sqrt{3}}{2}\cos x+\frac{1}{2}\sin x=1$, $\cos(x-\frac{\pi}{6})=1$, $x-\frac{\pi}{6}=2\pi n$, $x=\frac{\pi}{6}+2\pi n$;

г) $\sqrt{3}\cos x+\sin x=1$, $\cos(x-\frac{\pi}{6})=\frac{1}{2}$,

$x-\frac{\pi}{6}=\pm\frac{\pi}{3}+2\pi n$; $x=\pm\frac{\pi}{3}+\frac{\pi}{6}+2\pi n$.

439. а) $\sin 5x\cos 3x-\cos 5x\sin 3x>\frac{1}{2}$, $\sin 2x>\frac{1}{2}$,

$2x\in(\frac{\pi}{6}+2\pi n;\ \frac{5\pi}{6}+2\pi n)$, $x\in(\frac{\pi}{12}+\pi n;\ \frac{5\pi}{2}+\pi n)$;

б) $\cos x\cos\frac{x}{2}+\sin x\sin\frac{x}{2}<-\frac{2}{7}$, $\cos\frac{x}{2}<-\frac{2}{7}$,

$\frac{x}{2}\in(\pi-\arccos\frac{2}{7}+2\pi n;\ \pi-\arccos\frac{2}{7}+2\pi n)$,

$x\in(2\pi-2\arccos\frac{2}{7}+4\pi n;\ 2\pi-2\arccos\frac{2}{7}+4\pi n)$;

в) $\sin\frac{x}{4}\cos\frac{x}{2}-\cos\frac{x}{4}\sin\frac{x}{2}<\frac{1}{3}$, $\sin(\frac{x}{4}-\frac{x}{2})<\frac{1}{3}$, $\sin\frac{x}{4}>-\frac{1}{3}$,

$\frac{x}{4}\in(-\arcsin\frac{1}{3}+2\pi n;\ \pi+\arcsin\frac{1}{3}+2\pi n)$,

$x\in(-4\arcsin\frac{1}{3}+8\pi n;\ 4\pi+4\arcsin\frac{1}{3}+8\pi n)$;

г) $\sin 2x\sin 5x+\cos 2x\cos 5x>-\frac{\sqrt{3}}{2}$, $\cos 3x>-\frac{\sqrt{3}}{2}$,

$3x\in(-\frac{5\pi}{6}+2\pi n;\ \frac{5\pi}{6}+2\pi n)$, $x\in(-\frac{5\pi}{18}+\frac{2\pi n}{3};\ \frac{5\pi}{18}+\frac{2\pi n}{3})$.

§ 23. Тангенс суммы и разности аргументов

440. а) $\mathrm{tg}\,15° = \mathrm{tg}\,(45° – 30°) = \dfrac{\mathrm{tg}\,45° - \mathrm{tg}\,30°}{1 + \mathrm{tg}45°\ \ \mathrm{tg}30°} = \dfrac{1 - \frac{\sqrt{3}}{3}}{1 + \frac{\sqrt{3}}{3}}$.

б) $\mathrm{tg}\,75° = \mathrm{tg}\,(45° + 30°) = \dfrac{\mathrm{tg}\,45° + \mathrm{tg}\,30°}{1 - \mathrm{tg}45°\ \ \mathrm{tg}30°} = \dfrac{1 + \frac{\sqrt{3}}{3}}{1 - \frac{\sqrt{3}}{3}}$.

в) $\mathrm{tg}\,105° = \mathrm{tg}\,(-60° + 45°) = \dfrac{\mathrm{tg}\,60° + \mathrm{tg}\,45°}{1 - \mathrm{tg}60°\ \ \mathrm{tg}45°} = \dfrac{\sqrt{3}+1}{1-\sqrt{3}}$.

г) $\mathrm{tg}\,165° = -\,\mathrm{tg}\,15° = -\dfrac{1 - \frac{\sqrt{3}}{3}}{1 + \frac{\sqrt{3}}{3}}$.

441. а) $\dfrac{\mathrm{tg}\,25° + \mathrm{tg}\,20°}{1 - \mathrm{tg}25°\ \ \mathrm{tg}20°} = \mathrm{tg}\,45° = -\,1$; б) $\dfrac{1 - \mathrm{tg}\,70°\ \mathrm{tg}\,65°}{\mathrm{tg}70\,° + \mathrm{tg}65°} = \mathrm{ctg}\,135° = -\,1$;

в) $\dfrac{\mathrm{tg}\,9° + \mathrm{tg}\,51°}{1 - \mathrm{tg}9°\ \ \mathrm{tg}51°} = \mathrm{tg}\,60° = \sqrt{3}$; г) $\dfrac{1 + \mathrm{tg}\,54°\ \mathrm{tg}\,9°}{\mathrm{tg}54°\ \text{-}\ \mathrm{tg}9°} = \mathrm{ctg}\,45° = 1$.

442. а) $\mathrm{tg}\,(\frac{\pi}{4} - \alpha)$? $\mathrm{tg}\,\alpha = \frac{2}{3}$, $\mathrm{tg}\,(\frac{\pi}{4} - \alpha) = \dfrac{1 - \mathrm{tg}\alpha}{1 + \mathrm{tg}\alpha} = \dfrac{1}{3} \cdot \dfrac{3}{5} = \dfrac{1}{5}$;

б) $\mathrm{tg}\,\alpha = \frac{4}{5}$, $\mathrm{tg}\,(\alpha + \frac{\pi}{3}) = \dfrac{\mathrm{tg}\,\alpha + \mathrm{tg}\frac{\pi}{3}}{1 - \mathrm{tg}\,\alpha\,\mathrm{tg}\frac{\pi}{3}} = \dfrac{\frac{4}{5} + \frac{\sqrt{3}}{3}}{1 - \frac{4}{5}\cdot\frac{\sqrt{3}}{3}} = \dfrac{12 + 5\sqrt{3}}{15 - 4\sqrt{3}}$;

в) $\mathrm{ctg}\,\alpha = \frac{4}{3}$, $\mathrm{tg}\,(\frac{\pi}{2} + \alpha) = -\,\mathrm{ctg}\,\alpha = -\frac{4}{3}$;

г) $\mathrm{ctg}\,\alpha = \frac{8}{5}$, $\mathrm{tg}\,(\alpha - \frac{\pi}{6}) = \dfrac{\mathrm{tg}\alpha - \frac{\sqrt{3}}{3}}{1 + \mathrm{tg}\alpha\cdot\frac{\sqrt{3}}{3}} = \dfrac{\frac{8}{5} - \frac{\sqrt{3}}{3}}{1 + \frac{8}{5}\cdot\frac{\sqrt{3}}{3}} = \dfrac{24 - 5\sqrt{3}}{15 + 8\sqrt{3}}$.

443. $\mathrm{tg}\,\alpha = \frac{1}{2}$, $\mathrm{tg}\,\beta = \frac{1}{3}$;

а) $\mathrm{tg}\,(\alpha + \beta) = \dfrac{\mathrm{tg}\,\alpha + \mathrm{tg}\,\beta}{1 - \mathrm{tg}\,\alpha\,\mathrm{tg}\,\beta} = \dfrac{\frac{1}{2} + \frac{1}{3}}{1 - \frac{1}{6}} = \dfrac{5}{6} \cdot \dfrac{6}{5} = 1$.

б) $tg(\alpha-\beta) = \dfrac{\frac{1}{8}-\frac{1}{3}}{1+\frac{1}{6}} = \dfrac{1}{6}\cdot\dfrac{6}{7} = \dfrac{1}{7}$.

444. $tg\,\alpha = \dfrac{2}{5}$, $tg(\dfrac{\pi}{2}+\beta) = -3$, $-ctg\,\beta = -3$, $tg\,\beta = \dfrac{1}{3}$;

а) $tg(\alpha+\beta) = \dfrac{tg\,\alpha + tg\,\beta}{1 - tg\,\alpha\ tg\,\beta} = \dfrac{\frac{2}{3}+\frac{1}{3}}{1-\frac{2}{15}} = \dfrac{11}{15}\cdot\dfrac{15}{13} = \dfrac{11}{13}$;

б) $tg(\alpha-\beta) = \dfrac{tg\,\alpha - tg\,\beta}{1 + tg\,\alpha\ tg\,\beta} = \dfrac{\frac{2}{5}-\frac{1}{3}}{1-\frac{2}{15}} = \dfrac{11}{15}\cdot\dfrac{15}{17} = \dfrac{1}{17}$.

445. а) $\dfrac{tg\,2{,}22 + tg\,0{,}92}{1 - tg\,2{,}22\cdot tg\,0{,}92} = tg(2{,}22+0{,}92) = tg\,3{,}14$.

б) $\dfrac{tg\,1{,}47 - tg\,0{,}69}{1 + tg\,1{,}47\ tg\,0{,}69} = tg(1{,}47-0{,}69) = tg\,0{,}78$.

446. а) $\dfrac{tg\,x + tg\,3x}{1 - tg\,x\ tg\,3x} = 1$, $tg\,4x = 1$, $4x = \dfrac{\pi}{4} + \pi n$, $x = \dfrac{\pi}{16} + \dfrac{\pi}{4}n$;

б) $\dfrac{tg\,5x - tg\,3x}{1 + tg\,3x\ tg\,5x} = \sqrt{3}$, $2x = \dfrac{\pi}{3} + \pi n$, $x = \dfrac{\pi}{6} + \dfrac{\pi n}{2}$.

447. а) $tg(\alpha - \dfrac{\pi}{4}) = 3$, $\dfrac{tg\,\alpha - 1}{1 + tg\,\alpha} = 3$, $tg\,\alpha - 1 = 3 + 3\,tg\,\alpha$,

$2tg\,\alpha = -4$, $tg\,\alpha = -2$;

б) $tg(\alpha + \dfrac{\pi}{4}) = \dfrac{1}{5}$, $\dfrac{tg\,\alpha + 1}{1 - tg\,\alpha} = \dfrac{1}{5}$, $5\,tg\,\alpha + 5 = 1 - tg\,\alpha$,

$tg\,\alpha = -\dfrac{2}{3}$, $ctg\,\alpha = -\dfrac{3}{2}$.

448. а) $tg\,\alpha = 3$, $tg(\alpha+\beta) = 1$.

$\dfrac{tg\,\alpha + tg\beta}{1 - tg\,\alpha\ tg\beta} = 1$, $3 + tg\,\beta = 1 - 3\,tg\,\beta$, $tg\,\beta = -\dfrac{1}{2}$;

б) $tg\,\alpha = \dfrac{1}{4}$, $tg(\alpha-\beta) = 2$.

$\dfrac{tg\,\alpha - tg\beta}{1 + tg\,\alpha\ tg\beta} = 2$, $\dfrac{1}{4} - tg\,\beta = 2 + \dfrac{1}{2}tg\,\beta$,

$\dfrac{3}{2}tg\,\beta = -\dfrac{7}{4}$, $tg\,\beta = -\dfrac{7}{6}$.

449. $\sin\alpha = -\frac{12}{13}$, $\pi < \alpha < \frac{3\pi}{2}$;

$\cos\alpha = -\frac{5}{13}$, $\text{tg}\,\alpha\,\beta = \frac{12}{5}$;

а) $\text{tg}\,(\alpha + \frac{\pi}{4}) = \frac{\text{tg}\,\alpha + 1}{1 - \text{tg}\,\alpha} = \frac{\frac{12}{5}+1}{1-\frac{12}{5}} = -\frac{17}{5}\cdot\frac{5}{7} = -\frac{17}{7}$.

б) $\text{tg}\,(\alpha - \frac{\pi}{4}) = \frac{\text{tg}\,\alpha - 1}{1 + \text{tg}\,\alpha} = \frac{\frac{12}{5}-1}{1+\frac{12}{5}} = \frac{17}{5}\cdot\frac{5}{17} = \frac{7}{17}$.

450. $\cos\alpha = \frac{3}{5}$, $0 < \alpha < \frac{\pi}{2}$, $\sin\alpha = \frac{4}{5}$, $\text{tg}\,\alpha = \frac{4}{3}$;

а) $\text{tg}\,(\alpha + \frac{\pi}{3}) = \frac{\text{tg}\,\alpha + \sqrt{3}}{1 - \sqrt{3}\,\text{tg}\,\alpha} = \frac{\frac{4}{3}+\sqrt{3}}{1-\frac{4}{\sqrt{3}}} = \frac{4+3\sqrt{3}}{3}$

$\frac{\sqrt{3}}{\sqrt{3}-4} = \frac{4\sqrt{3}+9}{3\sqrt{3}-12} = -\frac{48+25\sqrt{3}}{39}$;

б) $\text{tg}\,(\alpha - \frac{\pi}{3}) = \frac{\text{tg}\,\alpha - \sqrt{3}}{1 + \sqrt{3}\,\text{tg}\,\alpha} = \frac{\frac{4}{3}-3\sqrt{3}}{3}\cdot\frac{3}{3+4\sqrt{3}} = \frac{4-3\sqrt{3}}{3+4\sqrt{3}}$.

451. а) $\frac{\text{tg}(\frac{\pi}{8}+\alpha)+\text{tg}(\frac{\pi}{8}-\alpha)}{1-\text{tg}(\frac{\pi}{8}+\alpha)\ \text{tg}(\frac{\pi}{8}-\alpha)} = \text{tg}\,(\frac{\pi}{8} + \alpha + \frac{\pi}{8} - \alpha) = \text{tg}\,\frac{\pi}{4} = 1.$

б) $\frac{tg(45^\circ+\alpha)-tg\,\alpha}{1+tg(45^\circ+\alpha)\ tg\,\alpha} = \text{tg}\,(45^\circ + \alpha - \alpha) = \text{tg}\,45^\circ = 1.$

452. а) $\frac{1-\text{tg}\alpha}{1+\text{tg}\alpha} = \text{tg}\,(45^\circ - \alpha)$, $\text{tg}\,(45^\circ - \alpha) = \frac{1-\text{tg}\alpha}{1+\text{tg}\alpha}$.

б) $\frac{\text{tg}\alpha+\text{tg}\beta}{\text{tg}(\alpha+\beta)} + \frac{\text{tg}\alpha-\text{tg}\beta}{\text{tg}(\alpha-\beta)} = 2.$

$\frac{\text{tg}\alpha+\text{tg}\beta}{\text{tg}(\alpha+\beta)} + \frac{\text{tg}\alpha-\text{tg}\beta}{\text{tg}(\alpha-\beta)} = 1 - \text{tg}\,\alpha\,\text{tg}\,\beta + 1 + \text{tg}\alpha\,\text{tg}\,\beta = 2.$

453. а) $\text{tg}\,(\frac{3\pi}{4} - x) + \text{tg}\,x = \text{tg}\,(\frac{3\pi}{4} - x)\,\text{tg}\,x - 1.$

$\frac{tg\,x+1}{tg\,x-1}+tgx=\frac{tg\,x+1}{tg\,x-1}tg\,x-1, \quad \frac{tg^2x+1}{tg\,x-1}=\frac{tg^2x+tg\,x-tg\,x+1}{tg\,x-1};$

б) $tg\,(\alpha+\frac{\pi}{4})-tg\,\alpha=1+tg\,(\frac{\pi}{4}+\alpha)\,tg\,\alpha.$

$\frac{tg\alpha+1}{1-tg\alpha}-tg\alpha=\frac{-tg\alpha+tg^2\alpha+tg\alpha+1}{1-tg\alpha}=\frac{1-tg\alpha+tg^2\alpha+tg\alpha}{1-tg\alpha}.$

$\frac{tg^2\alpha+1}{1-tg\alpha}=\frac{tg^2\alpha+1}{1-tg\alpha}.$

454. а) $tg\,(\alpha+\beta)-(tg\,\alpha+tg\,\beta)=tg\,(\alpha+\beta)\,tg\,\alpha\,tg\,\beta.$

$\frac{tg\,\alpha+tg\,\beta+tg^2\alpha\,tg\,\beta-tg\,\beta+tg\,\alpha\,tg^2\beta}{1-tg\,\alpha\,tg\,\beta}=\frac{tg^2\alpha\,tg\,\beta+tg\,\alpha\,tg^2\beta}{1-tg\,\alpha}=$

$=tg\,\alpha\,tg\,\beta\,tg\,(\alpha+\beta).$

б) $tg\,(\alpha-\beta)-(tg\,\alpha-tg\,\beta)=tg\,(\beta-\alpha)-tg\,\alpha\;\;tg\,\beta.$

$\frac{tg\,\alpha-tg\,\beta-tg\alpha-tg^2\alpha\,tg\,\beta+tg\,\beta+tg\,\alpha\,tg^2\beta}{1+tg\,\alpha\,tg\,\beta}=tg\,(\beta-\alpha)\,tg\alpha\,tg\,\beta.$

455. а) $\frac{\sqrt{3}-tgx}{1+\sqrt{3}tgx}=1, \quad tg\,(\frac{\pi}{3}-x)=1, \quad x-\frac{\pi}{3}=-\frac{\pi}{4}+\pi n, \quad x=\frac{\pi}{12}+\pi n;$

б) $\frac{tg\frac{\pi}{5}-tg2x}{tg\frac{\pi}{5}tg\;2x+1}=\sqrt{3}, \quad tg\,(2x-\frac{\pi}{5})=-\sqrt{3},$

$2x-\frac{\pi}{5}=-\frac{\pi}{3}+\pi n, \quad x=-\frac{\pi}{15}+\frac{\pi n}{2}.$

456. $tg\,2x=tg\,(x+x)=\frac{2tgx}{1-tg^2x}.$

457. $\alpha-\beta=\frac{\pi}{4}; \qquad \alpha=\frac{\pi}{4}+\beta; \qquad \beta=\alpha-\frac{\pi}{4};$

а) $\frac{1+tg\beta}{1-tg\beta}=tg\alpha, \quad tg\,\alpha=tg\,(\beta+\frac{\pi}{4})=\frac{tg\beta+1}{1-tg\beta};$

б) $\frac{tg\,\alpha-1}{tg\alpha+1}=tg\beta, \quad tg\,\beta=tg\,(\alpha-\frac{\pi}{4})=\frac{tg\,\alpha-1}{1+tg\,\alpha}.$

458. $\frac{tg(\alpha-\beta)-tg\,\alpha+tg\,\beta}{tg(\alpha-\beta)\,tg\,\beta}=\frac{tg(\beta-\alpha)\,tg\,\alpha+tg\,\beta}{tg\,(\alpha-\beta)\,tg\,\beta}=-tg\,\alpha.$

459. $\frac{tg\frac{\pi}{5}+tgx}{1-tg\frac{\pi}{5}tgx}<1$

а) $tg\left(x+\frac{\pi}{5}\right)<1$, $x+\frac{\pi}{5}\in\left(-\frac{\pi}{2}+\pi n;\ \frac{\pi}{4}+\pi n\right)$, $x\in\left(-\frac{7\pi}{10}+\pi n;\ \frac{\pi}{20}+\pi n\right)$;

б) $\frac{tg3x-1}{tg3x+1}>1$, $tg\left(3x-\frac{\pi}{4}\right)>1$,

$3x-\frac{\pi}{4}\in\left(\frac{\pi}{4}+\pi n;\ \frac{\pi}{2}+\pi n\right)$, $x\in\left(\frac{\pi}{6}+\frac{\pi n}{3};\ \frac{\pi}{4}+\frac{\pi n}{3}\right)$.

460. $y_1=3x+1$ $\qquad$ $y_2=6-2x$.

$tg\,y_1=3$ – тангенс угла наклона 1-й прямой.

$tg\,y_2$ – тангенс угла наклона 2-ой прямой.

$y_1=$ arctg 3

$y_2=-$ arctg 2

$y_1-y_2=$ arctg 3 + arctg 2

$tg\,(y_1-y_2)=\frac{3+2}{1-6}=-1$.

$y_1-y_2=\frac{3\pi}{4}$.

461. $tg\angle KBC=\frac{1}{2}$, $\qquad$ $\angle KBC=arctg\,\frac{1}{2}$. (см. рис. 144)

$\angle BKC=90°-\angle KBC$

$\angle KOC=180°-45°-90°+\angle KBC=45°+\angle KBC$.

$tg\angle KOC=tg\left(45°+arctg\,\frac{1}{2}\right)=\frac{1+\frac{1}{2}}{1-\frac{1}{2}}=\frac{3}{2}\cdot2=3$

$tg\angle KOC=arctg\,3$.

§ 24. Формулы двойного аргумента

462. а) $\frac{\sin2t}{\cos t}-\sin t=2\sin t-\sin t=\sin t$.

б) $\frac{\sin6t}{\cos^2 3t}=2\,tg\,3t$.

в) $\cos^2 t-\cos2t=\cos^2 t-\cos^2 t+\sin^2 t=\sin^2 t$.

г) $\frac{\cos2t}{\cos t-\sin t}-\sin t=\frac{(\cos-\sin t)(\cos t+\sin t)}{\cos t-\sin t}-\sin t=\cos t$.

463. а) $\frac{\sin40°}{\sin20°}=2\cos20°$; б) $\frac{\cos80°}{\cos40°+\sin40°}=\cos40°-\sin40°$;

в) $\frac{\sin100°}{2\cos50°}=\sin50°$; $\qquad$ г) $\frac{\cos36°+\sin^2 18°}{\cos18°}=\frac{\cos^2 18°}{\cos18°}=\cos18°$.

464. а) $2\sin 15°\cos 15° = \sin 30° = \frac{1}{2}$.

б) $(\cos 75° - \sin 75°)^2 = 1 - 2\sin 75°\cos 75° = 1 - \sin 150° = \frac{1}{2}$.

в) $\cos^2 15° - \sin^2 15° = \cos 30° = \frac{\sqrt{3}}{2}$.

г) $(\cos 15° + \sin 15°)^2 = 1 + 2\sin 15°\cos 15° = \frac{3}{2}$.

465. а) $2\sin\frac{\pi}{8}\cos\frac{\pi}{8} = \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$.

б) $\sin\frac{\pi}{8}\cos\frac{\pi}{8} + \frac{1}{4} = \frac{\sqrt{2}}{4} + \frac{1}{4} = \frac{\sqrt{2}+1}{4}$.

в) $\cos^2\frac{\pi}{8} - \sin^2\frac{\pi}{8} = \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$.

г) $\frac{\sqrt{2}}{2} - (\cos\frac{\pi}{8} + \sin\frac{\pi}{8})^2 = \frac{\sqrt{2}}{2} - 1 - \sin\frac{\pi}{4} = -1$.

466. а) $\frac{2\mathrm{tg}15^\circ}{1-\mathrm{tg}^2 15^\circ} = \mathrm{tg}\,30° = \frac{\sqrt{3}}{3}$.

б) $\frac{\mathrm{tg}\frac{\pi}{8}}{1-\mathrm{tg}^2\frac{\pi}{8}} = \frac{1}{2}\,\mathrm{tg}\frac{\pi}{4} = \frac{1}{2}$.

в) $\frac{\mathrm{tg}75^\circ}{1-\mathrm{tg}^2 75^\circ} = -\mathrm{tg}\frac{\pi}{3} = -\sqrt{3}$

г) $\frac{2\mathrm{tg}\frac{\pi}{6}}{\mathrm{tg}^2\frac{\pi}{6}-1} = -\mathrm{tg}\,150^\circ = -\frac{\sqrt{3}}{3}$.

467. а) $\sin\frac{x}{2}\cos\frac{x}{2} = \frac{1}{2}\sin x$, $\frac{1}{2}\sin x = \sin\frac{x}{2}\cos\frac{x}{2}$.

б) $\cos^2\frac{x}{4} - \sin^2\frac{x}{4} = \cos\frac{x}{2}$, $\cos\frac{x}{2} = \cos^2\frac{x}{4} - \sin^2\frac{x}{4}$.

в) $\sin 2x\cos 2x = \frac{1}{2}\sin 4x$, $\frac{1}{2}\sin 4x = \sin 2x\cos 2x$.

г) $\cos^2\frac{x}{2} - \sin^2\frac{x}{2} = \cos x$, $\cos x = \cos^2\frac{x}{2} - \sin^2\frac{x}{2}$.

468. а) $\cos(2\alpha + 2\beta) = \cos^2(\alpha + \beta) - \sin^2(\alpha + \beta)$.
$\cos(2\alpha + 2\beta) = \cos 2(\alpha + \beta) = \cos^2(\alpha + \beta) - \sin^2(\alpha + \beta)$.
б) $\sin(2\alpha + 2\beta) = 2\sin(\alpha + \beta)\cos(\alpha + \beta)$.
$\sin(2\alpha + 2\beta) = \sin 2(\alpha + \beta) = 2\sin(\alpha + \beta)\cos(\alpha + \beta)$.

469. а) $\mathrm{tg}(2\alpha + 2\beta) = \dfrac{2\mathrm{tg}(\alpha+\beta)}{1-\mathrm{tg}^2(\alpha+\beta)}$

$\mathrm{tg}(2\alpha + 2\beta) = \mathrm{tg}\, 2(\alpha + \beta) = \dfrac{2\mathrm{tg}(\alpha+\beta)}{1-\mathrm{tg}^2(\alpha+\beta)}$.

б) $\mathrm{tg}(\alpha + \beta) = \dfrac{2\mathrm{tg}(\frac{\alpha}{2}+\frac{\beta}{2})}{1-\mathrm{tg}^2(\frac{\alpha}{2}+\frac{\beta}{2})}$

$\mathrm{tg}(\alpha + \beta) = \mathrm{tg}\, 2\left(\frac{\alpha}{2} + \frac{\beta}{2}\right) = \dfrac{2\mathrm{tg}(\frac{\alpha}{2}+\frac{\beta}{2})}{1-\mathrm{tg}^2(\frac{\alpha}{2}+\frac{\beta}{2})}$.

470. $\sin t = \frac{5}{13}$, $\frac{\pi}{2} < t < \pi$; $\cos t = -\frac{12}{13}$; $\mathrm{tg}\, t = \frac{5}{13} \cdot \left(-\frac{13}{12}\right) = -\frac{5}{12}$;

а) $\sin 2t = 2\sin t\cos t = 2 \cdot \frac{5}{13} \cdot \left(-\frac{12}{13}\right) = -\frac{120}{169}$.

б) $\cos 2t = \cos^2 t - \sin^2 t = \frac{144}{169} - \frac{25}{169} = \frac{119}{169}$.

в) $\mathrm{tg}\, 2t = \dfrac{2\mathrm{tg}t}{1-\mathrm{tg}^2 t} = \dfrac{2\cdot\left(-\frac{5}{12}\right)}{1-\frac{25}{144}} = -\frac{5}{6}\cdot\frac{144}{119} = -\frac{120}{119}$.

г) $\mathrm{ctg}\, 2t = \dfrac{1}{\mathrm{tg}2t} = -\dfrac{119}{120}$.

471. $\cos x = \frac{4}{5}$, $0 < x < \frac{\pi}{2}$;

$\sin x = \frac{3}{5}$, $\mathrm{tg}\, x = \frac{3}{4}$, $\mathrm{ctg}\, x = \frac{4}{3}$;

а) $\sin 2x = 2 \cdot \frac{4}{5} \cdot \frac{3}{5} = \frac{24}{25}$; б) $\cos 2x = \frac{16}{25} - \frac{9}{25} = \frac{7}{25}$.

в) $\mathrm{tg}\, 2x = \dfrac{2\mathrm{tg}x}{1-\mathrm{tg}^2 x} = \dfrac{\frac{3}{2}}{1-\frac{9}{16}} = \frac{3}{2} \cdot \frac{16}{7} = \frac{24}{7}$; г) $\mathrm{ctg}\, 2x = \frac{7}{24}$.

472. а) $\dfrac{\sin t}{2\cos^2\frac{t}{2}} = \dfrac{2\sin\frac{t}{2}(\cos\frac{t}{2})}{2\cos^2\frac{t}{2}} = \text{tg}\ \dfrac{t}{2}$.

б) $\dfrac{\cos t}{\cos\frac{t}{2}+\sin\frac{t}{2}} = \dfrac{\cos^2\frac{t}{2}-\sin^2\frac{t}{2}}{\cos\frac{t}{2}+\sin\frac{t}{2}} = \cos\ \dfrac{t}{2} - \sin\ \dfrac{t}{2}$.

в) $\dfrac{\sin 4t}{\cos 2t} = \dfrac{2\sin 2t\cos 2t}{\cos 2t} = 2\sin 2t$.

г) $\dfrac{\cos 2t-\sin 2t}{\cos 4t} = \dfrac{\cos 2t-\sin 2t}{(\sin 2t-\sin 2t)(\cos 2t+\cos 2t)} = \dfrac{1}{(\cos 2t+\sin 2t)}$.

473. а) $\dfrac{\sin 2t-2\sin t}{\cos t-1} = \dfrac{2\sin t(\cos t-1)}{\cos t-1}. = 2\sin t$.

б) $\dfrac{\cos 2t-\cos^2 t}{1-\cos^2 t} = \dfrac{\cos^2 t-\sin^2 t-\cos^2 t}{\sin^2 t} = -1$

в) $\sin 2t\ \text{ctg}\ t - 1 = 2\sin t\cos t\cdot\dfrac{\cos t}{\sin t} - 1 = 2\cos^2 t - 1 = \cos 2t$.

г) $(\text{tg}\ t + \text{ctg}\ t)\sin 2t = \dfrac{1}{\sin t\cos t}\cdot 2\sin t\cos t = 2$.

474. а) $\dfrac{2}{\text{tg}t+\text{ctg}t} = \dfrac{2}{\frac{1}{\sin t\cos t}} = \sin 2t$.

б) $\dfrac{2}{\text{tg}t-\text{ctg}t} = \dfrac{2}{\frac{-\cos 2t}{\sin t\cos t}} = \dfrac{\sin 2t}{-\cos 2t} = -\text{tg}\ 2t$.

475. а) $(1-\text{tg}^2 t)\cos^2 t = \cos^2 t - \sin^2 t = \cos 2t$.

б) $2\cos^2\dfrac{\pi+t}{4} - 2\sin^2\dfrac{\pi+t}{4} = 2\cos\left(\dfrac{\pi}{2}+\dfrac{t}{2}\right) = -2\sin\ \dfrac{t}{2}$.

476. а) $(\sin t-\cos t)^2 = 1-\sin 2t$, $\sin^2 t - 2\sin t\cos t + \cos^2 t = 1 - \sin 2t$.
б) $2\cos^2 t = 1 + \cos 2t$, $1+\cos 2t = \sin^2 t + \cos^2 t - \sin^2 t + \cos^2 t = 2\cos^2 t$.
в) $(\sin t+\cos t)^2 = 1+\sin 2t$, $\sin^2 t + \cos^2 t + 2\sin t\cos t = 1 + \sin 2t$.
г) $2\sin^2 t = 1-\cos 2t$, $1-\cos 2t = \sin^2 t + \cos^2 t - \cos^2 t + \sin^2 t = 2\sin^2 t$.
477. а) $\cos^4 t - \sin^4 t = \cos 2t$.
$\cos 2t = (\cos^2 t - \sin^2 t)\cdot 1 = (\cos^2 t-\sin^2 t)(\cos^2 t+\sin^2 t) = \cos^4 t - \sin^4 t$.

б) $\cos^4 t + \sin^4 t = 1 - \dfrac{1}{2}\cdot\sin^2 2t$.

$\cos^4 t + \sin^4 t + 2\sin^2 t\cos^2 t - 2\sin^2 t\cos^2 t = (\sin^2 t+\cos^2 t)^2 - \dfrac{1}{2}\sin^2 2t = 1 - \dfrac{1}{2}\sin^2 2t$.

478. а) ctg t – sin 2t = ctg t cos 2t.

$$\frac{\cos t - 2\sin^2 t\cos t}{\sin t} = \frac{\cos t\,(\cos^2 t - \sin^2 t)}{\sin t} = \frac{\cos t \cdot \cos 2t}{\sin t} = \text{ctg } t \cos 2t$$

б) sin 2t – tg t = cos 2t tg t

$$\frac{2\sin t\cos^2 t - \sin t}{\cos t} = \frac{\left(\cos^2 t - \sin^2 t\right)\sin t}{\cos t} = \frac{\cos 2t \sin t}{\cos t} = \cos 2t \text{ tg } t$$

479. а) sin 2x – 2 cos x = 0.

$2 \cos x = (\sin x - 1) = 0$, $\cos x = 0$, $x = \frac{\pi}{2} + \pi n$, $\sin x = 1$, $x = \frac{\pi}{2} + 2\pi n$;

б) $2 \sin x = \sin 2x$, $2 \sin x (\cos x - 1) = 0$,
$\sin x = 0$, $\cos x = 1$; $x = \pi n$, $x = 2\pi n$.
в) $\sin 2x - \sin x = 0$, $\sin x (2 \cos x - 1) = 0$, $\sin x (2 \cos x - 1) = 0$,

$\sin x = 0$, $x = \pi n$, $\cos x = \frac{1}{2}$, $x = \pm \frac{\pi}{3} + 2\pi n$;

г) $\sin 2x - \cos x = 0$, $\cos x (2 \sin x - 1) = 0$, $\cos x = 0$,

$x = \frac{\pi}{2} + \pi n$, $\sin x = \frac{1}{2}$, $x = (-1)^k \frac{\pi}{6} + \pi k$.

480. а) sin x cos x = 1.

$\frac{1}{2} \sin x \cos x = \frac{1}{2}$, $\sin 2x = \frac{1}{2}$; $2x = (-1)^n \frac{\pi}{6} + 2\pi n$,

$x = (-1)^n \frac{\pi}{12} + \pi n$, $\sin 2x = 2$. решений нет.

б) $\sin 4x \cos 4x = \frac{1}{2}$.

$\sin 8x = 1$, $8x = \frac{\pi}{2} + \pi n$, $x = \frac{\pi}{16} + \frac{\pi n}{8}$;

в) $\cos^2 \frac{x}{3} - \sin^2 \frac{x}{3} = \frac{1}{2}$,

$\cos \frac{2x}{3} = \frac{1}{2}$, $\frac{2x}{3} = \pm \frac{\pi}{3} + 2\pi n$, $x = \pm \frac{\pi}{2} + 3\pi n$;

г) $\sin^2 x - \cos^2 x = \frac{1}{2}$,

$\cos 2x = -\frac{1}{2}$, $2x = \pm \frac{2\pi}{3} + 2\pi n$, $x = \pm \frac{\pi}{3} + \pi n$.

481. а) cos 2x + 3 sin x = 1.
$1 - 2 \sin^2 x + 3 \sin x = 1$, $\sin x (2\sin x - 3) = 0$, $\sin x = 0$, $x = \pi n$;
б) $\sin^2 x = -\cos 2x$,

$\sin^2 x + 1 - 2\sin^2 x = 0$, $\sin^2 x = 1$, $\sin x = \pm 1$, $x = \frac{\pi}{2} + \pi n$;

в) $\cos 2x = \cos^2 2x$, $2 \cos^2 2x - 1 - \cos^2 x = 0$, $\cos^2 2x = 1$, $\cos x = 1$, $x = \pi n$;

г) $\cos 2x = 2\sin^2 x$, $1 - 2\sin^2 x = 2\sin^2 x$, $\sin x = \pm\frac{1}{2}$,

$x = (-1)^{k+1}\frac{\pi}{6} + \pi$; $x = (-1)^{n+1}\frac{\pi}{6} + \pi n$.

482. а) $\sin 11°\,15' \cdot \cos 11°\,15' \cos 22°\;30' \cos 45° =$

$= \frac{1}{2}\sin 22°\;30' \cos 22°\,30' \cos 45° = \frac{1}{8}\sin 90° = \frac{1}{8}$.

б) $\sin\frac{\pi}{48}\cos\frac{\pi}{48}\cos\frac{\pi}{24}\cos\frac{\pi}{12} = \frac{1}{8}\sin\frac{\pi}{6} = \frac{1}{16}$.

483. а) $\frac{1+\cos 40^\circ + \cos 80^\circ}{\sin 80^\circ + \sin 40^\circ} tg 40^\circ = \frac{2\cos^2 40^\circ + \cos 40^\circ}{\sin 40^\circ(2\cos 40^\circ + 1)} \cdot tg 40^\circ =$

$=$ tg 40° ctg 40° = 1

б) $\frac{1-\cos 25^\circ + \cos 50^\circ}{\sin 50^\circ - \sin 25^\circ} - \text{tg}65^\circ = \frac{2\cos^2 25^\circ - \cos 25^\circ}{2\sin 25^\circ \cos 25^\circ - \sin 25^\circ} - tg 65^\circ =$

$= \frac{\cos 25^\circ}{\sin 25^\circ} \cdot \frac{2\cos 25^\circ - 1}{2\cos 25^\circ - 1} - \text{tg}65° = \text{ctg}25° - \text{tg}65° = \text{tg}65° - \text{tg}65° = 0$.

484. $\text{tg } x = \frac{3}{4}$, $\pi < x < \frac{3\pi}{2}$;

$\cos x = -\frac{4}{5}$, $\sin x = -\frac{3}{5}$;

а) $\sin 2x = 2 \cdot \frac{3}{5} \cdot \frac{4}{5} = \frac{24}{25}$;

б) $\cos 2x = \cos^2 x - \sin^2 x = \frac{16}{25} - \frac{9}{25} = \frac{7}{25}$;

в) $\text{tg } 2x = \frac{24}{25} \cdot \frac{25}{7} = \frac{24}{7}$;

г) $\text{ctg } 2x = \frac{7}{24}$.

485. $\text{ctg } x = -\frac{4}{3}$, $\frac{3\pi}{2} < x < 2\pi$; $\text{tg } x = -\frac{3}{4}$;

$\cos = \frac{4}{5}$ $\sin x = -\frac{3}{5}$

а) $\sin 2x = -2 \cdot \frac{4}{5} \cdot \frac{3}{5} = -\frac{24}{25}$;

б) $\cos 2x = \cos^2 x - \sin^2 x = \frac{16}{25} - \frac{9}{25} = \frac{7}{25}$;

в) $\text{tg } 2x = -\frac{24}{25} \cdot \frac{25}{7} = -\frac{24}{7}$; г) $\text{ctg } 2x = -\frac{7}{24}$.

486. а) $\frac{\cos 2t}{\sin t\cos t+\sin^2 t}=\mathrm{ctg}(\pi+t)-1$;

$\frac{\cos 2t}{\sin t(\cos t+\sin t)}=\frac{\cos t-\sin t}{\sin t}=ctg\,t-1=ctg(\pi+t)-1$.

б) (ctg t – tg t) sin 2t = 2 cos 2t;

$\frac{\cos t}{\sin t}\;2\sin t\;\cos t-\frac{\sin t}{\cos t}\;2\sin t\cos t=2\cos^2 t-2\sin^2 t=2\cos 2t.$

487. а) $\frac{\sin 2t-2\sin(\frac{\pi}{2}-t)}{\cos(\frac{\pi}{2}-t)-2\sin^2 t}=-2\,\mathrm{ctg}\,t$, $\frac{2\cos t(\sin t-1)}{-\sin t(\sin t-1)}=-2\,\mathrm{ctg}\,t$;

б) $\frac{1-\cos 2t+\sin 2t}{1+\cos 2t+\sin 2t}\;\mathrm{tg}(\frac{\pi}{2}-t)=1$,

$\frac{2\sin^2 2t+\sin 2t}{2\cos^2 t+\sin 2t}\,\mathrm{ctg}\,t=\frac{2\sin t\,(\sin t+\cos t)}{2\cos t\,(\cos t+\sin t)}\;\mathrm{ctg}\,t=1.$

488. а) $\sin 2\alpha=\frac{1}{3}$,

$\sin^4\alpha+\cos^4\alpha=1-2\sin^2\alpha\cos^2\alpha=1-2\cdot\frac{1}{36}=\frac{17}{18}$.

б) $\sin^4\alpha+\cos^4\alpha=\frac{49}{50}$, $\quad\frac{\pi}{2}<\alpha<\pi$;

$1-2\sin^2\alpha\cos^2\alpha=\frac{49}{50}$, $\sin^2\alpha\cos^2\alpha=\frac{1}{100}$, $\sin\alpha\cos\alpha=-\frac{1}{10}$,

$\sin 2\alpha=-\frac{1}{5}$.

489. а) $\sin 3x=3\sin x-4\sin^3 x$.

$\sin 3x=\sin(x+2x)=\sin x\cos 2x+\sin 2x\cos x=$

$\sin x-2\sin^3 x+2\sin x-2\sin^3 x=3\sin x-4\sin^3 x$.

б) $\cos 3x=4\cos^3 x-3\cos x$.

$\cos(x+2x)=\cos x\cos 2x-\sin x\sin 2x=2\cos^3 x-\cos x-2\cos x\sin x=$

$2\cos^3 x-\cos x-2\cos x+2\cos^3 x=4\cos^3 x-3\cos x$.

490. а) $\cos x\cos 2x=\frac{\sin 4x}{4\sin x}$

$\frac{\sin 4x}{4\sin x}=\frac{2\sin 2x\cos 2x}{4\sin x}=\cos x\cos 2x.$

б) $\cos x\cos 2x\cos 4x=\frac{\sin 8x}{8\sin x}$.

$\frac{2\sin 4x\cos 4x}{8\sin x}=\frac{8\sin x\cos x\cos 2x\cos 4x}{8\sin x}=\cos x\cos 2x\cos 4x.$

в) $\sin x \cos 2x = \frac{\sin 4x}{4\cos x}$.

$\frac{\sin 4x}{4\cos x} = \frac{4\sin x \cos x \cos 2x}{4\cos x} = \sin x \cos 2x$.

г) $\sin x \cos 2x \cos 4x = \frac{\sin 8x}{8\cos x}$

$\frac{\sin 8x}{8\cos x} = \frac{8\sin x \cos x \cos 2x \cos 4x}{8\cos x} = \sin x \cos 2x \cos 4x$.

491. а) $\sin 18° \cos 18° \cos 36° = \frac{1}{4} \sin 72°$.

$\frac{1}{2} \sin 36° \cos 36° = \frac{1}{4} \sin 72°$.

б) $\sin 18° \cos 36° = \frac{1}{4}$; т. к. $\sin 18° \cos 18° \cos 36° = \frac{1}{4} \sin 72°$.

$\sin 18° \cos 18° \cos 36° = \frac{1}{4} \cdot \cos 18° \Rightarrow \sin 18° \cos 36° = \frac{1}{4}$.

492. а) $\cos\frac{\pi}{33} \cos\frac{2\pi}{33} \cos\frac{4\pi}{33} \cos\frac{8\pi}{33} \cos\frac{16\pi}{33} = \frac{\sin\frac{32\pi}{33}}{32\sin\frac{\pi}{33}} = \frac{1}{32}$.

б) $\cos\frac{\pi}{7} \cos\frac{2\pi}{7} \cos\frac{4\pi}{7} = \frac{\sin\frac{8\pi}{7}}{8\sin\frac{\pi}{7}} = -\frac{1}{8}$.

493. а) $2 - \cos 2x + 3\sin x = 0$;

$2\sin^2 x - 1 + 2 + 3\sin x = 0$; $2\sin^2 x + 3\sin x + 1 = 0$;

$\sin x = \frac{-3-1}{4} = -1$; $x = -\frac{\pi}{2} + 2\pi n$; $\sin x = -\frac{1}{2}$; $x = (-1)^{k+1}\frac{\pi}{6} + \pi k$.

б) $\cos 6x - \cos 3x - 2 = 0$. $2\cos^2 3x - \cos 3x - 3 = 0$.

$\cos 3x = \frac{1+5}{4}$ решений нет.

$\cos 3x = -1$, $x = \frac{\pi}{3} + \frac{2\pi n}{3}$.

494. а) $26\sin x \cos x - \cos 4x + 7 = 0$;

$13\sin 2x - 1 + 2\sin^2 2x + 7 = 0$; $2\sin^2 2x + 13\sin 2x + 6 = 0$;

$\sin 2x = \frac{-13-11}{4}$ решений нет; $\sin 2x = -\frac{1}{2}$;

$2x = (-1)^{k+1}\frac{\pi}{6} + \pi k$; $x = (-1)^{k+1}\frac{\pi}{12} + \frac{\pi k}{2}$.

б) $\sin^4 x + \cos^4 x = \sin x \cos x$.

$\sin^4 x + 2\sin^2 x\cos^2 x + \cos^4 x = 2\sin^2 x\cos^2 x + \frac{1}{2}\sin 2x;$

$\frac{1}{2}\sin^2 2x + \frac{1}{2}\sin 2x - 1 = 0;$

1) $\sin 2x = 1;\ 2x = \frac{\pi}{2} + 2\pi n;\ x = \frac{\pi}{4} + \pi n.$

2) $\sin 2x = -2$; нет решений

495. а) $3\sin 2x + \cos 2x = 1.$

$6\sin x\cos x - 2\sin^2 x = 0;\ 2\sin x(3\cos x - \sin x) = 0;$

$\sin x = 3\cos x;\quad \operatorname{tg} x = 3;\ x = \operatorname{arctg} 3 + \pi n,\ \sin x = 0;\quad x = \pi n;$

б) $\cos 4x + 2\sin 4x = 1.$

$-2\sin^2 2x + 4\sin 2x\cos 2x = 0;\ \sin 2x(2\cos 2x - \sin 2x) = 0.$

$\cos 2x = \frac{1}{2}\sin 2x,\ \operatorname{tg} 2x = 2,\ 2x = \operatorname{arctg} 2 + \pi n,\ x = \frac{1}{2}\operatorname{arctg} 2 + \frac{\pi n}{2},$

$\sin 2x = 0,\ 2x = \pi n,\ x = \frac{\pi n}{2}.$

496. а) $4\sin x + \sin 2x = 0,\qquad x \in [0; 2\pi]$

$2\sin x(2 + \cos x) = 0,\ \sin x = 0,\ \cos x = -2$ – нет решений

$x = 0,\ x = \pi,\ x = 2\pi.$

б) $\cos^2(3x + \frac{\pi}{4}) - \sin^2(3x + \frac{\pi}{4}) + \frac{\sqrt{3}}{2} = 0,\ x \in [\frac{3\pi}{4}, \pi]$

$\cos(6x + \frac{2\pi}{4}) = -\frac{\sqrt{3}}{2},\quad \sin 6x = \frac{\sqrt{3}}{2},$

$6x = (-1)^k\frac{\pi}{3} + \pi k,\quad x = (-1)^k\frac{\pi}{18} + \frac{\pi k}{6}\ \Rightarrow\ x = \frac{7\pi}{18}.$

497. а) $(\cos x - \sin x)^2 = 1 - 2\sin x = 2x,\ x \in \left[\frac{20\pi}{9}, \frac{28\pi}{9}\right]$

$1 - \sin 2x = 1 - 2\sin 2x,\quad \sin 2x = 0,\ x = \frac{\pi n}{2},$ 2 корня.

б) $2\cos^2(2x - \frac{\pi}{4}) - 2\sin^2(2x - \frac{\pi}{4}) + 1 = 0,\ x \in \left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$

$\cos(4x - \frac{\pi}{2}) = -\frac{1}{2},\quad \sin 4x = -\frac{1}{2},\quad 4x = (-1)^{k+1}\frac{\pi}{6} + \pi k,$

$x = (-1)^{k+1}\frac{\pi}{24} + \frac{\pi k}{4},$ 3 корня.

498. а) $\sin x = \frac{2\operatorname{tg}\frac{x}{2}}{1 + \operatorname{tg}^2\frac{x}{2}} = 2\operatorname{tg}\frac{x}{2}\cos^2\frac{x}{2} = 2\sin\frac{x}{2}\cos\frac{x}{2} = \sin x$

б) $\cos x = \dfrac{1-\mathrm{tg}^2\frac{x}{2}}{1+\mathrm{tg}^2\frac{x}{2}} = (1-\mathrm{tg}^2\frac{x}{2})\cos^2\frac{x}{2} = \cos^2\frac{x}{2} - \sin^2\frac{x}{2} = \cos x.$

499. а) $\sin x + 7\cos x = 5.$

$$\frac{2\mathrm{tg}\frac{x}{2}}{1+\mathrm{tg}^2\frac{x}{2}} + \frac{7-7\mathrm{tg}^2\frac{x}{2}}{1+\mathrm{tg}^2\frac{x}{2}} = 2\,\mathrm{tg}\frac{x}{2} + 7 - 7\,\mathrm{tg}^2\frac{x}{2} = 5 + 5\,\mathrm{tg}^2\frac{x}{2}.$$

$12\,\mathrm{tg}^2\frac{x}{2} - 2\,\mathrm{tg}\frac{x}{2} - 2 = 0,\quad 6\,\mathrm{tg}^2\frac{x}{2} - \mathrm{tg}\frac{x}{2} - 1 = 0,$

$\mathrm{tg}\frac{x}{2} = \frac{1+5}{12} = \frac{1}{2},\qquad x = 2\,\mathrm{arctg}\frac{1}{2} + 2\pi n$

$\mathrm{tg}\frac{x}{2} = \frac{1-5}{12} = -\frac{1}{3},\qquad x = 2\,\mathrm{arctg}(-\frac{1}{3}) + 2\pi n$

$x = -2\,\mathrm{arctg}\frac{1}{3} + 2\pi n.$

б) $5\sin x + 10\cos x + 2 = 0,\quad \dfrac{10\mathrm{tg}\frac{x}{2}}{1+\mathrm{tg}^2\frac{x}{2}} + \dfrac{10-10\mathrm{tg}^2\frac{x}{2}}{1+\mathrm{tg}^2\frac{x}{2}} + 2 = 0.$

$10\,\mathrm{tg}\frac{x}{2} + 10 - 10\,\mathrm{tg}^2\frac{x}{2} + 2 + 2\,\mathrm{tg}^2\frac{x}{2} = 0.$

$4\,\mathrm{tg}^2\frac{x}{2} - 5\,\mathrm{tg}^2\frac{x}{2} - 6 = 0;\ \ \mathrm{tg}\frac{x}{2} = \frac{5+11}{8} = 2;\ \ x = 2\,\mathrm{arctg}\,2 + 2\pi n.$

$\mathrm{tg}\frac{x}{2} = -\frac{3}{4};\ \ x = -2\,\mathrm{arctg}\frac{3}{4} + 2\pi n.$

500. а) $\cos\frac{1}{x^2-\pi}\sin 2x = 8\sin x\cos x;$

$\cos\frac{1}{x^2-\pi}\sin 2x = 4\sin 2x;\ \ \sin 2x = 0.$

$x = \frac{\pi n}{2},\ n \neq \pm 2.\ \ \cos\frac{1}{x^2-\pi} = 4.$ решений нет.

б) $16\sin x\cos x + \sin 2x\sin\frac{1}{x} = 0;\ \ 8\sin 2x + \sin 2x\sin\frac{1}{x} = 0;$

$\sin 2x = 0;\ \ x = \frac{\pi n}{2},\ n \neq 0.\ \ \sin\frac{1}{x} = -8$ – решений нет

501. а) $\sin 2x + 2\sin x = 2 - 2\cos x,\ \ \sin x\cos x + \sin x + \cos x = 1.$

$(\sin x + \cos x) = t;\ \ \sin x\cos x = \frac{1}{2}t^2 - \frac{1}{2};$

$t^2 - 1 + 2t - 2 = 0$; $t^2 + 2t - 3 = 0$; $t = -3$ – решений нет.

$t = 1$, $\sin x + \cos x = 1$, $\sin (x + \frac{\pi}{4}) = \frac{\sqrt{2}}{2}$.

$x + \frac{\pi}{4} = (-1)^k \frac{\pi}{4} + \pi k$. $x = (-1)^k \frac{\pi}{4} + \frac{\pi}{4}\pi k$.

б) $4\sin 2x + 8 (\sin x - \cos x) = 7$.

$\sin x - \cos x = t$; $1 - \sin 2x = t^2$; $\sin 2x = 1 - t^2$;

$4 - 4t^2 + 8t - 7 = 0$; $4t^2 - 8t + 3 = 0$; $t = \frac{4+2}{4} = \frac{3}{2}$;

$\sin x + \cos x = \frac{3}{2}$ – решений нет.

$t = \frac{1}{2}$; $\sin x + \cos x = \frac{1}{2}$; $\sin (x + \frac{\pi}{4}) = \frac{\sqrt{2}}{4}$.

$x + \frac{\pi}{4} = (-1)^k \arcsin \frac{\sqrt{2}}{4} + \pi k$. $x = (-1)^k \arcsin \frac{\sqrt{2}}{4} + \pi k - \frac{\pi}{4}$.

502. а) $\sin 2x \cos 2x < \frac{1}{4}$. $\sin 4x < \frac{1}{2}$.

$4x \in (-\frac{7\pi}{6} + 2\pi n; \frac{\pi}{6} + 2\pi n)$, $x \in (-\frac{7\pi}{24} + \frac{\pi n}{2}; \frac{\pi}{24} + \frac{\pi n}{2})$.

б) $\cos^2 \frac{x}{4} - \sin^2 \frac{x}{4} > \frac{1}{2}$; $\cos \frac{x}{2} > \frac{1}{2}$;

$\frac{x}{2} \in (-\frac{\pi}{3} + 2\pi n; \frac{\pi}{3} + 2\pi n)$, $x \in (-\frac{2\pi}{3} + 4\pi n; \frac{2\pi}{3} + 4\pi n)$.

503. а) $\cos^2 2x - \sin^2 2x \le -1$.

$\cos 4x \le -1$. $4x = \pi + 2\pi n$. $x = \frac{\pi}{4} + \frac{\pi n}{2}$.

б) $\sin 5x \cos 5x \ge \frac{1}{2}$. $\sin 10x \ge 1$. $10x = \frac{\pi}{2} + 2\pi n$. $x = \frac{\pi}{20} + \frac{\pi n}{5}$.

в) $\sin^2 3x - \cos^2 3x \le -1$. $\cos 6x \ge 1$. $6x = 2\pi n$. $x = \frac{\pi n}{3}$.

г) $\sin \frac{2x}{3} \cos \frac{2x}{3} \le -\frac{1}{2}$. $\sin \frac{4x}{3} \le -1$. $\frac{4x}{3} = -\frac{\pi}{2} + 2\pi n$. $x = -\frac{3\pi}{8} + \frac{3\pi n}{2}$.

§ 25. Формулы понижения степени

504. а) $\sin^2 2t = \frac{1 - \cos 4t}{2}$

$\frac{1 - \cos 4t}{2} = \frac{1}{2}(1 - \cos^2 2t + \sin^2 2t) = \frac{1}{2}(2\sin^2 2t) = \sin^2 2t$

б) $2\sin^2\frac{t}{2}+\cos t=1$. $2(1-\cos t)\cdot\frac{1}{2}+\cos t=1$.

в) $2\sin^2 2t=1+\sin\left(\frac{3t}{2}-4t\right)$. $1-\cos 4t=1+\sin\left(\frac{3t}{2}-4t\right)$.

г) $2\cos^2 t-\cos 2t=1$. $2\cos^2 t-\cos 2t=2\cos^2 t-\cos^2 t+\sin^2 t=1$.

505. а) $\cos^2 3t=\frac{1+\sin(\frac{\pi}{2}-6t)}{2}$, $\cos^2 3t=\frac{1+\cos 6t}{2}=\frac{1+\sin(\frac{\pi}{3}-6t)}{2}$.

б) $\frac{1-\cos t}{1+\cos t}=\text{tg}^2\frac{t}{2}$. $\text{tg}^2\frac{t}{2}=\frac{\sin^2\frac{t}{2}}{\cos^2\frac{t}{2}}=\frac{1-\cos t}{1+\cos t}$.

в) $\cos^2 3t=\frac{1-\cos(6t+(-3\pi))}{2}=\cos^2 3t=\frac{1+\cos 6t}{2}=\frac{1-\cos(6t-3\pi)}{2}$

г) $\frac{1-\cos t}{\sin t}=\text{tg}\frac{t}{2}$, $\frac{1-\cos t}{2}=\frac{2\sin^2\frac{t}{2}}{2\sin\frac{t}{2}\cos\frac{t}{2}}=\text{tg}\frac{t}{2}$.

506. а) $1+\sin\alpha=2\cos^2(45°-\frac{1}{2})$

$2\cos^2(45°-\frac{\alpha}{2})=1+\cos(90°-\alpha)=1+\sin\alpha$.

б) $2\sin^2(45°-\alpha)+\sin 2\alpha=1$, $1-\cos(90°-2\alpha)+\sin 2\alpha=1$,
$1-\sin 2\alpha+\sin 2\alpha=1$;

в) $1-\sin\alpha=2\sin^2(45°-\frac{\alpha}{2})$,

$2\sin^2(45°-\frac{\alpha}{2})=1-\cos(90°-\alpha)=1-\sin\alpha$;

г) $2\cos^2(45°+\alpha)+\sin 2\alpha=1$,
$1+\cos(90°+2\alpha)+\sin 2\alpha=1-\sin 2\alpha+\sin 2\alpha=1$.

507. а) $\sin 22{,}5°=\sqrt{\frac{1-\cos 45°}{2}}=\sqrt{\frac{2-\sqrt{2}}{2}}$.

б) $\sin 22{,}5°=\sqrt{\frac{1+\cos 45°}{2}}=\sqrt{\frac{2+\sqrt{2}}{2}}$.

в) $\sin\frac{3\pi}{8}=\sqrt{\frac{1-\cos\frac{3\pi}{4}}{2}}=\sqrt{\frac{2-\sqrt{2}}{2}}$.

г) $\cos\frac{3\pi}{8}=\sqrt{\frac{1+\cos\frac{3\pi}{4}}{2}}=\sqrt{\frac{2+\sqrt{2}}{2}}$.

508. а) $1 - \cos x = 2\sin\frac{x}{2}$, $1 - (1 - 2\sin^2\frac{x}{2}) = 2\sin\frac{x}{2}$;

$\sin^2\frac{x}{2} = \sin\frac{x}{2}$; $(\sin\frac{x}{2} - 1)\cdot\sin\frac{x}{2} = 0$;

$\left[\begin{array}{l}\sin\frac{x}{2}=0\\ \sin\frac{x}{2}=1\end{array}\right.$; $\left[\begin{array}{l}x=2\pi n\\ x=\pi+4\pi k\end{array}\right.$.

б) $1 + \cos x = 2\cos\frac{x}{2}$.

$1 + (2\cos^2\frac{x}{2} - 1) = 2\cos\frac{x}{2}$; $\cos^2\frac{x}{2} = \text{cox}\frac{x}{2}$;

$(\cos\frac{x}{2} - 1)\cdot\cos\frac{x}{2} = 0$; $\left[\begin{array}{l}\cos\frac{x}{2}=0\\ \cos\frac{x}{2}=1\end{array}\right.$; $\left[\begin{array}{l}x=\pi+2\pi n\\ x=4\pi k\end{array}\right.$

509. а) $1 - \cos x = \sin x\sin\frac{x}{2}$.

$1 - (1 - 2\sin^2\frac{x}{2}) = \sin x\sin\frac{x}{2}$; $2\sin^2\frac{x}{2} = 2\sin^2\frac{x}{2}\cos\frac{x}{2}$;

$\sin^2\frac{x}{2}(1 - \cos\frac{x}{2}) = 0$; $\left[\begin{array}{l}\sin\frac{x}{2}=0\\ \cos\frac{x}{2}=1\end{array}\right.$; $\left[\begin{array}{l}x=2\pi n\\ x=4\pi k\end{array}\right.$; $x = 2\pi n$.

б) $\sin x = \mathrm{tg}^2\frac{x}{2}(1 + \cos x)$,

$\sin x = \frac{1-\cos x}{1+\cos x}(1+\cos x)$; $\left\{\begin{array}{l}\sin x = 1-\cos x\\ 1+\cos x \ne 0\end{array}\right.$;

$\left\{\begin{array}{l}\sqrt{2}\sin x\left(x+\frac{\pi}{4}\right)=1\\ x\ne\pi+2\pi n\end{array}\right.$; $\left\{\begin{array}{l}\left[\begin{array}{l}x+\frac{\pi}{4}=\frac{\pi}{4}+2\pi k\\ x+\frac{\pi}{4}=\frac{3\pi}{4}+2\pi k\end{array}\right.\\ x\ne\pi+2\pi n\end{array}\right.$; $\left\{\begin{array}{l}\left[\begin{array}{l}x=2\pi k\\ x=\frac{\pi}{2}+2\pi k\end{array}\right.\\ x\ne\pi+2\pi n\end{array}\right.$; $\left[\begin{array}{l}x=2\pi k\\ x\frac{\pi}{2}+2\pi k\end{array}\right.$

510. а)$\sin^2 2x = 1$

$\left[\begin{array}{l}\sin 2x=1\\ \sin 2x=-1\end{array}\right.$; $\left[\begin{array}{l}2x=\frac{\pi}{2}+2\pi n\\ 2x=-\frac{\pi}{2}+2\pi k\end{array}\right.$;

$2x = \frac{\pi}{2} + \pi\ell$; $x = \frac{\pi}{4} + \frac{\pi\ell}{2}$

б) $\cos^2 4x = \frac{1}{2}$, $2\cos^2 4x - 1 = 0$; $\cos 8x = 0$;

$8x = \frac{\pi}{2} + \pi n$; $x = \frac{\pi}{16} + \frac{\pi n}{8}$.

в) $\sin^2 \frac{x}{2} = \frac{3}{4}$.

$\left[\begin{array}{l} \text{six}\frac{x}{2} = \frac{\sqrt{3}}{2} \\ \sin\frac{x}{2} = -\frac{\sqrt{3}}{2} \end{array}\right.$; $\left[\begin{array}{l} \frac{x}{2} = (-1)^n\frac{\pi}{3} + \pi n \\ \frac{x}{2} = (-1)^k\left(-\frac{\pi}{3}\right) + \pi k \end{array}\right.$;

$\left[\begin{array}{l} x = (-1)^n\frac{2\pi}{3} + 2\pi n \\ x = (-1)^k\left(-\frac{2\pi}{3}\right) + 2\pi k \end{array}\right.$; $x = \pm\frac{2\pi}{3} + 2\pi n$

г) $\cos^2 \frac{x}{4} = \frac{1}{4}$.

$\left[\begin{array}{l} \cos\frac{x}{4} = \frac{1}{2} \\ \cos\frac{x}{4} = -\frac{1}{2} \end{array}\right.$; $\left[\begin{array}{l} \frac{x}{4} = \pm\frac{\pi}{3} + 2\pi n \\ \frac{x}{4} = \pm\frac{2\pi}{3} + 2\pi k \end{array}\right.$; $\frac{x}{4} = \pm\frac{\pi}{3} + \pi n$; $x = \pm\frac{4\pi}{3} + 4\pi n$

511. $2\cos^2 \frac{x}{2} - \cos\frac{\pi}{9} = 1$

$(2\cos^2 \frac{x}{2} - 1) = \cos\frac{\pi}{9}$; $\cos x = \cos\frac{\pi}{9}$; $x = \pm\arccos(\cos\frac{\pi}{9}) + 2\pi n$

Отсюда имеем, что уравнение имеет отрезок $[-2\pi, 2\pi]$ 4 корня:

$\frac{\pi}{9} - 2\pi, -\frac{\pi}{9}, \frac{\pi}{9}, 2\pi - \frac{\pi}{9}$.

512. а) Т.к. $0 < t < \frac{\pi}{2}$, то $\cos\frac{t}{2} = \sqrt{\frac{1}{2}(1+\cos t)} = \sqrt{\frac{1}{2}\left(1+\frac{3}{4}\right)} = \frac{\sqrt{7}}{2\sqrt{2}}$,

$\sin\frac{t}{2} = \sqrt{\frac{1}{2}(1-\cos t)} = \frac{1}{2\sqrt{2}}$, $\operatorname{tg}\frac{t}{2} = \frac{\sin\frac{t}{2}}{\cos\frac{t}{2}} = \frac{1}{\sqrt{7}}$, $\operatorname{ctg}\frac{t}{2} = \frac{1}{\operatorname{tg}\frac{t}{2}} = \sqrt{7}$.

б) Имеем: $\cos^2 t = \frac{\operatorname{ctg}^2 t}{1+\operatorname{ctg}^2 t} = \frac{9/16}{1+9/16} = \frac{9}{25}$.

Т.к. $\pi < t < \frac{3\pi}{2}$ (т.е. $\cos t < 0$), получим $\cos t = -\frac{3}{5}$

Поскольку $\frac{\pi}{2} < \frac{t}{2} < \frac{3\pi}{4}$, то $\sin\frac{t}{2} > 0$, $\cos\frac{t}{2} < 0$, т.е. имеем:

$\sin\frac{t}{2} = \sqrt{\frac{1}{2}(1-\cos t)} = \sqrt{\frac{1}{2}\left(1+\frac{3}{5}\right)} = \frac{2}{\sqrt{5}}$;

$\cos\frac{t}{2} = \sqrt{\frac{1}{2}(1+\cos t)} = -\sqrt{\frac{1}{2}\left(1-\frac{3}{5}\right)} = -\frac{1}{\sqrt{5}}$;

$\operatorname{tg}\frac{t}{2} = \frac{\sin\frac{t}{2}}{\cos\frac{t}{2}} = -2$; $\operatorname{ctg}\frac{t}{2} = \frac{1}{tg\frac{t}{2}} = -\frac{1}{2}$.

513. а) Рассмотрим два случая:

1. $\frac{\pi}{2} < x < \frac{3\pi}{4}$. Тогда $\pi < 2x < \frac{3\pi}{2}$ и $\cos 2x = -\sqrt{1-\sin^2 2x} = -\frac{4}{5}$.

$\sin x = \sqrt{\frac{1}{2}(1-\cos 2x)} = \sqrt{\frac{1}{2}\left(1+\frac{4}{5}\right)} = \frac{3}{\sqrt{10}}$;

$\cos x = -\sqrt{\frac{1}{2}(1+\cos 2x)} = -\sqrt{\frac{1}{2}\left(1-\frac{4}{5}\right)} = -\frac{1}{\sqrt{10}}$;

$\operatorname{tg} x = \frac{\sin x}{\cos x} = -3$; $\operatorname{ctg} x = \frac{1}{tgx} = -\frac{1}{3}$.

2. $\frac{3\pi}{4} < x < \pi$. Тогда $\frac{3\pi}{2} < 2x < 2\pi$ и $\cos 2x = \sqrt{1-\sin^2 2x} = \frac{4}{5}$.

$\sin x = \sqrt{\frac{1}{2}(1-\cos 2x)} = \sqrt{\frac{1}{2}\left(1-\frac{4}{5}\right)} = \frac{1}{\sqrt{10}}$;

$\cos x = -\sqrt{\frac{1}{2}(1+\cos 2x)} = -\sqrt{\frac{1}{2}\left(1+\frac{4}{5}\right)} = -\frac{3}{\sqrt{10}}$;

$\operatorname{tg} x = \frac{\sin x}{\cos x} = -\frac{1}{3}$; $\operatorname{ctg} x = \frac{1}{tgx} = -3$.

б) Т.к. $\pi < x < \frac{5\pi}{4}$, то $2\pi < 2x < \frac{5\pi}{2}$, т.е. $\cos 2x =$

$= \frac{1}{\sqrt{1+tg^2 2x}} = \frac{1}{\sqrt{1+9/16}} = \frac{4}{5}$

$\sin x = -\sqrt{\frac{1}{2}(1-\cos 2x)} = -\sqrt{\frac{1}{2}\left(1-\frac{4}{5}\right)} = -\frac{1}{\sqrt{10}}$;

$\cos x = -\sqrt{\frac{1}{2}(1+\cos 2x)} = -\sqrt{\frac{1}{2}\left(1+\frac{4}{5}\right)} = -\frac{3}{\sqrt{10}}$;

$\operatorname{tg} x = \frac{\sin x}{\cos x} = \frac{1}{3}$; $\operatorname{ctg} x = \frac{1}{tgx} = 3$.

514. а) $\frac{\sin 2t}{1+\cos 2t}\cdot\frac{\cos t}{1+\cos t}=tg\frac{t}{2}$.

$$\frac{\sin 2t}{1+\cos 2t}\cdot\frac{\cos t}{1+\cos t}=\frac{2\sin t\cos^2 t}{2\cos^2 t\cdot(1+\cos t)}=\frac{\sin t}{1+\cos t}=tg\frac{t}{2}$$

б) $\frac{\sin 2t}{1+\cos 2t}\cdot\frac{\cos t}{1+\cos t}\cdot\frac{\cos\frac{t}{2}}{1+\cos\frac{t}{2}}=tg\frac{t}{4}$.

$$\frac{\sin 2t}{1+\cos 2t}\cdot\frac{\cos t}{1+\cos t}\cdot\frac{\cos\frac{t}{2}}{1+\cos\frac{t}{2}}=\frac{2\sin t\cos^2 t}{2\cos^2 t(1+\cos t)}\cdot\frac{\cos\frac{t}{2}}{1+\cos\frac{t}{2}}=$$

$$=\frac{\sin t}{1+\cos t}\cdot\frac{\cos\frac{t}{2}}{1+\cos\frac{t}{2}}=tg\frac{t}{2}\cdot\frac{\cos\frac{t}{2}}{1+\cos\frac{t}{2}}=\frac{\sin\frac{t}{2}}{1+\cos\frac{t}{2}}=tg\frac{t}{4}$$

515. а) $\frac{1-\cos 2t+\sin 2t}{1+\sin 2t+\cos 2t}=tgt$.

$$\frac{1-(1-2\sin^2 t)+2\sin t\cos t}{1+2\sin t\cos t+2\cos^2 t-1}=\frac{\sin t(\sin t+\cos t)}{\cos t(\sin t+\cos t)}=\frac{\sin t}{\cos t}=\text{tg}\,t.$$

б) $\frac{1+\cos 2t-\sin 2t}{1+\sin 2t+\cos 2t}=\text{tg}\,(\frac{\pi}{4}-t)$.

$$\frac{1+\cos 2t-\sin 2t}{1+\sin 2t+\cos 2t}=\frac{1+2\cos^2 t-1-2\sin t\cos t}{1+2\sin t\cos t+2\cos^2 t-1}=$$

$$=\frac{\cos t-\sin t}{\cos t+\sin t}=\frac{\sqrt{2}\sin\left(\frac{\pi}{4}-t\right)}{\sqrt{2}\cos\left(\frac{\pi}{4}-t\right)}=tg\left(\frac{\pi}{4}-t\right).$$

516. а) $\cos^2 t-\cos^2(\frac{\pi}{4}-t).=\frac{1}{\sqrt{2}}\sin(\frac{\pi}{4}-2t)$.

$$\frac{1}{\sqrt{2}}\sin\left(\frac{\pi}{4}-2t\right)=\frac{1}{\sqrt{2}}\left(\sin\frac{\pi}{4}\cos 2t-\sin 2t\cos\frac{\pi}{4}\right)=$$

$$=\frac{1}{2}(\cos 2t-\sin 2t)=\frac{1}{2}\left(\cos 2t-\cos\left(\frac{\pi}{2}-2t\right)\right)=$$

$$=\frac{1}{2}\left(2\cos^2 t-1-2\cos^2\left(\frac{\pi}{4}-t\right)+1\right)=\cos^2 t-\cos^2\left(\frac{\pi}{4}-t\right).$$

б) $\sin^2 t - \sin^2 (\frac{\pi}{4} - t) = \frac{1}{\sqrt{2}} \sin (2t - \frac{\pi}{4})$.

$$\frac{1}{\sqrt{2}}\sin\left(2t-\frac{\pi}{4}\right)=\frac{1}{\sqrt{2}}\left(\sin 2t\cos\frac{\pi}{4}-\sin\frac{\pi}{4}\cos 2t\right)=$$

$$=\frac{1}{2}(\sin 2t-\cos 2t)=\frac{1}{2}\left(\cos\left(\frac{\pi}{2}-2t\right)-\cos 2t\right)=$$

$$=\frac{1}{2}\left(1-2\sin^2\left(\frac{\pi}{4}-t\right)-1+2\sin^2 t\right)=\sin^2 t-\sin^2\left(\frac{\pi}{4}-t\right).$$

517. а) $f(x) = 2\cos 2x + \sin^2 x = 2\cos 2x + \frac{1}{2}(1 - 2\cos 2x) = \frac{3}{2}\cos 2x + \frac{1}{2}$

Поскольку наибольшее значение функции $y = \cos 2x$ равно 1, а наименьшее –1, то наибольшее значение функции $f(x)$ равно 2, а наименьшее –1.
б) $f(x) = 2\sin^2 3x - \cos 6x = (1 - \cos 6x) - \cos 6x = 1 - 2\cos 6x$.
Поскольку наибольшее значение функции $y = \cos 6x$ равно 1, а наименьшее –1, то наибольшее значение функции $f(x)$ равно 3, а наименьшее –1.

518. а) $t \in [\frac{\pi}{2}; \pi]$, т.е. $\sin t \geq 0$, $\cos t \leq 0$.

$$\sqrt{1-\cos 2x}+\sqrt{1+\cos 2t}=\sqrt{2}\left(|\sin t|+|\cos t|\right)=\sqrt{2}(\sin t-\cos t)=2\sin\left(t-\frac{\pi}{4}\right);$$

б) $t \in [\frac{3\pi}{2}; 2\pi]$, т.е. $\sin t \leq 0$, $\cos t \geq 0$

$$\sqrt{1-\cos 2x}+\sqrt{1+\cos 2t}=\sqrt{2}\left(|\sin t|+|\cos t|\right)=\sqrt{2}(-\sin t+\cos t)=2\sin\left(\frac{\pi}{4}-t\right);$$

в) $t \in [0; \frac{\pi}{2}]$, т.е. $\sin t \geq 0$, $\cos t \geq 0$

$$\sqrt{1-\cos 2x}+\sqrt{1+\cos 2t}=\sqrt{2}\left(|\sin t|+|\cos t|\right)=\sqrt{2}(\sin t+\cos t)=2\sin\left(t+\frac{\pi}{4}\right)$$

г) $t \in [\pi; \frac{3\pi}{2}]$,т.е. $\sin t \leq 0$, $\cos t \leq 0$

$$\sqrt{1-\cos 2x}+\sqrt{1+\cos 2t}=\sqrt{2}\left(|\sin t|+|\cos t|\right)=-\sqrt{2}(\sin t+\cos t)=-2\sin\left(t+\frac{\pi}{4}\right).$$

519. $\cos 2x = \frac{5}{13} = 2\cos^2 x - 1 = 1 - 2\sin^2 x$,

откуда $\cos^2 x = \frac{1}{2}\left(1+\frac{5}{13}\right)=\frac{9}{13}$, $\sin^2 x = \frac{1}{2}\left(1-\frac{5}{13}\right)=\frac{4}{13}$.

а) $\sin^4 x + \cos^4 x = (\sin^2 x)^2 + (\cos^2 x)^2 = \left(\frac{4}{13}\right)^2 + \left(\frac{9}{13}\right)^2 = \frac{97}{169}$.

б) $\sin^8 x + \cos^8 x = (\sin^2 x)^4 + (\cos^2 x)^4 = \left(\frac{4}{13}\right)^4 + \left(\frac{9}{13}\right)^4 = \frac{6817}{28561}$.

520. а) $\sin^2 (2x - \frac{\pi}{6}) = \frac{3}{4}$.

$\frac{1}{2}\left(1-\cos\left(4x-\frac{\pi}{3}\right)\right)=\frac{3}{4}$; $\cos\left(4x-\frac{\pi}{3}\right)=-\frac{1}{2}$

$4x - \frac{\pi}{3} = \pm\frac{2\pi}{3} + 2\pi n$; $\left[\begin{array}{l} x=\frac{\pi}{4}+\frac{\pi n}{2} \\ x=-\frac{\pi}{12}+\frac{\pi k}{2} \end{array}\right.$.

б) $\cos^2 (x + \frac{\pi}{3}) = 1$.

$\frac{1}{2}\left(1+\cos\left(2x-\frac{2\pi}{3}\right)\right)=1$; $\cos\left(2x+\frac{2\pi}{3}\right)=1$;

$2x + \frac{2\pi}{3} = 2\pi n$; $x = -\frac{\pi}{3} + \pi n$.

в) $\sin^2\left(x+\frac{\pi}{2}\right)=\frac{1}{2}$, $\frac{1}{2}(1-\cos(2x+\pi))=\frac{1}{2}$; $\frac{1}{2}(1+\cos 2x)=\frac{1}{2}$;

$\cos 2x = -1$; $2x = \pi + 2\pi n$; $x = \frac{\pi}{2} + \pi n$

г) $\cos^2 (3x - \frac{\pi}{4}) = \frac{3}{4}$. $\frac{1}{2}\left(1+\cos\left(6x-\frac{\pi}{2}\right)\right)=\frac{3}{4}$; $\frac{1}{2}(1+\sin 6x)=\frac{3}{4}$;

$\sin 6x = \frac{1}{2}$; $6x = (-1)^n \frac{\pi}{6} + \pi n$; $x = (-1)^n \frac{\pi}{36} + \frac{\pi n}{6}$

521. а) $4 \sin^2 x + \sin^2 2x = 3$.
$2 (1 - \cos 2x) + 1 - \cos^2 2x = 3$; $\cos^2 2x + 2\text{cox } 2x = 0$;
$\cos 2x (\cos 2x + 2) = 0$; $\cos 2x = 0$ (т.к. $\cos 2x + 2 > 0$ для всех x);

$2x = \frac{\pi}{2} + \pi n$; $x = \frac{\pi}{4} + \frac{\pi n}{2}$.

б) $4 \cos^2 2x + 8 \cos^2 x = 7$. $4 \cos^2 2x + 4(1 + \cos 2x) = 7$;
$4 \cos^2 2x + 4 \cos 2x - 3 = 0$;

$\cos 2x = \frac{-2\pm\sqrt{4+12}}{4} = \frac{-2\pm 4}{4} = \left[\begin{array}{l} \frac{1}{2} \\ -\frac{3}{2} \end{array}\right.$;

$\cos 2x = \frac{1}{2}$ (т.к. $\cos 2x \neq -\frac{3}{2}$ при всех x);

$2x = \pm\frac{\pi}{3} + 2\pi n$; $x = \pm\frac{\pi}{6} + \pi n$.

522. а) $4 \sin^2 3x < 3$.

$2(1 - \cos 6x) < 3$; $\cos 6x > -\frac{1}{2}$; $-\frac{2\pi}{3} + 2\pi n < 6x < \frac{2\pi}{3} + 2\pi n$;

$-\frac{\pi}{9} + \frac{\pi n}{3} < x < \frac{\pi}{9} + \frac{\pi n}{3}$.

б) $4 \cos^2 \frac{x}{4} > 1$. $2(1 + \cos \frac{x}{2}) > 1$; $\cos \frac{x}{2} > -\frac{1}{2}$;

$-\frac{2\pi}{3} + 2\pi n < \frac{x}{2} < \frac{2\pi}{3} + 2\pi n$; $-\frac{4\pi}{3} + 4\pi n < x < \frac{4\pi}{3} + 4\pi n$;

§ 26. Преобразование сумм тригонометрических функций в произведение

523. а) $\sin 40° + \sin 16° = 2\sin \frac{40° + 16°}{2} \cos \frac{40° - 16°}{2} = 2\sin 28° \cos 12°$

б) $\sin 20° - \sin 40° = 2\sin \frac{20° - 40°}{2} \cos \frac{20° + 40°}{2} =$

$= -2\sin 10° \cos 30° = -\sqrt{3} \sin 10°$.

в) $\sin 10° + \sin 50° = 2\sin \frac{10° + 50°}{2} \cos \frac{50° - 10°}{2} = 2\sin 30° \cos 20° = \cos 20°$.

г) $\sin 52° - \sin 36° = 2\sin \frac{52° - 36°}{2} \cos \frac{52° + 36°}{2} = 2\sin 8° \cos 44°$.

524. а) $\cos 15° + \cos 45° = 2\cos \frac{15° + 45°}{2} \cos \frac{45° - 15°}{2} =$

$= 2 \cos 30° \cos 15° = \sqrt{3} \cos 15°$.

б) $\cos 46° - \cos 74° = 2\sin \frac{46° + 74°}{2} \sin \frac{74° - 46°}{2} = 2\sin 60° \sin 14° = \sqrt{3} \cos 14°$.

в) $\cos 20° + \cos 40° = 2\cos \frac{20° + 40°}{2} \cos \frac{40° - 20°}{2} =$

$= 2 \cos 30° \cos 10° = \sqrt{3} \cos 10°$.

г) $\cos 75° - \cos 15° = 2\sin \frac{75° + 15°}{2} \sin \frac{15° - 75°}{2} = -2 \sin 45° \sin 30° = -\frac{\sqrt{2}}{2}$.

525. а) $\sin\frac{\pi}{5} - \sin\frac{\pi}{10} = 2\sin\frac{\frac{\pi}{5}-\frac{\pi}{10}}{2}\cos\frac{\frac{\pi}{5}+\frac{\pi}{10}}{2} = 2\sin\frac{\pi}{20}\cos\frac{3\pi}{20}$.

б) $\sin\frac{\pi}{3} + \sin\frac{\pi}{4} = = 2\sin\frac{\frac{\pi}{3}+\frac{\pi}{4}}{2}\cos\frac{\frac{\pi}{3}-\frac{\pi}{4}}{2} = 2\sin\frac{7\pi}{24}\cos\frac{\pi}{24}$.

в) $\sin\frac{\pi}{6} + \sin\frac{\pi}{7} = 2\sin\frac{\frac{\pi}{6}+\frac{\pi}{7}}{2}\cos\frac{\frac{\pi}{6}-\frac{\pi}{7}}{2} = 2\sin\frac{13\pi}{84}\cos\frac{\pi}{84}$

г) $\sin\frac{\pi}{3} - \sin\frac{\pi}{11} = 2\sin\frac{\frac{\pi}{3}-\frac{\pi}{11}}{2}\cos\frac{\frac{\pi}{3}+\frac{\pi}{11}}{2} = 2\sin\frac{4\pi}{33}\cos\frac{7\pi}{33}$

526. а) $\cos\frac{\pi}{10} - \cos\frac{\pi}{20} = 2\sin\frac{\frac{\pi}{10}+\frac{\pi}{20}}{2}\sin\frac{\frac{\pi}{20}-\frac{\pi}{10}}{2} = -2\sin\frac{3\pi}{40}\sin\frac{\pi}{40}$

б) $\cos\frac{11\pi}{12} + \cos\frac{3\pi}{4} = 2\cos\frac{\frac{11\pi}{12}+\frac{3\pi}{4}}{2}\cos\frac{\frac{11\pi}{12}-\frac{3\pi}{4}}{2} =$

$= 2\cos\frac{5\pi}{6}\cos\frac{\pi}{12} = -\sqrt{3}\cos\frac{\pi}{12}$.

в) $\cos\frac{\pi}{5} - \cos\frac{\pi}{11} = 2\sin\frac{\frac{\pi}{5}+\frac{\pi}{11}}{2}\sin\frac{\frac{\pi}{11}-\frac{\pi}{5}}{2} = -2\sin\frac{8\pi}{55}\sin\frac{3\pi}{55}$.

г) $\cos\frac{3\pi}{8} + \cos\frac{5\pi}{4} = 2\cos\frac{\frac{3\pi}{8}+\frac{5\pi}{4}}{2}\cos\frac{\frac{3\pi}{8}-\frac{5\pi}{4}}{2} = 2\cos\frac{13\pi}{16}\cos\frac{7\pi}{16}$

527. а) $\sin 3t - \sin t = 2\sin\frac{3t-t}{2}\cos\frac{3t+t}{2} = 2\sin t\cos 2t$

б) $\cos(\alpha-2\beta) - \cos(\alpha+2\beta) = 2\sin\frac{\alpha-2\beta+\alpha+2\beta}{2}\sin\frac{\alpha+2\beta-\alpha+2\beta}{2} =$

$= 2\sin\alpha\sin 2\beta$.

в) $\cos 6t + \cos 4t = 2\cos\frac{6t+4t}{2}\cos\frac{6t-4t}{2} = 2\cos 5t\cos t$.

г) $\sin(\alpha-2\beta) - \sin(\alpha+2\beta) = 2\sin\frac{\alpha-2\beta-\alpha-2\beta}{2}\cos\frac{\alpha-2\beta+\alpha+2\beta}{2} =$

$= -2\sin 2\beta\cos\alpha$.

528. а) $\text{tg}\,25° + \text{tg}\,35° = \frac{\sin(25°+35°)}{\cos 25°\cos 35°} = \frac{\sin 60°}{\cos 25°\cos 35°} = \frac{\sqrt{3}}{2\cos 25°\cos 35°}$.

б) $\operatorname{tg}\frac{\pi}{5}-\operatorname{tg}\frac{\pi}{10}=\dfrac{\sin\left(\frac{\pi}{5}-\frac{\pi}{10}\right)}{\cos\frac{\pi}{5}\cos\frac{\pi}{10}}=\dfrac{\sin\frac{\pi}{10}}{\cos\frac{\pi}{5}\cos\frac{\pi}{10}}=\dfrac{tg\frac{\pi}{10}}{\cos\frac{\pi}{5}}$.

в) $\operatorname{tg}20°+\operatorname{tg}40°=\dfrac{\sin(20°+40°)}{\sin 20°\sin 40°}=\dfrac{\sin 60°}{\sin 20°\sin 40°}=\dfrac{\sqrt{3}}{2\sin 20^\circ\sin 40^\circ}$

г) $\operatorname{tg}\frac{\pi}{3}-\operatorname{tg}\frac{\pi}{4}=\dfrac{\sin\left(\frac{\pi}{3}-\frac{\pi}{4}\right)}{\cos\frac{\pi}{3}\cos\frac{\pi}{4}}=\dfrac{\sin\frac{\pi}{12}}{\frac{1}{2}\cdot\frac{\sqrt{2}}{2}}=2\sqrt{2}\sin\frac{\pi}{12}$.

529. а) $\dfrac{\cos 68^\circ-\cos 22^\circ}{\sin 68-\sin 22^\circ}=\dfrac{2\sin\frac{68^\circ+22^\circ}{2}\sin\frac{22^\circ-68^\circ}{2}}{2\sin\frac{68^\circ-22^\circ}{2}\sin\frac{68^\circ+22^\circ}{2}}=\dfrac{-\sin 45^\circ\sin 23^\circ}{2\sin 23^\circ\cos 45^\circ}=$

$=-\operatorname{tg}45°=-1$.

б) $\dfrac{\sin 130^\circ+\sin 110^\circ}{\cos 130^\circ+\cos 110^\circ}=\dfrac{2\sin\frac{130^\circ+110^\circ}{2}\cos\frac{130^\circ-110^\circ}{2}}{2\cos\frac{130^\circ+110^\circ}{2}\cos\frac{130^\circ-110^\circ}{2}}=$

$=\dfrac{\sin 120^\circ\sin 10^\circ}{\cos 120^\circ\cos 10^\circ}=\dfrac{\sin 120^\circ}{\cos 120^\circ}=\operatorname{tg}120°=-\sqrt{3}$.

530. а) $\sin 35°+\sin 25°=\cos 5°$.

$\sin 35°+\sin 25°=2\sin\frac{35^\circ+25^\circ}{2}\cos\frac{35^\circ-25^\circ}{2}=2\sin 30°\cos 5°=\cos 5°$

б) $\sin 40°+\cos 70°=\cos 10°$.

$\sin 40°+\cos 70°=\sin 40°+\sin 20°=2\sin\frac{40^\circ+20^\circ}{2}\cos\frac{40^\circ-20^\circ}{2}=$

$=2\sin 30°\cos 10°=\cos 10°$

в) $\cos 12°-\cos 48°=\sin 18°$.

$\cos 12°-\cos 48°=2\sin\frac{12^\circ+48^\circ}{2}\sin\frac{48^\circ-12^\circ}{2}=2\sin 30°\sin 18°=\sin 18°$

г) $\cos 20°-\sin 50°=\sin 10°$.

$\cos 20°-\sin 50°=\cos 20°-\cos 40°=2\sin\frac{20^\circ+40^\circ}{2}\sin\frac{40^\circ-20^\circ}{2}=$

$=2\sin 30°\sin 10°=\sin 10°$.

531. а) $\dfrac{\sin 2\alpha+\sin 6\alpha}{\cos 2\alpha+\cos 6\alpha}=\operatorname{tg}4\alpha$.

$$\frac{\sin 2\alpha+\sin 6\alpha}{\cos 2\alpha+\cos 6\alpha}=\frac{2\sin\frac{2\alpha+6\alpha}{2}\cos\frac{6\alpha-2\alpha}{2}}{2\cos\frac{2\alpha+6\alpha}{2}\cos\frac{6\alpha-2\alpha}{2}}=\frac{\sin 4\alpha\cos 2\alpha}{\cos 4\alpha\cos 2\alpha}=\frac{\sin 4\alpha}{\cos 4\alpha}=\text{tg } 4\alpha.$$

б) $\frac{\cos 2\alpha-\cos 4\alpha}{\cos 2\alpha+\cos 4\alpha}=\text{tg } 3\alpha \text{ tg } \alpha.$

$$\frac{\cos 2\alpha-\cos 4\alpha}{\cos 2\alpha+\cos 4\alpha}=\frac{2\sin\frac{2\alpha+4\alpha}{2}\cos\frac{4\alpha-2\alpha}{2}}{2\cos\frac{2\alpha+4\alpha}{2}\cos\frac{4\alpha-2\alpha}{2}}=\frac{\sin 3\alpha\cos\alpha}{\cos 3\alpha\cos\alpha}=\text{tg } 3\alpha \text{ tg } \alpha.$$

532. а) cos x + cos 3x = 0.

$2\cos\frac{x+3x}{2}\cos\frac{3x-x}{2}=0;$ $\cos 2x\cos x=0;$

$$\left[\begin{array}{l}\cos 2x=0\\ \cos x=0\end{array}\right.;\quad \left[\begin{array}{l}2x=\frac{\pi}{2}+\pi n\\ x=\frac{\pi}{2}+\pi k\end{array}\right.;\quad \left[\begin{array}{l}x=\frac{\pi}{4}+\frac{\pi n}{2}\\ x=\frac{\pi}{2}+\pi k\end{array}\right.$$

б) sin 12 x + sin 4x = 0.

$2\sin\frac{12x+4x}{2}\cos\frac{12x-4x}{2}=0;$ $\sin 8x\cos 4x=0;$

$$\left[\begin{array}{l}\sin 8x=0\\ \cos 4x=0\end{array}\right.;\quad \left[\begin{array}{l}8x=\pi n\\ 4x=\frac{\pi}{2}+\pi k\end{array}\right.;\quad \left[\begin{array}{l}x=\frac{\pi n}{8}\\ x=\frac{\pi}{8}+\frac{\pi k}{4}\end{array}\right.;\ x=\frac{\pi n}{8}$$

в) cos x = cos 5x.

$\cos x-\cos 5x=0;$ $2\sin\frac{x+5x}{2}\cos\frac{5x-x}{2}=0;$

$$\sin 3x\sin 2x=0;\ \left[\begin{array}{l}\sin 3x=0\\ \cos 2x=0\end{array}\right.;\quad \left[\begin{array}{l}3x=\pi n\\ 2x=\pi k\end{array}\right.;\quad \left[\begin{array}{l}x=\frac{\pi n}{3}\\ x=\frac{\pi k}{2}\end{array}\right.$$

г) sin 3x = sin 17 x.

$\sin 17x-\sin 3x=0;$ $2\sin\frac{17x-3x}{2}\cos\frac{17x+3x}{2}=0;$

$$\sin 7x\cos 10x=0;\ \left[\begin{array}{l}\sin 7x=0\\ \cos 10x=0\end{array}\right.;\quad \left[\begin{array}{l}7x=\pi n\\ 10x=\frac{\pi}{2}+\pi k\end{array}\right.;\quad \left[\begin{array}{l}x=\frac{\pi n}{7}\\ x=\frac{\pi}{20}+\frac{\pi k}{10}\end{array}\right.$$

533. а) sin x + sin 2x + sin 3x = 0.

$\sin 2x + 2\sin\frac{x+3x}{2}\cos\frac{3x-x}{2} = 0$; $\sin 2x + 2\sin 2x\cos x = 0$;

$\sin 2x\,(1 + 2\cos x) = 0$; $\left[\begin{array}{l}\sin 2x = 0\\ \cos x = -\frac{1}{2}\end{array}\right.$; $\left[\begin{array}{l}2x = \pi n\\ x = \pm\frac{2\pi}{3} + 2\pi k\end{array}\right.$; $\left[\begin{array}{l}x = \frac{\pi n}{2}\\ x = \pm\frac{2\pi}{3} + 2\pi k\end{array}\right.$

б) cos 3x – cos 5x = sin 4x.

$2\sin\frac{3x+5x}{2}\cos\frac{5x-3x}{2} = \sin 4x$; $2\sin 4x\sin x - \sin 4x = 0$;

$\sin 4x\,(2\sin x - 1) = 0$; $\left[\begin{array}{l}\sin 4x = 0\\ \cos x = \frac{1}{2}\end{array}\right.$; $\left[\begin{array}{l}4x = \pi n\\ x = (-1)^k\frac{\pi}{6} + \pi k\end{array}\right.$; $\left[\begin{array}{l}x = \frac{\pi n}{4}\\ x = (-1)^k\frac{\pi}{6} + \pi k\end{array}\right.$

534. а) $\frac{1}{2} - \cos t = \cos\frac{\pi}{3} - \cos t = 2\sin\frac{\frac{\pi}{3}+t}{2}\sin\frac{t-\frac{\pi}{3}}{2} = 2\sin\left(\frac{\pi}{6}+\frac{t}{2}\right)\sin\left(\frac{t}{2}-\frac{\pi}{6}\right)$.

б) $\frac{\sqrt{3}}{2} + \sin t = \sin\frac{\pi}{3} + \sin t = 2\sin\frac{\frac{\pi}{3}+t}{2}\cos\frac{\frac{\pi}{3}-t}{2} = 2\sin\left(\frac{\pi}{6}+\frac{t}{2}\right)\cos\left(\frac{\pi}{6}-\frac{t}{2}\right)$.

в) $1 + 2\cos t = 2\left(\frac{1}{2}+\cos t\right) = 2\left(\cos\frac{\pi}{3}+\cos t\right) =$

$= 4\cos\frac{\frac{\pi}{3}+t}{2}\cos\frac{\frac{\pi}{3}-t}{2} = 4\cos\left(\frac{\pi}{6}+\frac{t}{2}\right)\cos\left(\frac{\pi}{6}-\frac{t}{2}\right)$.

г) $\cos t + \sin t = \cos t + \cos\left(\frac{\pi}{2} - t\right) = 2\cos\frac{t+\frac{\pi}{2}-t}{2}\cos\frac{t-\frac{\pi}{2}+t}{2} =$

$= 2\cos\frac{\pi}{4}\cos\left(t-\frac{\pi}{4}\right) = \sqrt{2}\cos\left(t-\frac{\pi}{4}\right)$.

535. а) sin 5x + 2 sin 6x + sin 7x = (sin 5x + sin 7x) + 2 sin 6x =

$= 2\sin\frac{5x+7x}{2}\cos\frac{7x-5x}{2} + 2\sin 6x = 2\sin 6x\cos x + 2\sin 6x =$

$= 2\sin 6x\,(1 + \cos x) = 4\sin 6x\cos^2\frac{x}{2}$.

б) $2\cos x + \cos 2x + \cos 4x = 2\cos x + 2\cos\frac{2x+4x}{2}\cos\frac{4x-2x}{2} =$

$= 2\cos x + 2\cos x\cos 3x = 2\cos x\,(1+\cos 3x) = 4\cos x\cos^2\frac{3x}{2}$.

536. а) sin t + sin 2t + sin 3t + sin 4t = (sin t + sin 4t) + (sin 2t + sin 3t) =

$= 2\sin\frac{t+4t}{2}\cos\frac{4t-t}{2} + 2\sin\frac{2t+3t}{2}\cos\frac{3t-2t}{2} =$

$= 2\sin\frac{5t}{2}\cos\frac{3t}{2} + 2\sin\frac{5t}{2}\cos\frac{t}{2} = 2\sin\frac{5t}{2}\left(\cos\frac{3t}{2}+\cos\frac{t}{2}\right) =$

$= 4\sin\frac{5t}{2}\cos\frac{\frac{3t}{2}+\frac{t}{2}}{2}\cos\frac{\frac{3t}{2}-\frac{t}{2}}{2} = 4\sin\frac{5t}{2}\cos t\cos\frac{t}{2}.$

б) cos 2t–cos 4t–cos 6t+cos 8t=(cos 2t + cos 8t)–(cos 4t + cos 6t) =

$= 2\cos\frac{2t+8t}{2}\cos\frac{8t-2t}{2} - 2\cos\frac{4t+6t}{2}\cos\frac{6t-4t}{2} =$

= 2 cos 5t cos 3t – 2 cos 5t cos t = 2 cos 5t (cos 3t – cos t) =

$= 4\cos 5t\sin\frac{3t+t}{2}\sin\frac{t-3t}{2} = -4\cos 5t\sin 2t\sin t.$

537. а) sin 20° + sin 40° – cos 10° = 0.

$\sin 20°+\sin 40°-\cos 10° = 2\sin\frac{20°+40°}{2}\cos\frac{40°-20°}{2} - \cos 10° =$

$= 2\sin 30°\cos 10° - \cos 10° = 2\cdot\frac{1}{2}\cdot\cos 10° - \cos 10° = 0.$

б) cos 85° + cos 35° – cos 25° = 0.

$\cos 85°+\cos 35°-\cos 25° = 2\cos\frac{85°+35°}{2}\cos\frac{85°-35°}{2} - \cos 25° =$

$= 2\cos 60°\cos 25° - \cos 25° = 2\cdot\frac{1}{2}\cdot\cos 25° - \cos 25° = 0.$

538. а) sin 87° – sin 59° – sin 93° + sin 61° = sin 1°.

sin87°–sin59°–sin 93°+sin 61°=(sin 87°–sin 93°)+(sin61°–sin59°)=

$= 2\sin\frac{87°-93°}{2}\cos\frac{87°+93°}{2} + 2\sin\frac{61°-59°}{2}\cos\frac{61°+59°}{2} =$

$= -2\sin 3°\cos 90° + 2\sin 1°\cos 60° = 0 + 2\cdot\frac{1}{2}\cdot\sin 1° = \sin 1°.$

б) cos 115° – cos 35° + cos 65° + cos 25° = sin 5°.

cos 115° – cos 35° + cos 65° + cos 25° = (cos 115° + cos 65°) +

$+ (\cos 25° - \cos 35°) = 2\cos\frac{115°+65°}{2}\cos\frac{115°-65°}{2} +$

$+ 2\sin\frac{25°+35°}{2}\sin\frac{35°+25°}{2} = 2\sin 90°\cos 25° + 2\sin 30°\sin 5° =$

$= 0 + 2\cdot\frac{1}{2}\cdot\sin 5° = \sin 5°.$

539. а) $\dfrac{\sin(\alpha+\beta)+\sin(\alpha-\beta)}{\cos(\alpha+\beta)+\cos(\alpha-\beta)} = tg\alpha$

$$\frac{\sin(\alpha+\beta)+\sin(\alpha-\beta)}{\cos(\alpha+\beta)+\cos(\alpha-\beta)} = \frac{\sin\alpha\cos\beta+\cos\alpha\sin\beta+\sin\alpha\cos\beta-\cos\alpha\sin\beta}{\cos\alpha\cos\beta-\sin\alpha\sin\beta+\cos\alpha\cos\beta+\sin\alpha\sin\beta} =$$

$$= \frac{2\sin\alpha\cos\beta}{2\cos\alpha\cos\beta} = \frac{\sin\alpha}{\cos\beta} = tg\alpha$$

б) $\dfrac{\cos(\alpha-\beta)-\cos(\alpha+\beta)}{\sin(\alpha+\beta)-\sin(\alpha-\beta)} = tg\alpha$

$$\frac{\cos(\alpha-\beta)-\cos(\alpha+\beta)}{\sin(\alpha+\beta)-\sin(\alpha-\beta)} = \frac{\cos\alpha\cos\beta+\sin\alpha\sin\beta-\cos\alpha\cos\beta+\sin\alpha\sin\beta}{\sin\alpha\cos\beta+\cos\alpha\sin\beta-\sin\alpha\cos\beta+\cos\alpha\sin\beta} =$$

$$= \frac{2\sin\alpha\sin\beta}{2\cos\alpha\sin\beta} = \frac{\sin\alpha}{\cos\alpha} = tg\alpha$$

540. а) $\sin x + \sin y + \sin(x-y) = 4\sin\frac{x}{2}\cos\frac{x}{2}\cos\frac{x-y}{2}$

$\sin x + \sin y + \sin(x-y) = 2\sin\frac{x+y}{2}\cos\frac{x-y}{2} +$

$+ 2\sin\frac{x-y}{2}\cos\frac{x-y}{2} = 2\cos\frac{x-y}{2}\left(\sin\frac{x+y}{2} + \sin\frac{x-y}{2}\right) =$

$=4\cos\frac{x-y}{2}\sin\dfrac{\frac{x+y}{2}+\frac{x-y}{2}}{2}\cos\dfrac{\frac{x+y}{2}-\frac{x-y}{2}}{2} = 4\sin\frac{x}{2}\cos\frac{x}{2}\cos\frac{x-y}{2}$.

б) $\dfrac{\sin x+\sin 2x+\sin 3x}{\cos x+\cos 2x+\cos 3x} = tg 2x$

$$\frac{\sin x+\sin 2x+\sin 3x}{\cos x+\cos 2x+\cos 3x} = \frac{\sin 2x + 2\sin\frac{x+3x}{2}\cos\frac{3x-x}{2}}{\cos 2x + 2\cos\frac{x+3x}{2}\cos\frac{3x-x}{2}} =$$

$$= \frac{\sin 2x + 2\sin 2x\cos x}{\cos 2x + 2\cos 2x\cos x} = \frac{\sin 2x(1+2\cos x)}{\cos 2x(1+2\cos x)} = \frac{\sin 2x}{\cos 2x} = tg 2x$$

541. а) $\sin^2(\alpha+\beta) - \sin^2(\alpha-\beta) = \sin 2\alpha\ \sin 2\beta$

$\sin^2(\alpha+\beta) - \sin^2(\alpha-\beta) = (\sin(\alpha+\beta) - \sin(\alpha-\beta))$x

x $(\sin(\alpha+\beta) + \sin(\alpha-\beta)) = (2\cos\alpha\sin\beta)\cdot(2\sin\alpha\cos\beta) =$

$= (2\sin\alpha\cos\alpha)\cdot(2\sin\beta\cos\beta) = \sin 2\alpha\ \sin 2\beta$.

б) $\cos^2(\alpha-\beta) - \cos^2(\alpha+\beta) = \sin 2\alpha\ \sin 2\beta$

$\cos^2(\alpha-\beta) - \cos^2(\alpha+\beta) = (\cos(\alpha-\beta) - \cos(\alpha+\beta))$x

x $(\cos(\alpha-\beta) + \cos(\alpha+\beta)) = (2\sin\alpha\sin\beta)\cdot(2\cos\alpha\cos\beta) =$

$= (2\sin\alpha\cos\alpha)\cdot(2\sin\beta\cos\beta) = \sin 2\alpha\ \sin 2\beta$.

542.

$$\frac{\sin\alpha+\sin3\alpha+\sin5\alpha+\sin7\alpha}{\cos\alpha+\cos3\alpha+\cos5\alpha+\cos7\alpha}=\frac{(\sin\alpha+\sin7\alpha)+(\sin3\alpha+\sin5\alpha)}{(\cos\alpha+\cos7\alpha)+(\cos3\alpha+\cos5\alpha)}=$$

$$=\frac{2\sin4\alpha\cos3\alpha+2\sin4\alpha\cos\alpha}{2\cos4\alpha\cos3\alpha+2\cos4\alpha\cos\alpha}=\frac{2\sin4\alpha(\cos3\alpha+\cos\alpha)}{2\cos4\alpha(\cos3\alpha+\cos\alpha)}=$$

$$=\frac{\sin4\alpha}{\cos3\alpha}=\frac{1}{ctg4\alpha}=\frac{1}{0{,}2}=5$$

543. а) tg α + tg β + tg γ = tg α tg β tg γ (α + β + γ = π)

tg α + tg β + tg γ = tg α + tg β + tg (π – α – β) =

$$=(\text{tg }\alpha+\text{tg }\beta)-\text{tg }(\alpha+\beta)=\frac{\sin(\alpha+\beta)}{\cos\alpha\cos\beta}-\frac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)}=$$

$$=\sin(\alpha+\beta)\left(\frac{1}{\cos\alpha\cos\beta}-\frac{1}{\cos(\alpha+\beta)}\right)=$$

$$=\sin(\pi-\alpha-\beta)\left(\frac{1}{\cos\alpha\cos\beta}-\frac{1}{-\cos(\pi-\alpha-\beta)}\right)=$$

$$=\sin\gamma\left(\frac{1}{\cos\alpha\cos\beta}+\frac{1}{\cos\gamma}\right)=\sin\gamma\cdot\frac{\cos\gamma+\cos\alpha\cos\beta}{\cos\alpha\cos\beta\cos\gamma}=$$

$$=\sin\gamma\frac{-\cos(\alpha+\beta)+\cos\alpha\cos\beta}{\cos\alpha\cos\beta\cos\gamma}=\frac{\sin\alpha\sin\beta\sin\gamma}{\cos\alpha\cos\beta\cos\gamma}=tg\alpha tg\beta tg\gamma\,.$$

б) $\sin\alpha+\sin\beta+\sin\gamma=4\cos\frac{\alpha}{2}\cos\frac{\beta}{2}\cos\frac{\gamma}{2}$ (α + β + γ = π)

sin α + sin β + sin γ = sin α + sin β + sin (π – α – β) =

$$=\sin\alpha+\sin\beta+\sin(\alpha+\beta)=2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}+$$

$$+2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha+\beta}{2}=2\sin\frac{\alpha+\beta}{2}\left(\cos\frac{\alpha-\beta}{2}+\cos\frac{\alpha+\beta}{2}\right)=$$

$$=4\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha}{2}\cos\frac{\beta}{2}=4\cos\left(\frac{\pi}{2}-\frac{\alpha+\beta}{2}\right)\cos\frac{\alpha}{2}\cos\frac{\beta}{2}=$$

$$=4\cos\frac{\alpha}{2}\cos\frac{\beta}{2}\cos\frac{\gamma}{2}.$$

544. а) $\sin^2 10^\circ+\sin^2 130^\circ+\sin^2 110^\circ=\frac{1}{2}(1-\cos 20^\circ+1-\cos 260^\circ+$

$$+1-\cos 220^\circ)=\frac{3}{2}-\frac{1}{2}\cos 20^\circ+\frac{1}{2}\cos 80^\circ+\frac{1}{2}\text{cso } 40^\circ=$$

$$=\frac{3}{2}-\frac{1}{2}\cos 20^\circ+\cos\frac{80^\circ+40^\circ}{2}\cos\frac{80^\circ-40^\circ}{2}=$$

$$= \frac{3}{2} - \frac{1}{2}\cos 20^\circ + \cos 60^\circ \cos 20^\circ = \frac{3}{2} - \frac{1}{2}\cos 20^\circ + \frac{1}{2}\cos 20^\circ = \frac{3}{2}.$$

б) $\cos^2 35^\circ + \cos^2 25^\circ - \cos^2 5^\circ = \frac{1}{2}(1 + \cos 70^\circ + 1 + \cos 50^\circ -$

$- 1 - \cos 10^\circ) = \frac{1}{2}(1 + \cos 70^\circ + \cos 50^\circ - \cos 10^\circ) = \frac{1}{2} +$

$+ \cos\frac{70^\circ + 50^\circ}{2}\cos\frac{70^\circ - 50^\circ}{2} - \frac{1}{2}\cos 10^\circ = \frac{1}{2} + \cos 60^\circ \cos 10^\circ -$

$- \frac{1}{2}\cos 10^\circ = \frac{1}{2} + \frac{1}{2}\cos 10^\circ - \frac{1}{2}\cos 10^\circ = \frac{1}{2}.$

545. а) $\sin 3x = \cos 2x$

$\sin 3x - \sin\left(\frac{\pi}{2} - 2x\right) = 0;$

$$2\sin\frac{3x - \frac{\pi}{2} + 2x}{2}\cos\frac{3x + \frac{\pi}{2} - 2x}{2} = 0;\ \sin\left(\frac{5x}{2} - \frac{\pi}{4}\right)\cos\left(\frac{x}{2} + \frac{\pi}{4}\right) = 0;$$

$$\left[\begin{array}{l} \sin\left(\frac{5x}{2} - \frac{\pi}{4}\right) = 0 \\ \cos\left(\frac{x}{2} + \frac{\pi}{4}\right) = 0 \end{array}\right.;\quad \left[\begin{array}{l} \frac{5x}{2} - \frac{\pi}{4} = \pi n \\ \frac{x}{2} + \frac{\pi}{4} = \frac{\pi}{2} + \pi k \end{array}\right.;\quad \left[\begin{array}{l} x = \frac{\pi}{10} + \frac{2\pi n}{5} \\ x = \frac{\pi}{2} + 2\pi k \end{array}\right.;\quad x = \frac{\pi}{10} + \frac{2\pi n}{5}$$

б) $\sin(5x - x) = \cos(2x + 7\pi).$

$\sin x = -\cos 2x;\quad \cos 2x + \cos\left(\frac{\pi}{2} - x\right) = 0;$

$$2\cos\frac{2x + \frac{\pi}{2} - x}{2}\cos\frac{2x - \frac{\pi}{2} + x}{2} = 0;\ \cos\left(\frac{x}{2} + \frac{\pi}{4}\right)\cos\left(\frac{3x}{2} - \frac{\pi}{4}\right) = 0;$$

$$\left[\begin{array}{l} \cos\left(\frac{x}{2} + \frac{\pi}{4}\right) = 0 \\ \cos\left(\frac{3x}{2} - \frac{\pi}{4}\right) = 0 \end{array}\right.;\quad \left[\begin{array}{l} \frac{x}{2} + \frac{\pi}{4} = \frac{\pi}{2} + \pi n \\ \frac{3x}{2} - \frac{\pi}{4} = \frac{\pi}{2} + \pi k \end{array}\right.;\quad \left[\begin{array}{l} x = \frac{\pi}{2} + 2\pi n \\ x = \frac{\pi}{2} + \frac{2\pi k}{3} \end{array}\right.;\quad x = \frac{\pi}{2} + \frac{2\pi k}{3}.$$

в) $\cos 5x = \sin 15x$

$$\cos 5x - \cos\left(\frac{\pi}{2} - 15x\right) = 0;\ 2\sin\frac{5x + \frac{\pi}{2} - 15x}{2}\sin\frac{\frac{\pi}{2} - 15x - 5x}{2} = 0;$$

$$\sin\left(\frac{\pi}{4} - 5x\right)\sin\left(\frac{\pi}{4} - 10x\right) = 0;\ \sin\left(5x - \frac{\pi}{4}\right)\sin\left(10x - \frac{\pi}{4}\right) = 0;$$

$$\left[\begin{array}{l}\sin\left(5x-\frac{\pi}{4}\right)=0\\ \cos\left(10x-\frac{\pi}{4}\right)=0\end{array}\right.;\quad \left[\begin{array}{l}5x-\frac{\pi}{4}=\pi n\\ 10x-\frac{\pi}{4}=\pi k\end{array}\right.;\quad \left[\begin{array}{l}x=\frac{\pi}{10}+\frac{2\pi n}{5}\\ x=\frac{\pi}{2}+2\pi k\end{array}\right..$$

г) sin (7π + x) = cos (9π + 2x).

$-\sin x=-\cos 2x$; $\cos 2x-\sin x=0$; $\cos 2x-\cos\left(\frac{\pi}{2}-x\right)=0$;

$$2\sin\frac{2x+\frac{\pi}{2}-x}{2}\sin\frac{\frac{\pi}{2}-x-2x}{2}=0;$$

$\sin\left(\frac{x}{2}+\frac{\pi}{4}\right)\sin\left(\frac{\pi}{4}-\frac{3x}{2}\right)=0$; $\sin\left(\frac{x}{2}+\frac{\pi}{4}\right)\sin\left(\frac{3x}{2}-\frac{\pi}{4}\right)=0$;

$$\left[\begin{array}{l}\sin\left(\frac{x}{2}+\frac{\pi}{4}\right)=0\\ \sin\left(\frac{3x}{2}-\frac{\pi}{4}\right)=0\end{array}\right.;\quad \left[\begin{array}{l}\frac{x}{2}+\frac{\pi}{4}=\pi n\\ \frac{3x}{2}-\frac{\pi}{4}=\pi k\end{array}\right.;\quad \left[\begin{array}{l}x=-\frac{\pi}{2}+2\pi n\\ x=\frac{\pi}{6}+\frac{2\pi k}{3}\end{array}\right.;\quad x=\frac{\pi}{6}+\frac{2\pi k}{3}.$$

546. а) $1+\cos 6x=2\sin^2 5x$

$1+\cos 6x=1-\cos 10x$; $\cos 6x+\cos 10x=0$;

$2\cos\frac{6x+10x}{2}\cos\frac{10x-6x}{2}=0$; $\cos 8x\cos 2x=0$;

$$\left[\begin{array}{l}\cos 8x=0\\ \cos 2x=0\end{array}\right.;\quad \left[\begin{array}{l}8x=\frac{\pi}{2}+\pi k\\ 2x=\frac{\pi}{2}+\pi n\end{array}\right.;\quad \left[\begin{array}{l}x=\frac{\pi}{16}+\frac{\pi k}{8}\\ x=\frac{\pi}{4}+\frac{\pi n}{2}\end{array}\right..$$

б) $\cos^2 2x=\cos^2 4x$.

$\frac{1}{2}(1+\cos 4x)=\frac{1}{2}(1+\cos 8x)$; $\cos 4x-\cos 8x=0$;

$2\sin\frac{4x+8x}{2}\cos\frac{8x-4x}{2}=0$; $\sin 6x\sin 2x=0$;

$$\left[\begin{array}{l}\sin 6x=0\\ \sin 2x=0\end{array}\right.;\quad \left[\begin{array}{l}6x=\pi n\\ 2x=\pi k\end{array}\right.;\quad \left[\begin{array}{l}x=\frac{\pi n}{6}\\ x=\frac{\pi k}{2}\end{array}\right.;\quad x=\frac{\pi n}{6}.$$

в) $\sin^2\frac{x}{2}=\cos^2\frac{7x}{2}$

$\frac{1}{2}(1-\cos x)=\frac{1}{2}(1+\cos 7x)$; $\cos 7x+\cos x=0$;

$$2\cos\frac{7x+x}{2}\cos\frac{7x-x}{2}=0; \qquad \cos 4x \cos 3x = 0;$$

$$\left[\begin{array}{l}\cos 4x = 0\\ \cos 3x = 0\end{array}\right.; \quad \left[\begin{array}{l}4x = \frac{\pi}{2}+\pi n\\ 3x = \frac{\pi}{2}+\pi k\end{array}\right.; \quad \left[\begin{array}{l}x = \frac{\pi}{8}+\frac{\pi n}{4}\\ x = \frac{\pi}{6}+\frac{\pi k}{3}\end{array}\right..$$

г) $\sin^2 x+\sin^2 3x = 1$.

$$\frac{1}{2}(1-\cos 2x)+\frac{1}{2}(1-\cos 6x); \qquad \cos 2x + \cos 6x = 0;$$

$$2\cos\frac{2x+6x}{2}\cos\frac{6x-2x}{2}=0; \qquad \cos 4x \cos 2x = 0;$$

$$\left[\begin{array}{l}\cos 4x = 0\\ \cos 2x = 0\end{array}\right.; \quad \left[\begin{array}{l}4x = \frac{\pi}{2}+\pi n\\ 2x = \frac{\pi}{2}+\pi k\end{array}\right.; \quad \left[\begin{array}{l}x = \frac{\pi}{8}+\frac{\pi n}{4}\\ x = \frac{\pi}{4}+\frac{\pi k}{2}\end{array}\right..$$

547. а) $2\sin 2x + \cos 5x = 1$.

$1-\cos 2x+\cos 5x = 1; \qquad \cos 5x - \cos 2x = 0;$

$$\left[\begin{array}{l}\sin\frac{7x}{2} = 0\\ \sin\frac{3x}{2} = 0\end{array}\right.; \quad \left[\begin{array}{l}\frac{7x}{2} = \pi n\\ \frac{3x}{2} = \pi k\end{array}\right.; \quad \left[\begin{array}{l}x = \frac{2\pi n}{7}\\ x = \frac{2\pi k}{3}\end{array}\right..$$

б) $2\sin^2 3x - 1 = \cos^2 4x - \sin^2 4x$

$-\cos 6x = \cos 8x; \quad \cos 6x + \cos 8x = 0;$

$$2\cos\frac{6x+8x}{2}\cos\frac{8x-6x}{2}=0; \qquad \cos 7x \cos x = 0;$$

$$\left[\begin{array}{l}\cos 7x = 0\\ \cos x = 0\end{array}\right.; \quad \left[\begin{array}{l}7x = \frac{\pi}{2}+\pi n\\ x = \frac{\pi}{2}+\pi k\end{array}\right.; \quad \left[\begin{array}{l}x = \frac{\pi}{14}+\frac{\pi n}{7}\\ x = \frac{\pi}{2}+\pi k\end{array}\right.; \quad x = \frac{\pi}{14}+\frac{\pi n}{7}.$$

548. а) $\mathrm{tg}\, x + \mathrm{tg}\, 5x = 0$.

$$\frac{\sin(x+5x)}{\cos x\cos 5x}=0; \quad \frac{\sin 6x}{\cos x \cos 5x}=0;$$

$$\left\{\begin{array}{l}\sin 6x = 0\\ \cos x \neq 0\\ \cos 5x \neq 0\end{array}\right.; \quad \left\{\begin{array}{l}6x = \pi n\\ x \neq \frac{\pi}{2}+\pi k\\ 5x \neq \frac{\pi}{2}+\pi \ell\end{array}\right.; \quad \left\{\begin{array}{l}x = \frac{\pi n}{6}\\ x \neq \frac{\pi}{2}+\pi k\\ x \neq \frac{\pi}{10}+\frac{\pi \ell}{5}\end{array}\right.; \quad \left\{\begin{array}{l}x = \frac{\pi n}{6}\\ n \neq 3+6k\\ 5n \neq 3+6\ell\end{array}\right..$$

б) tg 3x = ctg x.

$$\frac{\sin 3x}{\cos 3x}=\frac{\cos x}{\sin x}\ ;\quad \begin{cases}\sin 3x\sin x=\cos 3x\cos x\\ \cos 3x\neq 0\\ \sin x\neq 0\end{cases};$$

$$\begin{cases}\cos 3x\cos x-\sin 3x\sin x=0\\ \cos 3x\neq 0\\ \sin x\neq 0\end{cases};\quad \begin{cases}\cos(3x+x)=0\\ \cos 3x\neq 0\\ \sin x\neq 0\end{cases};$$

$$\begin{cases}\cos 4x=0\\ \cos 3x\neq 0\\ \sin x\neq 0\end{cases};\quad \begin{cases}4x=\frac{\pi}{2}+\pi n\\ 3x\neq\frac{\pi}{2}+\pi k\\ x\neq\pi\ell\end{cases};\quad \begin{cases}x=\frac{\pi}{8}+\frac{\pi n}{4}\\ x\neq\frac{\pi}{6}+\frac{\pi k}{3}\\ x\neq\pi\ell\end{cases};\quad x=\frac{\pi}{8}+\frac{\pi n}{4}.$$

в) tg 2x = tg 4x.

$$\frac{\sin 2x}{\cos 2x}=\frac{\sin 4x}{\cos 4x}\ ;\quad \begin{cases}\sin 2x\cos 4x=\sin 4x\cos 2x\\ \cos 2x\neq 0\\ \cos 4x\neq 0\end{cases};$$

$$\begin{cases}\sin(4x-2x)=0\\ \cos 2x\neq 0\\ \cos 4x\neq 0\end{cases};\quad \begin{cases}\sin 2x=0\\ \cos 2x\neq 0\\ \cos 4x\neq 0\end{cases};$$

$$\begin{cases}2x=\pi n\\ 2x\neq\frac{\pi}{2}+\pi k\\ 4x\neq\frac{\pi}{2}+\pi\ell\end{cases};\quad \begin{cases}x=\frac{\pi n}{2}\\ x\neq\frac{\pi}{4}+\frac{\pi k}{2}\\ x\neq\frac{\pi}{8}+\frac{\pi\ell}{4}\end{cases};\quad x=\frac{\pi n}{2}.$$

г) ctg $\frac{x}{2}$ + ctg $\frac{3x}{2}$ = 0.

$$\frac{\cos\frac{x}{2}}{\sin\frac{x}{2}}+\frac{\cos\frac{3x}{2}}{\sin\frac{3x}{2}}=0\ ;$$

$$\begin{cases}\cos\frac{x}{2}\sin\frac{3x}{2}+\cos\frac{3x}{2}\sin\frac{x}{2}=0\\ \sin\frac{x}{2}\neq 0\\ \sin\frac{3x}{2}\neq 0\end{cases};\quad \begin{cases}\sin\left(\frac{3x}{2}+\frac{x}{2}\right)=0\\ \sin\frac{x}{2}\neq 0\\ \sin\frac{3x}{2}\neq 0\end{cases};$$

$$\begin{cases}\sin 2x = 0\\ \sin\frac{x}{2} \neq 0\\ \sin\frac{3x}{2} \neq 0\end{cases};\quad \begin{cases}2x = \pi n\\ \frac{x}{2} \neq \pi k\\ \frac{3x}{2} \neq \pi\ell\end{cases};\quad \begin{cases}x = \frac{\pi n}{2}\\ x \neq 2\pi k\\ x \neq \frac{2\pi\ell}{3}\end{cases};\quad \begin{cases}x = \frac{\pi n}{2}\\ n \neq 4k\\ 3n \neq 4\ell\end{cases}.$$

549. а) sin x + sin 3x + cos x + cos 3x = 0.

$$2\sin\frac{x+3x}{2}\cos\frac{3x-x}{2} + 2\cos\frac{x+3x}{2}\cos\frac{3x-x}{2} = 0;$$

sin 2x cos x + cos 2x cos x = 0; cos x (sin 2x + cos 2x) = 0;

$$\sqrt{2}\cos x \cdot \sin\left(\frac{\pi}{4} + 2x\right) = 0;$$

$$\left[\begin{array}{l}\cos x = 0\\ \sin\left(\frac{\pi}{4} + 2x\right) = 0\end{array}\right.;\quad \left[\begin{array}{l}x = \frac{\pi}{2} + \pi n\\ \frac{\pi}{4} + 2x = \pi k\end{array}\right.;\quad \left[\begin{array}{l}x = \frac{\pi}{2} + \pi n\\ x = -\frac{\pi}{8} + \frac{\pi k}{2}\end{array}\right..$$

б) sin 5x + sin x + 2sin^2 x = 1.

$$\sin 5x + \sin x - \cos 2x = 0;\quad 2\sin\frac{5x+x}{2}\cos\frac{5x-x}{2} - \cos 2x = 0;$$

2sin 3x cos 2x – cos 2x = 0; cos 2x (2sin 3x – 1) = 0;

$$\left[\begin{array}{l}\cos 2x = 0\\ \sin 3x = \frac{1}{2}\end{array}\right.;\quad \left[\begin{array}{l}2x = \frac{\pi}{2} + \pi n\\ 3x = (-1)^k\frac{\pi}{6} + \pi k\end{array}\right.;\quad \left[\begin{array}{l}x = \frac{\pi}{4} + \frac{\pi n}{2}\\ x = (-1)^k\frac{\pi}{18} + \frac{\pi k}{3}\end{array}\right..$$

550. а) sin 2x + sin 6x = cos 2x

$$2\sin\frac{2x+6x}{2}\cos\frac{6x-2x}{2} - \cos 2x = 0;$$

2sin 4x cos 2x – cos 2x = 0; cos 2x (2sin 4x – 1) = 0;

$$\left[\begin{array}{l}\cos 2x = 0\\ \sin 4x = \frac{1}{2}\end{array}\right.;\quad \left[\begin{array}{l}2x = \frac{\pi}{2} + \pi n\\ 4x = (-1)^k\frac{\pi}{6} + \pi k\end{array}\right.;\quad \left[\begin{array}{l}x = \frac{\pi}{4} + \frac{\pi n}{2}\\ x = \frac{\pi}{24} + 2\pi k\\ x = \frac{5\pi}{24} + 2\pi k\end{array}\right..$$

Отсюда видно, что данное уравнение имеет на отрезке $[0, \frac{\pi}{2}]$ 3 корня: $x = \frac{\pi}{4}$, $x = \frac{\pi}{24}$ и $x = \frac{5\pi}{24}$.

б) $2\cos^2 x - 1 = \sin 3x$

$\cos 2x = \sin 3x;\quad \cos 2x - \cos\left(\frac{\pi}{2} - 3x\right) = 0;$

$$2\sin\frac{2x + \frac{\pi}{2} - 3x}{2}\sin\frac{\frac{\pi}{2} - 3x - 2x}{2} = 0;$$

$\sin\left(\frac{\pi}{4} - \frac{x}{2}\right)\sin\left(\frac{\pi}{4} - \frac{5x}{2}\right) = 0;\quad \sin\left(\frac{x}{2} - \frac{\pi}{4}\right)\sin\left(\frac{5x}{2} - \frac{\pi}{4}\right) = 0;$

$$\left[\begin{array}{l}\sin\left(\frac{x}{2} - \frac{\pi}{4}\right) = 0\\ \cos\left(\frac{5x}{2} - \frac{\pi}{4}\right) = 0\end{array}\right.;\quad \left[\begin{array}{l}\frac{x}{2} - \frac{\pi}{4} = \pi n\\ \frac{5x}{2} - \frac{\pi}{4} = \pi k\end{array}\right.;\quad \left[\begin{array}{l}x = \frac{\pi}{2} + 2\pi n\\ x = \frac{\pi}{10} + \frac{2\pi k}{5}\end{array}\right..$$

Отсюда видно, что данное уравнение имеет на отрезке $[0, \frac{\pi}{2}]$ 2 корня: $x = \frac{\pi}{10}$ и $x = \frac{\pi}{2}$.

551. а) $\cos 6x + \cos 8x = \cos 10x + \cos 12x$

$2\cos\frac{6x + 8x}{2}\cos\frac{8x - 6x}{2} = 2\cos\frac{10x + 12x}{2}\cos\frac{12x - 10x}{2};$

$\cos 7x \cos x = \cos 11x \cos x;\quad \cos x(\cos 7x - \cos 11x) = 0;$

$2\cos x \sin\frac{7x + 11x}{2}\sin\frac{11x - 7x}{2} = 0;\quad \cos x \sin 9x \sin 2x = 0;$

$$\left[\begin{array}{l}\cos 8x = 0\\ \cos 2x = 0\end{array}\right.;\quad \left[\begin{array}{l}8x = \frac{\pi}{2} + \pi k\\ 2x = \frac{\pi}{2} + \pi n\end{array}\right.;\quad \left[\begin{array}{l}x = \frac{\pi}{16} + \frac{\pi k}{8}\\ x = \frac{\pi}{4} + \frac{\pi n}{2}\end{array}\right..$$

Отсюда видно, что данное уравнение имеет на отрезке $[0, \pi]$ 9 корней: $x = \frac{\pi k}{9}$ (k=1, …, 8) и $x = \frac{\pi}{2}$.

б) $\sin 2x + 5\sin 4x + \sin 6x = 0.\ (\sin 2x + \sin 6x) + 5\sin 4x = 0;$

$2\sin 4x \cos 2x + 5\sin 4x = 0;\quad \sin 4x(2\cos 2x + 5) = 0;$

$\sin 4x = 0$ (т.к. $2\cos 2x + 5 > 0$ при всех x); $4x = \pi$; $x = \frac{\pi n}{4}$.

Отсюда видно, что данное уравнение имеет на отрезке $[0, \pi]$ 3 корня: $x = \frac{\pi n}{4}$ (n=1, 2, 3).

552. а) $a = \cos 7x$, $b = \cos 2x$, $c = \cos 11x$.

a, b, c образуют арифметическую прогрессию, если $b - a = c - b$, т.е. $\cos 2x - \cos 7x = \cos 11x - \cos 2x$.

2 cos 2x – (cos 7x + cos 11x) = 0; 2 cos 2x – 2 cos 9x cos 2x = 0;
cos 2x (1 – cos 9x) = 0;

$\left[\begin{array}{l}\cos 2x=0\\ \cos 9x=1\end{array}\right.$; $\left[\begin{array}{l}2x=\pi n+\dfrac{\pi}{2}\\ 9x=2\pi k\end{array}\right.$; $\left[\begin{array}{l}x=\dfrac{\pi n}{2}+\dfrac{\pi}{4}\\ x=\dfrac{2\pi k}{9}\end{array}\right.$.

б) a = sin 3x, b = cos x, c = sin 5x.
a, b, c образуют арифметическую прогрессию, если b – a = c –b, т.е. cos x – sin 3x = sin 5x – cos x.
2 cos x – (sin 3x + sin 5x) = 0; 2 cos x – 2 sin 4x cos x = 0;
cos x (1 – sin 4x) = 0;

$\left[\begin{array}{l}\cos x=0\\ \sin 4x=1\end{array}\right.$; $\left[\begin{array}{l}x=\dfrac{\pi}{2}+\pi n\\ 4x=\dfrac{\pi}{2}+2\pi k\end{array}\right.$; $\left[\begin{array}{l}x=\dfrac{\pi}{2}+\pi n\\ x=\dfrac{\pi}{8}+\dfrac{\pi k}{2}\end{array}\right.$.

§ 27. Преобразование произведений тригонометрических функций в сумму

553. а) $\sin 23^\circ \sin 32^\circ = \frac{1}{2}(\cos(32^\circ - 23^\circ) - \cos(32^\circ + 23^\circ)) =$

$= \frac{1}{2}(\cos 9^\circ - \cos 55^\circ)$.

б) $\cos\frac{\pi}{12}\cos\frac{\pi}{8} = \frac{1}{2}\left(\cos\left(\frac{\pi}{8}-\frac{\pi}{12}\right)+\cos\left(\frac{\pi}{8}+\frac{\pi}{12}\right)\right) =$

$= \frac{1}{2}\left(\cos\frac{\pi}{24}+\cos\frac{5\pi}{24}\right)$.

в) $\sin 14^\circ \sin 16^\circ = \frac{1}{2}(\cos(16^\circ - 14^\circ) - \cos(16^\circ + 14^\circ)) =$

$= \frac{1}{2}(\cos 2^\circ - \cos 30^\circ) = \frac{1}{2}\left(\cos 2^\circ - \frac{\sqrt{3}}{2}\right)$.

г) $2\sin\frac{\pi}{8}\cos\frac{\pi}{5} = \sin\left(\frac{\pi}{8}+\frac{\pi}{5}\right) + \sin\left(\frac{\pi}{8}-\frac{\pi}{5}\right) = \sin\frac{13\pi}{40} - \sin\frac{3\pi}{40}$.

554. а) $\sin(\alpha+\beta)\sin(\alpha-\beta) = \sin^2\alpha - \sin^2\beta = \cos^2\beta - \cos^2\alpha$.
б) $\cos(\alpha+\beta)\cos(\alpha-\beta) = \cos^2\beta - \sin^2\alpha = \cos^2\alpha - \sin^2\beta$.

в) $\cos\left(\frac{\alpha}{2}+\frac{\beta}{2}\right)\cos\left(\frac{\alpha}{2}-\frac{\beta}{2}\right) = \cos^2\frac{\beta}{2} - \sin^2\frac{\alpha}{2} = \cos^2\frac{\alpha}{2} - \sin^2\frac{\beta}{2}$.

г) $\cos\left(\alpha+\frac{\pi}{4}\right)\cos\left(\alpha-\frac{\pi}{4}\right) = \cos^2\frac{\pi}{4} - \sin^2\alpha = \frac{1}{2} - \sin^2\alpha$.

555. а) $\cos\alpha \sin(\alpha+\beta) = \frac{1}{2}(\sin(2\alpha+\beta) + \sin(\alpha+\beta-\alpha)) =$

$= \frac{1}{2}(\sin(2\alpha+\beta) + \sin\beta))$.

б) $\sin(60^\circ+\alpha)\sin(60^\circ-\alpha) = \sin^2 60^\circ - \sin^2\alpha = \frac{3}{2} - \sin^2\alpha$.

в) $\sin\beta\cos(\alpha+\beta) = \frac{1}{2}(\sin(\alpha+2\beta) + \sin(\beta-\alpha-\beta)) = \frac{1}{2}(\sin(\alpha+2\beta) - \sin\alpha))$.

г) $\cos\left(\alpha+\frac{\pi}{4}\right)\cos\left(\alpha-\frac{\pi}{4}\right) = \cos^2\frac{\pi}{4} - \sin^2\alpha = \frac{1}{2} - \sin^2\alpha$.

556. а) $\cos\left(x+\frac{\pi}{3}\right)\cos\left(x-\frac{\pi}{3}\right) - 0{,}25 = 0$;

$\frac{1}{2}\left(\cos\left(x+\frac{\pi}{3}-x+\frac{\pi}{3}\right)+\cos\left(x+\frac{\pi}{3}+x-\frac{\pi}{3}\right)\right) = \frac{1}{4}$;

$\frac{1}{2}\left(\cos^2\frac{\pi}{3}+\cos 2x\right) = \frac{1}{4}$; $\frac{1}{2}\left(-\frac{1}{2}+\cos 2x\right) = \frac{1}{4}$;

$\cos 2x = 1$; $2x = 2\pi n$; $x = \pi n$;

б) $\sin\left(x+\frac{\pi}{3}\right)\cos\left(x-\frac{\pi}{6}\right) = 1$.

$\frac{1}{2}\left(\sin\left(x+\frac{\pi}{3}-x-\frac{\pi}{6}\right)+\sin\left(x+\frac{\pi}{3}-x+\frac{\pi}{6}\right)\right) = 1$;

$\frac{1}{2}\left(\sin\left(2x+\frac{\pi}{6}\right)+\sin\frac{\pi}{2}\right) = 1$;

$\frac{1}{2}\sin\left(2x+\frac{\pi}{6}\right)+\frac{1}{2} = 1$; $\sin\left(2x+\frac{\pi}{6}\right) = 1$;

$2x + \frac{\pi}{6} = \frac{\pi}{2} + 2\pi n$; $x = \frac{\pi}{6} + \pi n$

557. а) $2\sin x\cos 3x + \sin 4x = 0$.
$\sin(x+3x) + \sin(x-3x) + \sin 4x = 0$;
$2\sin 4x - \sin 2x = 0$;
$4\sin 2x\cos 2x - \sin 2x = 0$;
$\sin 2x(4\cos 2x - 1) = 0$;

$\left[\begin{array}{l}\sin 2x = 0 \\ \cos 2x = \frac{1}{4}\end{array}\right.$; $\left[\begin{array}{l}2x = \pi n \\ 2x = \pm\arccos\frac{1}{4} + 2\pi k\end{array}\right.$; $\left[\begin{array}{l}x = \frac{\pi n}{2} \\ x = \pm\frac{1}{2}\arccos\frac{1}{4} + 2\pi k\end{array}\right.$.

б) $\sin\frac{x}{2}\sin\frac{3x}{2}=\frac{1}{2}$, $\frac{1}{2}\left(\cos\left(\frac{x}{2}-\frac{3x}{2}\right)-\cos\left(\frac{x}{2}+\frac{3x}{2}\right)\right)=\frac{1}{2}$;

$\cos x-\cos 2x=1$; $2\cos^2 x-\cos x=0$; $\cos x(2\cos x-1)=0$;

$\left[\begin{array}{l}\cos x=0\\ \cos x=\frac{1}{2}\end{array}\right.$; $\left[\begin{array}{l}x=\frac{\pi}{2}+\pi n\\ x=\pm\frac{\pi}{3}+2\pi k\end{array}\right.$.

558. а) $\sin 10^\circ\cos 8^\circ\cos 6^\circ=\frac{1}{2}\sin 10^\circ(\cos(8^\circ-6^\circ)+\cos(8^\circ+6^\circ))=$

$=\frac{1}{2}\sin 10^\circ\cos 2^\circ+\frac{1}{2}\sin 10^\circ=\frac{1}{4}(\sin 12^\circ+\sin 8^\circ+\sin 24^\circ-\sin 4^\circ)$.

б) $4\sin 25^\circ\cos 15^\circ\sin 5^\circ=2\sin 25^\circ(\sin 20^\circ+\sin(-10^\circ))=$
$=2(\sin 25^\circ\sin 20^\circ-\sin 25^\circ\sin 10^\circ)=\cos 5^\circ-\cos 45^\circ-\cos 15^\circ+$

$+\cos 35^\circ=\cos 5^\circ-\cos 15^\circ+\cos 35^\circ-\frac{1}{\sqrt{2}}$

559. а) $2\sin t\sin 2t+\cos 3t=\cos t$.
$2\sin t\sin 2t+\cos 3t=\cos(2t-t)-\cos(2t+t)+\cos 3t=\cos t$.

б) $\sin\alpha-2\sin\left(\frac{\alpha}{2}-15^\circ\right)\cos\left(\frac{\alpha}{2}+15^\circ\right)=\frac{1}{2}$.

$\sin\alpha-2\sin\left(\frac{\alpha}{2}-15^\circ\right)\cos\left(\frac{\alpha}{2}+15^\circ\right)=\sin\alpha-$

$-\sin\left(\frac{\alpha}{2}-15^\circ+\frac{\alpha}{2}-15^\circ\right)-\sin\left(\frac{\alpha}{2}-15^\circ-\frac{\alpha}{2}-15^\circ\right)=$

$=\sin\alpha-\sin\alpha-\sin(-30^\circ)=\sin 30^\circ=\frac{1}{2}$.

560. а) $\cos^2 3^\circ+\cos^2 1^\circ-\cos 4^\circ\cos 2^\circ=\cos^2 3^\circ+\cos^2 1^\circ-\cos(3^\circ+1^\circ)\cos(3^\circ-1^\circ)=$
$=\cos^2 3^\circ+\cos^2 1^\circ-\cos^2 1^\circ+\sin^2 3^\circ=\cos^2 3^\circ+\sin^2 3^\circ=1$

б) $\sin^2 10^\circ+\cos 50^\circ\cos 70^\circ=\sin^2 10^\circ=\cos(60^\circ-10^\circ)\cos(60^\circ+10^\circ)=$
$=\sin^2 10^\circ+\cos^2 60^\circ-\sin^2 10^\circ=\cos^2 60^\circ=\frac{1}{4}$.

561. а) $\frac{1}{2\sin 10^\circ}-2\sin 70^\circ=\frac{1-4\sin 10^\circ\sin 70^\circ}{2\sin 10^\circ}=$

$=\frac{1-2\cos(70^\circ-10^\circ)+2\cos(70^\circ+10^\circ)}{2\sin 10^\circ}=\frac{1-2\cos 60^\circ+2\cos 80^\circ}{2\sin 10^\circ}=$

$=\frac{1-2\cdot\frac{1}{2}+2\sin 10^\circ}{2\sin 10^\circ}=1$.

б) $\frac{tg60^\circ}{\sin 40^\circ} + 4\cos 100^\circ = \frac{tg60^\circ + 4\sin 40^\circ \cos 100^\circ}{\sin 40^\circ} =$

$= \frac{tg60^\circ + 2\sin 140^\circ + 2\sin(-60^\circ)}{\sin 40^\circ} = \frac{\sqrt{3} + 2\sin 40^\circ - 2\cdot\frac{\sqrt{3}}{2}}{\sin 40^\circ} = \frac{2\sin 40^\circ}{\sin 40^\circ} = 2.$

562. а) $\sin 3x \cos x = \sin \frac{5x}{2} \cos \frac{3x}{2}$.

$\frac{1}{2}(\sin(3x + x) + \sin(3x - x)) = \frac{1}{2}\left(\sin\left(\frac{5x}{2} + \frac{3x}{2}\right) + \sin\left(\frac{5x}{2} - \frac{3x}{2}\right)\right)$;

$\sin 4x + \sin 2x = \sin 4x + \sin x$;

$\sin 2x - \sin x = 0$; $2\sin \frac{2x - x}{2} \cos \frac{2x + x}{2} = 0$;

$\sin \frac{x}{2} \cos \frac{3x}{2} = 0$; $\left[\begin{array}{l}\sin\frac{x}{2} = 0\\ \cos\frac{3x}{2} = 0\end{array}\right.$; $\left[\begin{array}{l}\frac{x}{2} = \pi n\\ \frac{3x}{2} = \frac{\pi}{2} + \pi k\end{array}\right.$; $\left[\begin{array}{l}x = 2\pi n\\ x = \frac{\pi}{3} + \frac{2\pi k}{3}\end{array}\right.$.

б) $2\sin\left(\frac{\pi}{4} + x\right)\sin\left(\frac{\pi}{4} - x\right) + \sin^2 x = 0$.

$2\left(\sin^2\frac{\pi}{4} - \sin^2 x\right) + \sin^2 x = 0$;

$1 - \sin^2 x = 0$; $-\frac{1}{2}(1 - \cos 2x) + 1 = 0$; $\cos 2x = -1$;

$2x = \pi + 2\pi n$; $x = \frac{\pi}{2} + \pi n$.

в) $\sin 2x \cos x = \sin x \cos 2x$.

$\frac{1}{2}(\sin 3x + \sin x) = \frac{1}{2}(\sin 3x - \sin x)$; $\sin x = 0$; $x = \pi n$.

г) $\cos 2x \cos x = \cos 2{,}5x \cos 0{,}5x$.

$\frac{1}{2}(\cos x + \cos 3x) = \frac{1}{2}(\cos 2x + \cos 3x)$;

$\cos x = \cos 2x$; $\cos x - \cos 2x = 0$;

$2\sin\frac{x + 2x}{2}\sin\frac{2x - x}{2} = 0$; $\sin\frac{3x}{2}\sin\frac{x}{2} = 0$;

$\left[\begin{array}{l}\sin\frac{3x}{2} = 0\\ \sin\frac{x}{2} = 0\end{array}\right.$; $\left[\begin{array}{l}\frac{3x}{2} = \pi n\\ \frac{x}{2} = \pi k\end{array}\right.$; $\left[\begin{array}{l}x = \frac{2\pi n}{3}\\ x = 2\pi k\end{array}\right.$.

563. а) sin x sin 3x = 0,5.

$\frac{1}{2}$ (cos 2x – cos 4x) = $\frac{1}{2}$;   cos 2x = 1+ cos 4x;

cos 2x = 2 $\cos^2$ 2x; cos 2x (2 cos 2x – 1) = 0;

$$\left[\begin{array}{l}\cos 2x=0\\ \cos 2x=\frac{1}{2}\end{array}\right.;\quad \left[\begin{array}{l}2x=\frac{\pi}{2}+\pi n\\ 2x=\pm\frac{\pi}{3}+2\pi k\end{array}\right.;\quad \left[\begin{array}{l}x=\frac{\pi}{4}+\frac{\pi n}{2}\\ x=\pm\frac{\pi}{6}+\pi k\end{array}\right..$$

б) cos x cos 3x + 0,5

$\frac{1}{2}$ (cos 2x + cos 4x) = $\frac{1}{2}$;   cos 2x + (1+ cos 4x) = 0;

cos 2x + 2 $\cos^2$ 2x = 0;   cos 2x (2 cos 2x + 1) = 0;

$$\left[\begin{array}{l}\cos 2x=0\\ \cos 2x=-\frac{1}{2}\end{array}\right.;\quad \left[\begin{array}{l}2x=\frac{\pi}{2}+\pi n\\ 2x=\pm\frac{2\pi}{3}+2\pi k\end{array}\right.;\quad \left[\begin{array}{l}x=\frac{\pi}{4}+\frac{\pi n}{2}\\ x=\pm\frac{\pi}{3}+\pi k\end{array}\right..$$

564. а) $f(x) = \sin\left(x+\frac{\pi}{8}\right)\cos\left(x-\frac{\pi}{24}\right) =$

$= \frac{1}{2}\left(\sin\left(x+\frac{\pi}{8}+x-\frac{\pi}{24}\right)+\sin\left(x+\frac{\pi}{8}-x+\frac{\pi}{24}\right)\right) =$

$= \frac{1}{2}\sin\left(2x+\frac{\pi}{12}\right)+\frac{1}{2}\sin\frac{\pi}{6} = \frac{1}{2}\sin\left(2x+\frac{\pi}{12}\right)+\frac{1}{4}.$

Поскольку наибольшее и наименьшее значения функции

$y = \sin\left(2x+\frac{\pi}{12}\right)$ равны 1 и –1 соответственно, то наибольшее и наименьшее значения функции f(x) равны $\frac{3}{4}$ и $-\frac{1}{4}$ соответственно.

б) $f(x) = \sin\left(x-\frac{\pi}{3}\right)\cos\left(x+\frac{\pi}{3}\right) =$

$= \frac{1}{2}\left(\sin\left(x+\frac{\pi}{8}+x-\frac{\pi}{24}\right)+\sin\left(x+\frac{\pi}{8}-x+\frac{\pi}{24}\right)\right) =$

$= \frac{1}{2}\left(\cos\frac{2\pi}{3}-\cos 2x\right) = -\frac{1}{4} - \frac{1}{2}$ cos 2x.

Поскольку наибольшее и наименьшее значения функции y = cos 2x равны 1 и –1 соответственно, то наибольшее и наименьшее значения функции f(x) равны $\frac{1}{4}$ и $-\frac{3}{4}$ соответственно.

565. $\cos^2(45^\circ - \alpha) - \cos^2(60^\circ + \alpha) - \cos 75^\circ \sin(75^\circ - 2\alpha) = \sin 2\alpha$.

$\cos^2(45^\circ - \alpha) - \cos^2(60^\circ + \alpha) - \cos 75^\circ \sin(75^\circ - 2\alpha) =$

$= \cos^2(45^\circ - \alpha) - \cos^2(60^\circ + \alpha) - \frac{1}{2}(\sin(75^\circ - 2\alpha + 75^\circ) +$

$+ \sin(75^\circ - 2\alpha - 75^\circ)) = \frac{1}{2}(1 + \cos(90^\circ - 2\alpha)) - \frac{1}{2}(1 + \cos(120^\circ +$

$+ 2\alpha)) - \sin(150^\circ - 2\alpha) + \sin 2\alpha) = \sin 2\alpha - \frac{1}{2}(\cos 120^\circ \cos 2\alpha -$

$- \sin 120^\circ \sin 2\alpha) - \frac{1}{2}(\sin 150^\circ \cos 2\alpha - \sin 2\alpha \cos 150^\circ) = \sin 2\alpha -$

$- \frac{1}{2}(-\frac{1}{2}\cos 2\alpha - \frac{\sqrt{3}}{2}\sin 2\alpha + \frac{1}{2}\cos 2\alpha + \frac{\sqrt{3}}{2}\sin 2\alpha) = \sin 2\alpha$.

566. Числа a, b, c образуют геометрическую прогрессию, если $a \neq 0$, $b \neq 0$, $c \neq 0$ и $\frac{b}{a} = \frac{c}{b}$.

а) a = cos 6x, b = cos 4x, c = cos 2x.

$$\begin{cases} \frac{\cos 4x}{\cos 6x} = \frac{\cos 2x}{\cos 4x} \\ \cos 2x \neq 0 \\ \cos 4x \neq 0 \\ \cos 6x \neq 0 \end{cases}; \quad \begin{cases} \cos^2 4x = \cos 2x \cos 6x \\ \cos 2x \neq 0 \\ \cos 4x \neq 0 \\ \cos 6x = 0 \end{cases};$$

$$\begin{cases} \frac{1}{2}(1 + \cos 8x) = \frac{1}{2}(\cos 8x + \cos 4x) \\ \cos 2x \neq 0 \\ \cos 4x \neq 0 \\ \cos 6x \neq 0 \end{cases}; \quad \begin{cases} \cos 4x = 1 \\ \cos 2x \neq 0 \\ \cos 4x \neq 0 \\ \cos 6x \neq 0 \end{cases};$$

$$\begin{cases} 4x = 2\pi n \\ \cos 2x \neq 0 \\ \cos 6x \neq 0 \end{cases}; \quad \begin{cases} x = \frac{\pi n}{2} \\ 2x \neq \frac{\pi}{2} + \pi k \\ 6x \neq \frac{\pi}{2} + \pi \ell \end{cases}; \quad \begin{cases} x = \frac{\pi n}{2} \\ x \neq \frac{\pi}{n} + \frac{\pi k}{2} \\ x \neq \frac{\pi}{12} + \frac{\pi \ell}{6} \end{cases}; \quad x = \frac{\pi n}{2}.$$

б) a = sin 2x, b = sin 3x, c = sin 4x.

$$\begin{cases} \frac{\sin 3x}{\sin 2x} = \frac{\sin 4x}{\sin 3x} \\ \sin 2x \neq 0 \\ \sin 3x \neq 0 \\ \sin 4x \neq 0 \end{cases}; \quad \begin{cases} \sin^2 3x = \sin 2x \sin 4x \\ \sin 2x \neq 0 \\ \sin 3x \neq 0 \\ \sin 4x = 0 \end{cases};$$

$$\begin{cases} \frac{1}{2}(1-\cos 6x) = \frac{1}{2}(\cos 2x - \cos 6x) \\ \sin 2x \neq 0 \\ \sin 3x \neq 0 \\ \sin 4x \neq 0 \end{cases}; \quad \begin{cases} \cos 2x = 1 \\ \sin 2x \neq 0 \\ \sin 3x \neq 0 \\ \sin 4x \neq 0 \end{cases};$$

$$\begin{cases} 2x = 2\pi n \\ 2x \neq \pi k \\ 3x \neq \pi \ell \\ 4x \neq \pi m \end{cases}$$ $\Rightarrow$ ни при каких x a, b, c не образуют геометрическую прогрессию.

§ 28. Преобразование выражения *A* sin *x* + *B* cos *x*

567. а) $\sqrt{3}\sin x + \cos x = 2\left(\frac{\sqrt{3}}{2}\sin x + \frac{1}{2}\cos x\right) =$

$= 2\left(\sin x\cos\frac{\pi}{6} + \cos x\sin\frac{\pi}{6}\right) = 2\sin\left(x + \frac{\pi}{6}\right)$.

б) $\sin x + \sqrt{3}\cos x = 2\left(\frac{1}{2}\sin x + \frac{\sqrt{3}}{2}\cos x\right) =$

$= 2\left(\cos x\cos\frac{\pi}{6} + \sin x\sin\frac{\pi}{6}\right) = 2\cos\left(x - \frac{\pi}{6}\right)$.

в) $\sin x - \cos x = \sqrt{2}\left(\frac{1}{\sqrt{2}}\sin x - \frac{1}{\sqrt{2}}\cos x\right) =$

$= \sqrt{2}\left(\sin x\cos\frac{\pi}{4} - \cos x\sin\frac{\pi}{4}\right) = \sqrt{2}\sin\left(x - \frac{\pi}{4}\right)$.

г) $2\sin x - \sqrt{12}\cos x = 4\left(\frac{1}{2}\sin x - \frac{\sqrt{3}}{2}\cos x\right) =$

$= 4\left(\sin x\cos\frac{\pi}{3} - \cos x\sin\frac{\pi}{3}\right) = 4\sin\left(x - \frac{\pi}{3}\right)$.

568. а) $3\sin x + 4\cos x = 5\left(\frac{3}{5}\sin x + \frac{4}{5}\cos x\right) =$

$= 5\left(\sin x\cos\left(\arcsin\frac{4}{5}\right) + \cos x\sin\left(\arcsin\frac{4}{5}\right)\right) = 5\sin\left(x + \arcsin\frac{4}{5}\right)$.

б) 5 cos x – 12 sin x = 13 $\left(\frac{5}{13}\cos x-\frac{12}{13}\sin x\right)$ =

= 13 $\left(\cos x\cos\left(\arcsin\frac{12}{13}\right)-\sin x\sin\left(\arcsin\frac{12}{13}\right)\right)$ = 13 cos $\left(x+\arcsin\frac{12}{13}\right)$.

в) 7 sin x – 24 cos x = 25 $\left(\frac{7}{25}\sin x-\frac{24}{25}\cos x\right)$ =

= 25 $\left(\cos\left(\arcsin\frac{24}{25}\right)\sin x-\cos x\sin\left(\arcsin\frac{24}{25}\right)\right)$ = 25 sin $\left(x-\arcsin\frac{24}{25}\right)$.

г) 8 cos x + 15 sin x = 17 $\left(\frac{8}{17}\cos x+\frac{15}{17}\sin x\right)$ =

= 17 $\left(\cos x\cos\left(\arcsin\frac{15}{17}\right)+\sin x\sin\left(\arcsin\frac{15}{17}\right)\right)$ = 17 cos $\left(x-\arcsin\frac{15}{17}\right)$.

569. а) $\sqrt{3}$ sin x + cos x = 1.

2 $\left(\frac{\sqrt{3}}{2}\sin x+\frac{1}{2}\cos x\right)$ = 1;

sin x sin $\frac{\pi}{3}$ + cos x cos $\frac{\pi}{3}$ = $\frac{1}{2}$; cos $\left(x-\frac{\pi}{3}\right)$ = $\frac{1}{2}$;

$$\left[\begin{array}{l} x-\frac{\pi}{3}=\frac{\pi}{3}+2\pi k \\ x-\frac{\pi}{3}=-\frac{\pi}{3}+2\pi n \end{array}\right. ;\quad \left[\begin{array}{l} x=\frac{2\pi}{3}+2\pi k \\ x=2\pi n \end{array}\right. .$$

б) sin x + cos x = $\sqrt{2}$.

$\frac{1}{\sqrt{2}}$ sin x + $\frac{1}{\sqrt{2}}$ cos x = 1; sin x cos $\frac{\pi}{4}$ + cos x sin $\frac{\pi}{4}$ = 1;

sin (x + $\frac{\pi}{4}$) = 1; x + $\frac{\pi}{4}$ = $\frac{\pi}{2}$ + 2πn; x = $\frac{\pi}{4}$ + 2πn.

в) sin x - $\sqrt{3}$ cos x = $\sqrt{3}$.

$\frac{1}{2}$ sin x - $\frac{\sqrt{3}}{2}$ cos x = $\frac{\sqrt{3}}{2}$; sin x cos $\frac{\pi}{3}$ - cox x sin $\frac{\pi}{3}$ = $\frac{\sqrt{3}}{2}$;

sin (x - $\frac{\pi}{3}$) = $\frac{\sqrt{3}}{2}$; $\left[\begin{array}{l} x-\frac{\pi}{3}=\frac{\pi}{3}+2\pi n \\ x-\frac{\pi}{3}=\frac{2\pi}{3}+2\pi k \end{array}\right.$; $\left[\begin{array}{l} x=\frac{2\pi}{3}+2\pi n \\ x=\pi+2\pi k \end{array}\right.$.

г) sin x – cos x = 1.

$\frac{1}{\sqrt{2}}$ sin x – $\frac{1}{\sqrt{2}}$ cos x = $\frac{1}{\sqrt{2}}$; sin x cos $\frac{\pi}{4}$ – cos x sin $\frac{\pi}{4}$ = $\frac{1}{\sqrt{2}}$;

sin (x – $\frac{\pi}{4}$) = $\frac{1}{\sqrt{2}}$; $\left[\begin{array}{l} x-\frac{\pi}{4}=\frac{\pi}{4}+2\pi n \\ x-\frac{\pi}{4}=\frac{3\pi}{4}+2\pi k \end{array}\right.$ $\left[\begin{array}{l} x=\frac{\pi}{2}+2\pi n \\ x=\pi+2\pi k \end{array}\right.$

570. а) y = $\sqrt{3}$ sin x + cos x = 2 $\left(\frac{\sqrt{3}}{2}\sin x+\frac{1}{2}\cos x\right)$ = 2 (sinx cos $\frac{\pi}{6}$ +

+ cos x sin $\frac{\pi}{6}$) = 2 sin (x + $\frac{\pi}{6}$).

Наибольшее значение 2, наименьшее –2.

б) y = sin x – $\sqrt{3}$ cos x = 2 $\left(\frac{1}{2}\sin x-\frac{\sqrt{3}}{2}\cos x\right)$ = 2 (sin x cos $\frac{\pi}{3}$ –

– cos x sin $\frac{\pi}{3}$) = 2 sin (x – $\frac{\pi}{3}$).

Наибольшее значение 2, наименьшее - –2.

в) y = sin x – cos x = $\sqrt{2}$ $\left(\frac{1}{\sqrt{2}}\sin x-\frac{1}{\sqrt{2}}\cos x\right)$ = $\sqrt{2}$ sin (x – $\frac{\pi}{4}$).

Наибольшее значение $\sqrt{2}$, наименьшее - –$\sqrt{2}$.

г) y = $\sqrt{6}$ sin x – $\sqrt{2}$ cos x = 2$\sqrt{2}$ $\left(\frac{\sqrt{3}}{2}\sin x-\frac{1}{2}\cos x\right)$ =

= 2$\sqrt{2}$ (sinx cos $\frac{\pi}{6}$ – cos x sin $\frac{\pi}{6}$) = 2$\sqrt{2}$ sin (x – $\frac{\pi}{6}$).

Наибольшее значение 2$\sqrt{2}$, наименьшее - –2$\sqrt{2}$.

571. а) y = 3 sin 2x – 4 cos 2x = 5 $\left(\frac{3}{5}\sin 2x-\frac{4}{5}\cos 2x\right)$ =

5 sin 2x cos (arcsin $\frac{4}{5}$) = cos 2x sin (arcsin $\frac{4}{5}$)) = 5 sin (2x - arcsin $\frac{4}{5}$).

Область значений – [–5, 5].

б) y = 5 cos 3x + 12 sin 3x = 13 $\left(\frac{5}{13}\cos 3x+\frac{12}{13}\sin 3x\right)$ =

= 13 (cos 3x cos (arcsin $\frac{12}{13}$) + sin 3x sin (arcsin $\frac{12}{13}$)) =13 cos (3x - arcsin $\frac{12}{13}$).

Область значений – [–13, 13].

в) y = 7 sin $\frac{x}{2}$ + 24 cos $\frac{x}{2}$ = 25 $\left(\frac{7}{25}\sin\frac{x}{2}+\frac{24}{25}\cos\frac{x}{2}\right)$ =

=25 (sin $\frac{x}{2}$ sin (arcsin $\frac{7}{25}$) + cos $\frac{x}{2}$ cos (arcsin $\frac{7}{25}$))=25 cos ($\frac{x}{2}$ – arcsin $\frac{7}{25}$).

Область значений – [–25, 25].

г) y = 8 cos $\frac{x}{3}$ - 15 sin $\frac{x}{3}$ = 17 $\left(\frac{8}{17}\cos\frac{x}{3}-\frac{15}{17}\sin\frac{x}{3}\right)$ =

=17(cos $\frac{x}{3}$ cos (arcsin $\frac{15}{17}$) – sin $\frac{x}{3}$ sin (arcsin $\frac{15}{17}$))=17 cos ($\frac{x}{3}$ + arcsin $\frac{15}{17}$).

Область значений – [–17, 17].

572. а) sin 5x + cos 5x = 1,5.

$\sqrt{2}$ sin (5x + $\frac{\pi}{4}$) = $\frac{3}{2}$; sin (5x + $\frac{\pi}{4}$) = $\frac{3}{2\sqrt{2}}=\sqrt{\frac{9}{8}}$.

Это равенство не выполняется ни при одном значении x,

т.к. sin (5x + $\frac{\pi}{4}$) ≤ 1 при всех x, а $\sqrt{\frac{9}{8}}$ > 1.

б) 3 sin 2x – 4 cos 2x = $\sqrt{26}$. 5 ($\frac{3}{5}$ sin 2x - $\frac{4}{5}$ cos 2x) = $\sqrt{26}$;

5 (sin 2x cos (arcsin $\frac{4}{5}$) – cos 2x sin (arcsin $\frac{4}{5}$)) = $\sqrt{26}$;

5 sin (2x - arcsin $\frac{4}{5}$) = $\sqrt{26}$; sin (2x - arcsin $\frac{4}{5}$) = $\frac{\sqrt{26}}{5}$.

Это равенство не выполняется ни при одном значении x,

т.к. sin (2x - arcsin $\frac{4}{5}$) ≤ 1 при всех x, а $\frac{\sqrt{26}}{5}$ > 1.

в) sin 7x – $\sqrt{3}$ cos 7x = $\frac{\pi}{2}$. $\frac{1}{2}$ sin 7x - $\frac{\sqrt{3}}{2}$ cos 7x = $\frac{\pi}{4}$;

sin 7x cos $\frac{\pi}{3}$ – cos 7x sin $\frac{\pi}{3}$ = $\frac{\pi}{4}$; sin (7x - $\frac{\pi}{3}$) = $\frac{\pi}{4}$.

Значения x, при которых выполняется это равенство, существуют, т.к. область значений функции y = sin (7x - $\frac{\pi}{3}$) – отрезок [-1;1] и –1< $\frac{\pi}{4}$ < 1.

г) 5 sin x + 12 cos x = $\sqrt{170}$. $\frac{5}{13}$ sin x + $\frac{12}{13}$ cos x = $\frac{\sqrt{170}}{13}$;

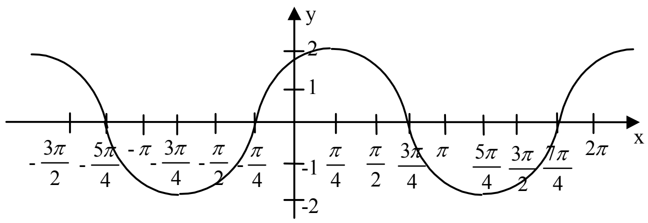
sin x cos (arcsin $\frac{12}{13}$) + cos x sin (arcsin $\frac{12}{13}$) = $\frac{\sqrt{170}}{13}$;

$\sin\left(x + \arcsin\frac{12}{13}\right) = \frac{\sqrt{170}}{13}$.

Это равенство не выполняется ни при одном значении x,

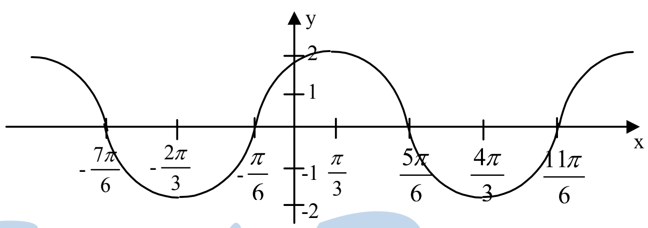
т.к. $\sin\left(x + \arcsin\frac{12}{13}\right) \le 1$ при всех x, а $\frac{\sqrt{170}}{13} > 1$.

573. а) $y = \sqrt{2}\,(\sin x + \cos x) = 2\sin\left(x + \frac{\pi}{4}\right)$



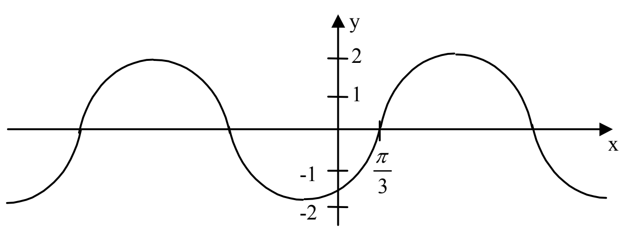
б) $y = \sqrt{3}\sin x + \cos x = 2\left(\sin x \cdot \frac{\sqrt{3}}{2} + \cos x \cdot \frac{1}{2}\right) =$

$= 2\left(\sin x \cos\frac{\pi}{6} + \cos x \sin\frac{\pi}{6}\right) = 2\sin\left(x + \frac{\pi}{6}\right)$.



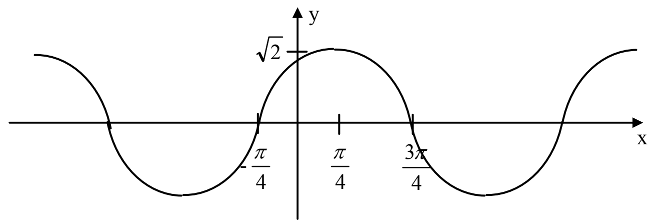
в) $y = \sin x - \sqrt{3}\cos x = 2\left(\frac{1}{2}\sin x - \frac{\sqrt{3}}{2}\cos x\right) =$

$= 2\left(\sin x \cos\frac{\pi}{3} - \cos x \sin\frac{\pi}{3}\right) = 2\sin\left(x - \frac{\pi}{3}\right)$.



г) $y = \sin x - \cos x = \sqrt{2}\ (\frac{1}{\sqrt{2}}\ \sin x - \frac{1}{\sqrt{2}}\ \cos x) =$

$= \sqrt{2}\ (\sin x \cos \frac{\pi}{4} - \cos x \sin \frac{\pi}{4}) = \sqrt{2}\ \sin (x - \frac{\pi}{4})$.



574. а) $\cos 2x + \sqrt{3}\ \sin 2x = \sqrt{2}$; $\frac{1}{2}\ \cos 2x + \frac{\sqrt{3}}{2}\ \sin 2x = \frac{1}{\sqrt{2}}$;

$\cos 2x \cos \frac{\pi}{3} + \sin \frac{\pi}{3}\ \sin 2x = \frac{1}{\sqrt{2}}$;

$\cos (2x - \frac{\pi}{3}) = \frac{1}{\sqrt{2}}$; $\left[\begin{array}{l} 2x - \frac{\pi}{3} = \frac{\pi}{4} + 2\pi n \\ 2x - \frac{\pi}{3} = -\frac{\pi}{4} + 2\pi k \end{array}\right.$; $\left[\begin{array}{l} x = \frac{7\pi}{24} + \pi n \\ x = \frac{\pi}{24} + \pi k \end{array}\right.$;

б) $\sin 5x - \cos 5x = \frac{\sqrt{6}}{2}$, $\frac{1}{\sqrt{2}}\ \sin 5x - \frac{1}{\sqrt{2}}\ \cos 5x = \frac{\sqrt{3}}{2}$;

$\sin 5x \cos \frac{\pi}{4} - \cos 5x \sin \frac{\pi}{4} = \frac{\sqrt{3}}{2}$;

$\sin (5x - \frac{\pi}{4}) = \frac{\sqrt{3}}{2}$; $\left[\begin{array}{l} 5x - \frac{\pi}{4} = \frac{\pi}{3} + 2\pi n \\ 5x - \frac{\pi}{4} = \frac{2\pi}{3} + 2\pi k \end{array}\right.$; $\left[\begin{array}{l} x = \frac{7\pi}{60} + \frac{2\pi n}{5} \\ x = \frac{3\pi}{20} + \frac{2\pi k}{5} \end{array}\right.$.

в) $\cos \frac{x}{2} - \sqrt{3}\ \sin \frac{x}{2} + 1 = 0$, $\frac{1}{2}\ \cos \frac{x}{2} - \frac{\sqrt{3}}{2}\ \sin \frac{x}{2} = -\frac{1}{2}$,

$\sin \frac{x}{2}\ \cos \frac{\pi}{6} - \cos \frac{x}{2}\ \sin \frac{\pi}{6} = \frac{1}{2}$;

$\sin (\frac{x}{2} - \frac{\pi}{6}) = \frac{1}{2}$; $\left[\begin{array}{l} \frac{x}{2} - \frac{\pi}{6} = \frac{\pi}{6} + 2\pi n \\ \frac{x}{2} - \frac{\pi}{6} = \frac{5\pi}{6} + 2\pi k \end{array}\right.$; $\left[\begin{array}{l} x = \frac{2\pi}{3} + 4\pi n \\ x = 2\pi + 2\pi k \end{array}\right.$.

г) $\sin\frac{x}{3}+\cos\frac{x}{3}=1$, $\frac{1}{\sqrt{2}}\sin\frac{x}{3}+\frac{1}{\sqrt{2}}\cos\frac{x}{3}=\frac{1}{\sqrt{2}}$,

$\sin\frac{x}{3}\cos\frac{\pi}{4}+\cos\frac{x}{3}\sin\frac{\pi}{4}=\frac{1}{\sqrt{2}}$;

$\sin\left(\frac{x}{3}+\frac{\pi}{4}\right)=\frac{1}{\sqrt{2}}$; $\left[\begin{array}{l}\frac{x}{3}+\frac{\pi}{4}=\frac{\pi}{4}+2\pi k\\ \frac{x}{3}+\frac{\pi}{4}=\frac{3\pi}{4}+2\pi n\end{array}\right.$; $\left[\begin{array}{l}x=6\pi k\\ x=\frac{3\pi}{2}+6\pi n\end{array}\right.$.

575. а) $4\sin x-3\cos x=5$.

$\frac{4}{5}\sin x-\frac{3}{5}\cos x=1$;

$\sin x\cos\left(\arcsin\frac{3}{5}\right)-\cos x\sin\left(\arcsin\frac{3}{5}\right)=1$;

$\sin\left(x-\arcsin\frac{3}{5}\right)=1$; $x-\arcsin\frac{3}{5}=\frac{\pi}{2}+2\pi n$;

$x=\frac{\pi}{2}+\arcsin\frac{3}{5}+2\pi n$.

б) $3\sin 2x+4\cos 2x=2{,}5$, $\frac{3}{5}\sin 2x+\frac{4}{5}\cos 2x=\frac{1}{2}$,

$\sin 2x\cos\left(\arcsin\frac{4}{5}\right)+\cos 2x\sin\left(\arcsin\frac{4}{5}\right)=\frac{1}{2}$;

$\sin\left(2x+\arcsin\frac{4}{5}\right)=\frac{1}{2}$; $2x+\arcsin\frac{4}{5}=(-1)^n\frac{\pi}{6}+\pi n$;

$x=(-1)^n\frac{\pi}{12}-\frac{1}{2}\arcsin\frac{4}{5}+\frac{\pi n}{2}$.

в) $12\sin x+5\cos x+13=0$; $\frac{12}{13}\sin x+\frac{5}{13}\cos x=-1$;

$\sin x\cos\left(\arcsin\frac{5}{13}\right)+\cos x\sin\left(\arcsin\frac{5}{13}\right)=-1$;

$\sin\left(x+\arcsin\frac{5}{13}\right)=-1$; $x+\arcsin\frac{5}{13}=-\frac{\pi}{2}+2\pi n$;

$x=-\frac{\pi}{2}-\arcsin\frac{5}{13}+2\pi n$.

г) $5\cos\frac{x}{2}-12\sin\frac{x}{2}=6{,}5$, $\frac{5}{13}\cos\frac{x}{2}-\frac{12}{13}\sin\frac{x}{2}=\frac{1}{2}$;

$\cos\frac{x}{2}\cos\left(\arcsin\frac{12}{13}\right)-\sin\frac{x}{2}\sin\left(\arcsin\frac{12}{13}\right)=\frac{1}{2}$;

$\cos\left(\frac{x}{2} + \arcsin\frac{12}{13}\right) = \frac{1}{2}$;

$\frac{x}{2} + \arcsin\frac{12}{13} = \pm\frac{\pi}{3} + 2\pi n$; $x = \pm\frac{2\pi}{3} - 2\arcsin\frac{12}{13} + 4\pi n$.

576. а) $\sin x + \cos x + \sqrt{2} = 2\sqrt{2}\cos^2\left(\frac{x}{2} - \frac{\pi}{8}\right)$.

$\sin x + \cos x + \sqrt{2} = \sqrt{2}\cos\left(x - \frac{\pi}{4}\right) + \sqrt{2} =$

$= \sqrt{2}\left(1 + \cos\left(x - \frac{\pi}{4}\right)\right) = 2\sqrt{2}\cos^2\left(\frac{x}{2} - \frac{\pi}{8}\right)$.

б) $\cos 2x - \sin 2x - \sqrt{2} = -2\sqrt{2}\sin^2\left(x + \frac{\pi}{8}\right)$.

$\cos 2x - \sin 2x - \sqrt{2} = \sqrt{2}\cos\left(2x + \frac{\pi}{4}\right) - \sqrt{2} =$

$= -\sqrt{2}\left(1 - \cos\left(2x + \frac{\pi}{4}\right)\right) = -2\sqrt{2}\sin^2\left(\frac{2x + \frac{\pi}{4}}{2}\right) = -2\sqrt{2}\sin^2\left(x + \frac{\pi}{8}\right)$.

577. а) $2\sin 17x + \sqrt{3}\cos 5x + \sin 5x = 0$.

$2\sin 17x + 2\left(\frac{\sqrt{3}}{2}\cos 5x + \frac{1}{2}\sin 5x\right) = 0$;

$2\sin 17x + 2\sin\left(5x + \frac{\pi}{3}\right) = 0$; $\sin 17x + \sin\left(5x + \frac{\pi}{3}\right) = 0$;

$2\sin\frac{17x + 5x + \frac{\pi}{3}}{2}\cos\frac{17x - 5x - \frac{\pi}{3}}{2} = 0$;

$\sin\left(11x + \frac{\pi}{6}\right)\cos\left(6x - \frac{\pi}{6}\right) = 0$;

$\left[\begin{array}{l}\sin(11x + \frac{\pi}{6}) = 0 \\ \cos(6x - \frac{\pi}{6}) = 0\end{array}\right.$; $\left[\begin{array}{l}11x + \frac{\pi}{6} = \pi n \\ 6x - \frac{\pi}{6} = \frac{\pi}{2} + \pi k\end{array}\right.$; $\left[\begin{array}{l}x = -\frac{\pi}{66} + \frac{\pi n}{11} \\ x = \frac{\pi}{9} + \frac{\pi k}{6}\end{array}\right.$.

б) $5\sin x - 12\cos x + 13\sin 3x = 0$; $\frac{5}{13}\sin x - \frac{12}{13}\cos x + \sin 3x = 0$;

$\sin x\cos\left(\arcsin\frac{12}{13}\right) - \cos x\sin\left(\arcsin\frac{12}{13}\right) + \sin 3x = 0$;

$\sin(x - \arcsin\frac{12}{13}) + \sin 3x = 0;$

$2\sin\frac{x - \arcsin\frac{12}{13} + 3x}{2}\cos\frac{x - \arcsin\frac{12}{13} - 3x}{2} = 0;$

$\sin(2x - \frac{1}{2}\arcsin\frac{12}{13})\cos(x + \frac{1}{2}\arcsin\frac{12}{13}) = 0;$

$\left[\begin{array}{l} \sin(2x - \frac{1}{2}\arcsin\frac{12}{13}) = 0 \\ \cos(x + \frac{1}{2}\arcsin\frac{12}{13}) = 0 \end{array}\right.; \quad \left[\begin{array}{l} 2x - \frac{1}{2}\arcsin\frac{12}{13} = \pi n \\ x + \frac{1}{2}\arcsin\frac{12}{13} = \frac{\pi}{2} + \pi k \end{array}\right.;$

$\left[\begin{array}{l} x = \frac{1}{4}\arcsin\frac{12}{13} + \frac{\pi n}{2} \\ x = \frac{\pi}{2} - \frac{1}{2}\arcsin\frac{12}{13} + \pi k \end{array}\right..$

578. а) $(\sin x + \sqrt{3}\cos x)^2 - 5 = \cos(\frac{\pi}{6} - x).$

$4(\frac{1}{2}\sin x + \frac{\sqrt{3}}{2}\cos x)^2 - 5 = \cos(\frac{\pi}{6} - x);$

$4(\sin x\cos\frac{\pi}{3} + \cos x\sin\frac{\pi}{3})^2 - 5 = \cos(\frac{\pi}{6} - x);$

$4\sin^2(x + \frac{\pi}{3}) - 5 = \cos(\frac{\pi}{6} - x);$ $4\cos^2(\frac{\pi}{6} - x) - 5 = \cos(\frac{\pi}{6} - x);$

$4\cos^2(\frac{\pi}{6} - x) - \cos(\frac{\pi}{6} - x) - 5 = 0;$

$\cos(\frac{\pi}{6} - x) = \frac{1 \pm \sqrt{1 + 80}}{8} = \frac{1 \pm 9}{8} = \left[\begin{array}{l} \frac{5}{4} \\ -1 \end{array}\right.;$

$\cos(\frac{\pi}{6} - x) = -1$ $\quad (\cos(\frac{\pi}{6} - x) \neq \frac{5}{4}$ при всех x, т.к.

$\cos(\frac{\pi}{6} - x) \leq 1);$ $\cos(x - \frac{\pi}{6}) = -1;$ $\quad x - \frac{\pi}{6} = \pi + 2\pi n;$ $x = \frac{7\pi}{6} + 2\pi n.$

б) $(\sqrt{3}\sin x - \cos x)2 + 1 = 4\cos(x + \frac{\pi}{3}).$

$4(\frac{\sqrt{3}}{2}\sin x - \frac{1}{2}\cos x)2 + 1 = 4\cos(x + \frac{\pi}{3});$

$4\cos2\left(x+\frac{\pi}{3}\right)+1-4\cos\left(x+\frac{\pi}{3}\right)=0$; $\left(2\cos x\left(x+\frac{\pi}{3}\right)-1\right)2=0$;

$\cos\left(x+\frac{\pi}{3}\right)=\frac{1}{2}$;

$$\left[\begin{array}{l} x+\frac{\pi}{3}=\frac{\pi}{3}+2\pi k \\ x+\frac{\pi}{3}=-\frac{\pi}{3}+2\pi n \end{array}\right. ; \quad \left[\begin{array}{l} x=2\pi k \\ x=-\frac{2\pi}{3}+2\pi n \end{array}\right. .$$

579. а) $\sqrt{3}\sin x+\cos x+2=\frac{12}{\pi}x$.

$\frac{\sqrt{3}}{2}\sin x+\frac{1}{2}\cos x+1=\frac{6}{\pi}x$; $\sin x\cos\frac{\pi}{6}+\cos x\sin\frac{\pi}{6}+1=\frac{6}{\pi}x$;

$\sin\left(x+\frac{\pi}{6}\right)=\frac{6}{\pi}x-1$; $x=\frac{\pi}{3}$.

б) $\sqrt{2}(\cos x-\sin x)=2x-\frac{\pi}{2}$.

$2\left(\cos x\cos\frac{\pi}{4}-\sin x\sin\frac{\pi}{4}\right)=2x-\frac{\pi}{2}$; $\cos\left(x+\frac{\pi}{4}\right)=x-\frac{\pi}{4}$; $x=\frac{\pi}{4}$.

580. а) $\sqrt{3}\sin x+\cos x>1$.

$\frac{\sqrt{3}}{2}\sin x+\frac{1}{2}\cos x>\frac{1}{2}$; $\sin x\cos\frac{\pi}{6}+\cos x\sin\frac{\pi}{6}>\frac{1}{2}$;

$\sin\left(x+\frac{\pi}{6}\right)>\frac{1}{2}$; $\frac{\pi}{6}+2\pi n<x+\frac{\pi}{6}<\frac{5\pi}{6}+2\pi n$; $2\pi n<x<\frac{2\pi}{3}+2\pi n$.

б) $3\sin x-4\cos x<2{,}5$; $\frac{3}{5}\sin x-\frac{4}{5}\cos x<\frac{1}{2}$;

$\sin x\cos\left(\arcsin\frac{4}{5}\right)-\cos x\sin\left(\arcsin\frac{4}{5}\right)<\frac{1}{2}$; $\sin\left(x-\arcsin\frac{4}{5}\right)<\frac{1}{2}$;

$-\frac{7\pi}{6}+2\pi n<x-\arcsin\frac{4}{5}<\frac{\pi}{6}+2\pi n$;

$-\frac{7\pi}{6}+2\pi n+\arcsin\frac{4}{5}<x<\frac{\pi}{6}+2\pi n+\arcsin\frac{4}{5}$.

Глава 4. Производная

§ 29. Числовые последовательности

581. а) $y = 3x^2 + 5$, $x \in Z$, является.

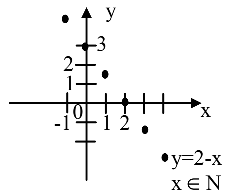
б) $y = \sin x$, $x \in [0; 2\pi]$, не является.

в) $y = 7 - x^2$, $x \in Q$, не является.

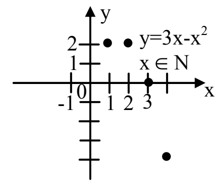
г) $y = \cos\frac{x}{2}$, $x \in N$, является.

582.

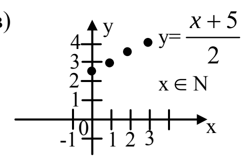
а)



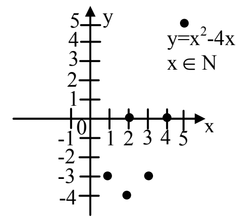
б)



в)

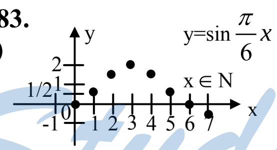


г)

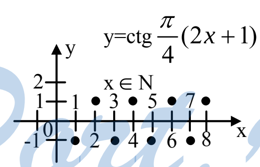


583.

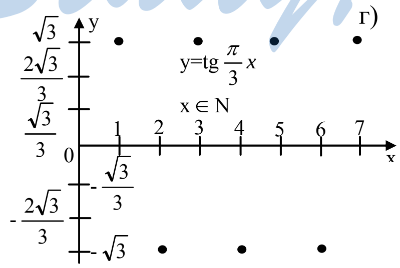
а)



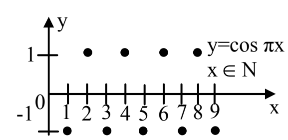
б)



в)



г)



584. а) $y_n = 2^n$; б) 1, 2, 4, 8, 16…; в) $b_1 = 2$, $b_{n+1} = b_n \cdot 2$.

585. а) $y_n = -n$, $n \in N$, $y_1 = -1$, $y_2 = -2$, $y_3 = -3$, $y_4 = -4$, $y_5 = -5$.

б) $y_n = \sqrt{n}$, $n \in N$, $y_1 = 1$, $y_2 = \sqrt{2}$, $y_3 = \sqrt{3}$, $y_4 = 2$, $y_5 = \sqrt{5}$.

в) $y_n = -5$, $y_1 = -5$, $y_2 = -5$, $y_3 = -5$, $y_4 = -5$, $y_5 = -5$.

г) $y_n = \frac{n^2}{2}$, $n \in N$, $y_1 = \frac{1}{2}$, $y_2 = 2$, $y_3 = \frac{9}{2}$, $y_4 = 8$, $y_5 = \frac{25}{2}$.

586. а) $y_7 = 42$, $y_9 = 54$, $y_7 = 72$, $6 \cdot n = y_n$.

б) $y_6 = 42$, $y_{10} = 70$, $y_{31} = 217$, $7 \cdot n = y_n$.

587. а) $y_1 = 2$, $y_2 = 7$, $y_3 = 12$, $y_4 = 17$, $y_5 = 22$.

б) $S_6 = 3 + 7 + 11 + 15 + 19 + 23 = 78$

588. а) $y_n = n^5$, $n \in N$, $y_1 = 1$, $y_2 = 32$, $y_n = n^5$, $y_{n+1} = (n + 1)^5$;

б) $y_n = 3^n$, $y_5 = 3^5 = 243$, $y_7 = 3^7 = 2187$, $y_{21} = 3^{21}$, $y_{2n}=3^{2n}$. $y_{2n+5} = 3^{2n+5}$.

589. а) y_{732} и y_{745}, $y_{733}, y_{734}, \ldots, y_{744}$.

б) y_{n-1} и y_{n+2}, y_n, y_{n+1}.

в) y_{998} и y_{1003}, y_{999}, y_{1000}, y_{1001}, y_{1002}.

г) y_{2n-2} и y_{2n+3}. y_{2n-1}, y_{2n}, y_{2n+1}, y_{2n+2}.

590. а) $y_n = 3 - 2n$.

$y_1 = 1$, $y_2 = -1$, $y_3 = -3$, $y_4 = -5$, $y_5 = -7$.

б) $y_n = 2n^2 - n$.

$y_1 = 1$, $y_2 = 6$, $y_3 = 15$, $y_4 = 28$, $y_5 = 45$.

в) $y_n = n^3 - 1$.

$y_1 = 0$, $y_2 = 7$, $y_3 = 26$, $y_4 = 63$, $y_5 = 124$.

г) $y_n = \frac{3n-1}{2n} = \frac{3}{2} - \frac{1}{2n}$.

$y_1 = 1$, $y_2 = \frac{5}{4} = 1\frac{1}{4}$, $y_3 = \frac{8}{6} = 1\frac{1}{3}$,

$y_4 = \frac{11}{8} = 1\frac{3}{8}$, $y_5 = \frac{7}{5} = 1\frac{2}{5}$.

591. а) $y_n = (-1)^n$.

$y_1 = -1$, $y_2 = 1$, $y_3 = -1$, $y_4 = 1$, $y_5 = -1$.

б) $y_n = \frac{(-2)^n}{n^2+1}$.

$y_1 = -1$, $y_2 = \frac{4}{5}$, $y_3 = -\frac{8}{10} = -\frac{4}{5}$, $y_4 = \frac{16}{17}$, $y_5 = -\frac{32}{26} = -\frac{16}{13} = -1\frac{3}{13}$.

в) $y_n = (-1)^n \frac{1}{10^n}$.

$y_1 = -\frac{1}{10}$, $y_2 = \frac{1}{100}$, $y_3 = -\frac{1}{1000}$,

$y_4 = \frac{1}{10000}$, $y_5 = -\frac{1}{100000}$.

г) $y_n = \frac{(-1)^n + 2}{3n-2}$.

$y_1 = 1$, $y_2 = \frac{3}{4}$, $y_3 = \frac{1}{7}$, $y_4 = \frac{3}{10}$, $y_5 = \frac{1}{13}$.

592. а) $y_n = 3\cos\frac{2\pi}{n}$.

$y_1 = 3$, $y_2 = -3$, $y_3 = -\frac{3}{2}$, $y_4 = 0$, $y_5 = 3\cos\frac{2\pi}{5}$.

б) $y_n = \operatorname{tg}\left((-1)^n\frac{\pi}{4}\right)$. $y_1 = -1$, $y_2 = 1$, $y_3 = -1$, $y_4 = 1$, $y_5 = -1$.

в) $y_n = 1 - \cos^2\frac{\pi}{n}$. $y_1 = 0$, $y_2 = 1$, $y_3 = \frac{3}{4}$, $y_4 = \frac{1}{2}$, $y_5 = \sin^2\frac{\pi}{5}$.

г) $y_n = \sin \pi n - \cos \pi n = -\cos \pi n$.

$y_1 = 1$, $y_2 = -1$, $y_3 = 1$, $y_4 = -1$, $y_5 = 1$.

593. $2^2 + 3^2 + 5^2 + 7^2 + 11^2 + 13^2 + 17^2 + 19^2 =$
$= 4 + 9 + 25 + 49 + 121 + 169 + 289 + 361 = 1027$

594. а) $y_n = n$, $n = 1, 2, 3 \ldots$
б) $y_n = -n$, $n \in N$.
в) $y_n = n + 4$, $n \in N$.
г) $y_n = 11 - n$, $n \in N$.

595. а) $y_n = 5n$, $n \in N$.
б) $y_n = 6n$, $n \in N$.
в) $y_n = 4n$, $n \in N$.
г) $y_n = 3n$, $n \in N$.

596. а) $y_n = 3^n$, $n \in N$.
б) $y_n = (n + 2)^2$, $n \in N$.
в) $y_n = n^3$, $n \in N$.
г) $y_n = n^3 + 1$, $n \in N$.

597. а) $y_n = \frac{2}{2^n} = \frac{1}{2^{n-1}}$; б) $y_n = \frac{2n+1}{2n+2}$;

в) $y_n = \frac{1}{n^3}$; г) $y_n = \frac{1}{(2n+1)(2n+3)}$.

598. а) $y_n = 1{,}5n$, $y_1 = 1{,}5$, $y_2 = 3$, $y_3 = 4{,}5$.
б) $y_n = (-1)^n$, $y_1 = -1$, $y_2 = 1$, $y_3 = -1$.

в) $y_n = \frac{8}{n}$, $y_1 = 8$, $y_2 = 4$, $y_3 = \frac{8}{3}$ …

г) $y_n = (-1)^{n+1}n$. $y_1 = 1$, $y_2 = -2$, $y_3 = 3$, $y_4 = -4$.

599. а) 1; 1,4; 1,41; 1,414; 1,4142.
б) 2; 1,5; 1,42; 1,415; 1,4143.

600. $y_n = \frac{2-n}{5n+1}$.

а) $y_n = 0$, $n = 2$.

б) $y_n = -\frac{3}{26}$, $n = 5$.

в) $y_n = -\frac{1}{6}$, $\frac{2-n}{5n+1} = -\frac{1}{6}$.

$6n - 12 = 5n + 1$, $n = 13$.

г) $-\frac{43}{226} = y_n$. $\frac{2-n}{5n+1} = -\frac{43}{226}$

$226n - 452 = 215n + 43$, $11n = 495$, $n = 45$.

601. $a_n = (2n - 1)(3n + 2)$

а) $a_n = 0$. нет, т.к. n не может быть равным $\frac{1}{2}, -\frac{2}{3}$.

б) $a_n = 24$. $6n^2 + n - 2 = 24$. $6n^2 + n - 26 = 0$.

$n = \frac{-1+25}{12} = 2$, или $n = -\frac{26}{12}$. (не подходит, т.к. $n \in N$).

Ответ: является.

в) $a_n = 153$. $6n^2 + n - 155 = 0$. $n = \frac{-1+61}{12} = 5$. $n = -\frac{62}{12}$ (не подходит).

Ответ: является.

г) $a_n = -2$. $6n^2 + n = 0$.

$n(6n + 1) = 0$. решения в натуральных числах нет $\Rightarrow$ не является.

602. а) 1; $\frac{1}{2}$; $\frac{1}{3}$; $\frac{1}{4}$…

ограничена снизу и сверху.

б) –1; 2; -3; 4; -5… не ограничена.

в) $\frac{1}{2}$; $\frac{2}{3}$; $\frac{3}{4}$; $\frac{4}{5}$… снизу ограничена.

г) 5; 4; 3; 2; 1; 0; -1

не ограничена снизу.

603. а) -3; -2; -1; 0; 1…

не ограничена сверху.

б) 1; -1; 1; -2; 1; -3…

ограничена сверху.

в) $\frac{1}{2}$; $\frac{1}{3}$; $\frac{1}{4}$; $\frac{1}{5}$…

ограничена сверху.

г) $\frac{1}{2}$; $\frac{2}{3}$; $\frac{3}{4}$; $\frac{4}{5}$…

ограничена сверху.

604. а) $\frac{1}{2}$; $\frac{1}{3}$; $\frac{1}{4}$;… $\frac{1}{n}$ … ограничена, т.к. $0 < y_n < 1$.

б) $\frac{1}{2}$; $\frac{3}{4}$; $\frac{5}{6}$;… $\frac{2n-1}{2n}$ … ограничена, т.к. $0 < y_n < 1$.

в) 5; -5; 5; -5; $(-1)^{n-1}5$ ограничена, т.к. $-10 < y_n < 10$.

г) -2; 3; -4; 5; $(-1)^n(n+1)$ не ограничена.

605. а) $x_n = 3 \cdot n + 2$ возьмем $n_2 > n_1$

$x_{n_2} = 3 \cdot n_2 + 2 > 3 \cdot n_1 + 2 \Rightarrow$ последов. возрастающая.

б) $x_n = \frac{5}{n+3}$ убывающая, т.к. при $n_2 > n_1$ $x_{n_2} < x_{n_1}$.

в) $x_n = n^3$ возрастающая, т.к. при $n_2 > n_1$ $x_{n_2} > x_{n_1}$.

г) $x_n = (-1)^{n-1}$

ни возрастающая, ни убывающая.

606. а) $x_n = \left(\frac{1}{3}\right)^{n+1}$ убывающая.

б) $x_n = 7^{n-5}$ возрастающая.

в) $x_n = 6^{1-n}$ убывающая.

г) $x_n = \left(-\frac{1}{5}\right)^{2n-1}$ возрастает, т.к. степень нечетная.

607. а) $y_{n+1} - y_n > 0$.

$y_{n+1} > y_n \Rightarrow$ по определению

она возрастающая.

б) $\frac{y_{n+1}}{y_n} < 1$, $y_n > 0$

$y_{n+1} < y_n \Rightarrow$ убывающая.

в) $y_{n+1} - y_n < 0$.

$y_{n+1} < y_n \Rightarrow$ убывающая.

г) $\frac{y_{n+1}}{y_n} < 1$, $(y_n < 0)$

$y_{n+1} > y_n \Rightarrow$ возрастающая.

608. а) $y_n = 2n - 1$

$y_{n+1} = 2n + 1$

$y_{n+1} > y_n \Rightarrow$ возрастающая.

б) $y_n = 5^{-n}$

$y_{n+1} = 5^{-n-1} = \frac{5^{-n}}{5}$

$y_{n+1} < y_n \Rightarrow$ убывающая.

в) $y_n = n^2 + 8$, $y_{n+1} = n^2 + 2n + 9$

$y_{n+1} > y_n \Rightarrow$ возрастающая.

г) $y_n = \frac{2}{3n+1}$, $y_{n+1} = \frac{2}{3n+4}$, $y_{n+1} < y_n \Rightarrow$ убывающая.

609. а) $x_n = (-2)^n$ $x_1 = -2$, $x_2 = 4$, $x_3 = -8$

$\Rightarrow$ последовательность не является монотонной.

б) $y_n = \cos\frac{\pi}{n+5}$

$y_1 = \cos\frac{\pi}{6}$, $y_2 = \cos\frac{\pi}{7}$, $y_3 = \cos\frac{\pi}{8}$.

$y_1 < y_2 < y_3 \ldots$ $\Rightarrow$ возраст.

в) $y_n = n^3 - 5$

$y_1 = -4$, $y_2 = 3$, $y_3 = 22$. $\Rightarrow$ возрастает.

г) $y_n = \sqrt{n+8}$, $y_1 = 3$, $y_2 = \sqrt{10}$, $y_3 = \sqrt{11}$.

610. а) $y_n = 2^n$; б) $y_n = 2^n$; в) $y_n = \left(\frac{1}{2}\right)^n$; г) $y_n = -n$.

611. а) $y_n = \sin\frac{\pi n}{2} - \operatorname{ctg}\frac{\pi}{4}(2n+1)$

$y_1 = 2$, $y_2 = -1$, $y_3 = 0$, $y_4 = 1$, $y_5 = 2$.

б) $y_n = \cos\frac{\pi n}{2} + \operatorname{tg}\frac{\pi}{4}(2n+1)$

$y_1 = -1$, $y_2 = 0$, $y_3 = -1$, $y_4 = 2$, $y_5 = -1$.

в) $y_n = n\sin\frac{\pi n}{2} + n^2\cos\frac{\pi n}{2}$

$y_1 = 1$, $y_2 = -4$, $y_3 = -3$, $y_4 = 16$, $y_5 = 5$.

г) $y_n = \sin\frac{\pi n}{4} - n\cos\frac{\pi n}{4}$

$y_1 = 0$, $y_2 = 1$, $y_3 = 2\sqrt{2}$, $y_4 = 4$, $y_5 = 2\sqrt{2}$.

612. а) $y_n = \frac{1\cdot 2\cdot 3\cdot 4\cdots n}{n^3+1}$

$y_1 = \frac{1}{2}$, $y_2 = \frac{2}{9}$, $y_3 = \frac{6}{28} = \frac{3}{14}$, $y_4 = \frac{24}{65}$, $y_5 = \frac{120}{126} = \frac{20}{21}$.

б) $y_n = \frac{1\cdot 3\cdot 5\cdot \ldots \cdot(2n-1)}{2\cdot 4\cdot 6\cdot \ldots \cdot 2n}$

$y_1 = \frac{1}{2}$, $y_2 = \frac{3}{8}$, $y_3 = \frac{3\cdot 5}{2\cdot 4\cdot 2\cdot 3} = \frac{5}{16}$,

$y_4 = \frac{5\cdot 7}{16\cdot 8} = \frac{35}{128}$, $y_5 = \frac{35\cdot 9}{128\cdot 10} = \frac{7\cdot 9}{128\cdot 5} = \frac{63}{256}$.

613.

а) $x_1 = 2$, $x_n = 5 - x_{n-1}$.

$x_1 = 2$, $x_2 = 5 - 2 = 3$, $x_3 = 2$, $x_4 = 3$, $x_5 = 2$.

б) $x_1 = 2$, $x_n = x_{n-1} + 10$.

$x_1 = 2$, $x_2 = 12$, $x_3 = 22$, $x_4 = 32$, $x_5 = 42$.

в) $x_1 = -1$, $x_n = 2 + x_{n-1}$

$x_1 = -1$, $x_2 = 1$, $x_3 = 3$, $x_4 = 5$, $x_5 = 7$.

г) $x_1 = 4$, $x_n = x_{n-1} - 3$.

$x_1 = 4$, $x_2 = 1$, $x_3 = -2$, $x_4 = -5$, $x_5 = -8$.

614. а) $x_1 = 2$, $x_n = nx_{n-1}$.

$x_1 = 2$, $x_2 = 4$, $x_3 = 12$, $x_4 = 48$, $x_5 = 240$.

б) $x_1 = -5$, $x_n = -\frac{1}{2}x_{n-1}$.

$x_1 = -5$, $x_2 = \frac{5}{2}$, $x_3 = -\frac{5}{4}$, $x_4 = \frac{5}{8}$, $x_5 = -\frac{5}{16}$.

в) $x_1 = -2$, $x_n = -x_{n-1}$

$x_1 = -2$, $x_2 = 2$, $x_3 = -2$, $x_4 = 2$, $x_5 = -2$.

г) $x_1 = 1$, $x_n = 10x_{n-1}$.

$x_1 = 1$, $x_2 = 10$, $x_3 = 100$, $x_4 = 1000$, $x_5 = 10000$.

615. а) $-\frac{1}{2}$; $\frac{3}{4}$; $-\frac{5}{6}$; $\frac{7}{8}$; $-\frac{9}{10}$...

$\frac{(-1)^n \cdot (2n-1)}{2n} = y_n$.

б) $\frac{2}{\sqrt{3}}$; $\frac{4}{3}$; $\frac{6}{3\sqrt{3}}$; $\frac{8}{9}$; $\frac{10}{9\sqrt{3}}$... $y_n = \frac{2n}{(\sqrt{3})^n}$.

в) $\frac{3}{4}$; $\frac{9}{16}$; $\frac{27}{64}$; $\frac{81}{256}$... $y_n = \left(\frac{3}{4}\right)^n$.

г) $\frac{1}{\sqrt{2}}$; $\frac{3}{2}$; $\frac{5}{2\sqrt{2}}$; $\frac{7}{4}$; $\frac{9}{4\sqrt{2}}$... $y_n = \frac{2n-1}{(\sqrt{2})^n}$.

616. а) $\frac{1}{\sqrt{1\cdot 2}}$; $-\frac{4}{\sqrt{2\cdot 3}}$; $\frac{9}{\sqrt{3\cdot 4}}$; $-\frac{16}{\sqrt{4\cdot 5}}$... $y_n = (-1)^{n+1}\frac{n^2}{\sqrt{n(n+1)}}$.

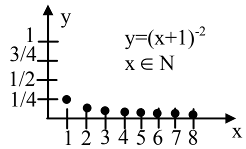
б) $-\frac{5}{2^2\cdot 3^2}$; $\frac{7}{3^2\cdot 4^2}$; $-\frac{9}{4^2\cdot 5^2}$; $\frac{1}{5^2\cdot 6^2}$... $y_n = (-1)^n\frac{2n+3}{(n+1)^2\cdot(n+2)^2}$.

в) $\frac{4}{1\cdot 2\cdot 3}$; $-\frac{9}{2\cdot 3\cdot 4}$; $\frac{14}{3\cdot 4\cdot 5}$; $-\frac{19}{4\cdot 5\cdot 6}$... $y_n = (-1)^{n+1}\frac{5n-1}{n(n+1)\cdot(n+2)}$.

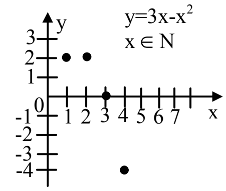
г) 0; $\frac{2}{1\cdot 2}$; 0; $\frac{2}{1\cdot 2\cdot 3\cdot 4}$; 0; $\frac{2}{1\cdot 2\cdot 3\cdot 4\cdot 5\cdot 6}$... $y_n = \frac{(-1)^n + 1}{1\cdot 2\cdot 3\cdot 4\cdots n}$.

617.

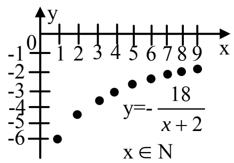
а)



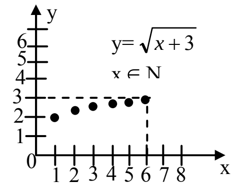
б)



в)

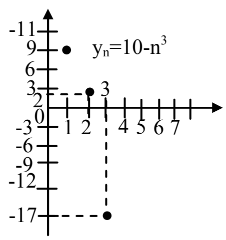


г)

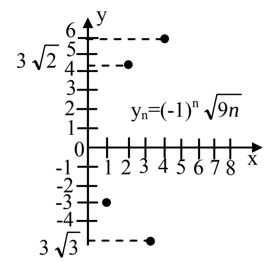


618.

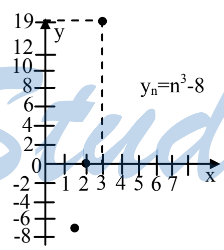
а)



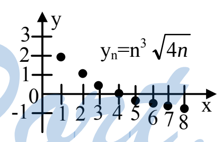
б)



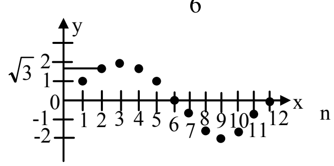
в)



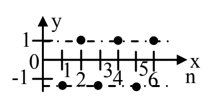
г)



619. а) $y_n = 2\sin\frac{\pi n}{6}$



б) $y_n = (-1)^n \operatorname{tg}\frac{\pi}{4}(2n-1)$



620. $x_1 = 5$, $x_2 = -3$, $x_3 = 1$, $x_4 = -1$, $x_5 = 0$, $x_6 = -\frac{1}{2}$.

$x_n = \frac{x_{n-2} + x_{n-1}}{2}$.

621. $a_1 = 1$; $S_1 = 1$; $P_1 = 4$.

$a_2 = \sqrt{2}$; $S_2 = 2$; $P_2 = 4\sqrt{2}$.

$a_3 = 2$; $S_3 = 4$; $P_3 = 8$.

$a_4 = 2\sqrt{2}$; $S_4 = 8$; $P_4 = 8\sqrt{2}$.

$a_5 = 4$; $S_5 = 16$; $P_5 = 16$.

$a_n = \left(\sqrt{2}\right)^{n-1}$; $S_n = 2^{n-1}$; $P_n = 4 \cdot \left(\sqrt{2}\right)^{n-1}$.

$a_{11} = \left(\sqrt{2}\right)^{10} = 2^5 = 32$.

$S_{17} = 2^{16}$.

622. $y_n = 2n^2 - 7n + 5$.

$\begin{cases} 2n^2 - 7n + 5 \le 5. \\ 2n^2 - 7n + 5 \ge 2. \end{cases}$ $\begin{cases} n(2n - 7) \le 0. \\ 2n^2 - 7n + 3 \ge 0. \end{cases}$ $(x - 3)(2x - 1) \ge 0$

$\begin{cases} n \in \left[0; \frac{7}{2}\right] \\ n \in \left(-\infty; \frac{1}{2}\right] \cup [3; +\infty) \end{cases}$

$\Downarrow$

$n \in \left[0; \frac{1}{2}\right] \cup \left[3; \frac{7}{2}\right]$, т.к. $x \in N$ то $n = 3 \Rightarrow y_3 \in [2; 5]$.

623. а) $x_n = 3n - 2$ $A = 15$.

$3n - 2 > 15$, $n > \frac{17}{3}$.

$n \ge 6$. начиная с x_6.

б) $x_n = 5^{n-1}$ $A = 125$.

$5^{n-1} > 125$. $5^{n-1} > 5^3$. $n > 4$. начиная с x_5.

в) $x_n = n^2 - 17$ $A = -2$.

$n^2 - 17 > -2$. $n^2 > 15$. начиная с x_4.

г) $x_n = 3^{n-5}$ $A = 27$.

$3^{n-5} > 3^3$. $n > 8$. начиная с x_9.

624. а) $x_1 = -14$ $x_n = x_{n-1} + 7$ $A = 25$.

$x_1 = -14$ $x_2 = -7$ $x_3 = 0$.

$x_n = -21 + 7n$. $-21 + 7n > 25$. $n > \frac{46}{7}$.

начиная с x_7.

б) $x_1 = 3$ $x_n = 6x_{n-1}$ A = 168.
$x_1 = 3$ $x_2 = 18$ $x_3 = 108$.
$x_n = 3\cdot6^{n-1}$. $3\cdot6^{n-1} > 168$. $6^{n-1} > 56$. начиная с x_4.
в) $x_1 = 0$ $x_n = x_{n-1} + 3$ A = 28.

$x_n = 3(n-1)$. $3(n-1) > 28$. $n > \frac{28}{3} + 1$. начиная с x_{11}.

г) $x_1 = 1$ $x_n = 7x_{n-1}$ A = 285.
$x_n = 7^{n-1}$. $7^{n-1} > 285$. начиная с x_4.

625. а) $\frac{1}{3125}$; $\frac{1}{625}$; $\frac{1}{125}$...

$x_n = \frac{1}{3125}\cdot5^{(n-1)}$. $5^{-5}\cdot5^{n-1} \le 1$. $n - 1 \le 5$. $n \le 6$. шесть членов.

б) $\frac{6}{377}$; $\frac{11}{379}$; $\frac{16}{381}$... $\frac{6+5(n-1)}{377+2(n-1)} = x_n$. $\frac{5n+1}{375+2n} \le 1$.

$3n \le 374$. $n \le \frac{374}{3}$. 124 члена.

в) $\frac{2}{729}$; $\frac{2}{243}$; $\frac{2}{81}$...

$\frac{2\cdot3^{n-1}}{729} = x_n$. $2\cdot3^{n-1} \le 729$. $2\cdot3^{n-1} \le 3^6$. $n < 7$ шесть членов.

г) $\frac{2}{219};\frac{9}{222};\frac{16}{225}$

$\frac{2+7(n-1)}{219+3(n-1)} = x_n$

$\frac{7n-5}{3n+216} \le 1$

$7n-5 \le 3n+216$

$4n \le 221$

$n \le \frac{221}{4}$

55 членов.

626. а) $y_n = \frac{n^2}{n+1} = n+1-\frac{2n+1}{n+1} = n+1-2+\frac{1}{n+1} =$

$= n-1+\frac{1}{n+1}$. Ограничена снизу.

б) $y_n = \frac{(-1)^n+1}{2n}$. $y_1 = 0$ $y_2 = \frac{1}{2}$ $y_3 = 0$ $y_4 = \frac{1}{4}$.
Ограничена снизу.

в) $y_n = ((-1)^n + 1)n^2$ $y_1 = 0$ $y_2 = 8$ $y_3 = 0$ $y_4 = 32$.
Ограничена снизу.

г) $y_n = \frac{1-n^2}{n^n}$. $y_1 = 0$ $y_2 = -\frac{3}{4}$ $y_3 = -\frac{8}{27}$.
Ограничена снизу.

627. а) $x_n = \frac{n}{n+1} = 1 - \frac{1}{n+1}$. $x_1 = \frac{1}{2}$ $x_2 = \frac{2}{3}$ $x_3 = \frac{3}{4}$.
Ограничена сверху.

б) $x_n = \frac{(-1)^n + 1}{n}$. $x_1 = 0$ $x_2 = 1$ $x_3 = 0$ $x_4 = \frac{1}{2}$.
Ограничена сверху.

в) $x_n = ((-1)^n + 1)^n$. $x_1 = 0$ $x_2 = 4$ $x_3 = 0$ $x_4 = 16$.
Не ограничена сверху.

г) $x_n = \frac{n^2-1}{n^2+2} = 1 - \frac{3}{n^2+2}$. $x_1 = 0$ $x_2 = \frac{3}{6}$ $x_3 = \frac{8}{11}$ $x_4 = \frac{15}{18}$.
Ограничена сверху.

628. а) cos1, cos2, cos3…cosn… Ограничена.

б) $\frac{\sin 1}{1}$, $-\frac{\sin 2}{2}$, $\frac{\sin 3}{3}$... $\frac{(-1)^{n-1}\sin n}{n}$... Ограничена.

в) $\operatorname{tg}\frac{\pi}{4}$; $\operatorname{tg}\frac{3\pi}{4}$; $\operatorname{tg}\frac{5\pi}{4}$... $\operatorname{tg}\frac{\pi}{4}(2n-1)$... Ограничена.

г) $\operatorname{ctg}\frac{\pi}{2}$; $\operatorname{ctg}\frac{\pi}{3}$; $\operatorname{ctg}\frac{\pi}{4}$... $\operatorname{ctg}\frac{\pi}{n+1}$... Не ограничена.

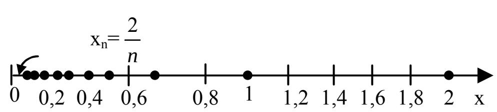
629. а) $x_n = \frac{2n-1}{n} = 2 - \frac{1}{n}$. $1 \le x_n < 2$. Ограничена.

б) $x_n = \frac{3}{5^n}$. $0 < x_n \le \frac{3}{5}$. Ограничена.

в) $x_n = \frac{n^2}{n^2+1} = 1 - \frac{1}{n^2+1}$. $\frac{1}{2} \le x_n < 1$. Ограничена.

г) $x_n = \sin\pi n$. $-1 \le x_n \le 1$. Ограничена.

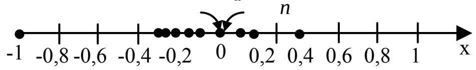
630. а)



$x_n \in [0;2]$

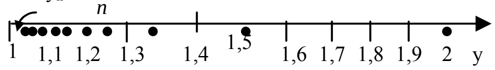
б)

$x_n = \frac{(-1)^n}{n}$



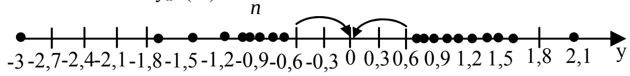
$x_n \in [-1; \frac{1}{2}]$

в) $y_n = \frac{n+1}{n}$



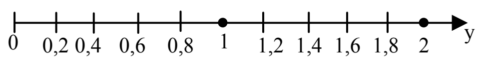
$x_n \in [1;2]$

г) $y_n = (-1)^n \frac{n+2}{n}$

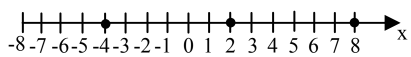


$x_n \in [-3;2]$

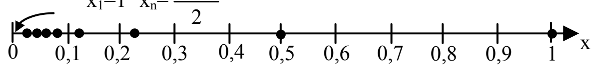
631. а) $x_1=1 \quad x_n=3-x_{n-1}$



б) $x_n=-2x_{n-1} \quad x_1=2$

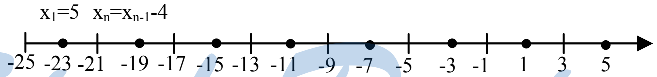


в) $x_1=1 \quad x_n=\frac{x_{n-1}}{2}$



$x_n \in [0;1]$

г) $x_1=5 \quad x_n=x_{n-1}-4$



632. а) $a_n = 7 - \frac{1}{n}$. $6 \le a_n \le 7$. б) $b_n = 2 + \frac{1}{2^n}$. $2 \le b_n \le 3$.

в) $p_n = \frac{2n+1}{2n-1}$. $1 \le p_n \le 2$. г) $q_n = \frac{2n-1}{2n+1}$. $0 \le q_n \le 1$.

§ 30. Предел числовой последовательности

633. а) a = 0, r = 0,1. (-0,1; 0,1); б) a = -3, r = 0,5. (-3,5; -2,5)
в) a = 2, r = 1. (1; 3); г) a = 0,2, r = 0,3. (-0,1; 0,5)
634. а) (1; 3), a = 2, r = 1. б) (-0,2; 0,2), a = 0, r = 0,2.
в) (2,1; 2,3), a = 2,2, r = 0,1. г) (-7; -5), a = -6, r = 1.
635. а) $x_1 = 1$, a = 2, r = 0,5. $1 \notin (1,5; 2,5)$;
б) $x_1 = 1,1$, a = 1, r = 0,2. $1,1 \in (0,8; 1,2)$;

в) $x_1 = -0{,}2$, $a = 0$, $r = 0{,}3$. $-0{,}2 \in (-0{,}3; 0{,}3)$;
г) $x_1 = 2{,}75$, $a = 2{,}5$, $r = 0{,}3$. $2{,}75 \in (2{,}2; 2{,}8)$.

636. а) $x_n = \dfrac{1}{n^2}$, $a = 0$, $r = 0{,}1$. $\dfrac{1}{n^2} \in (-0{,}1; 0{,}1)$. при $n_0 \ge 4$.

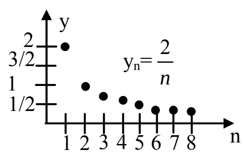
б) $x_n = \dfrac{1}{n^2}$, $a = 1$, $r = 0{,}1$. такого n_0 не существует.

в) $x_n = \dfrac{n}{n+1}$, $a = 0$, $r = 0{,}1$. такого n_0 не существует.

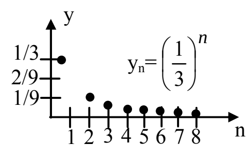
г) $x_n = \dfrac{n}{n+1}$, $a = 1$, $r = 0{,}1$. $x_n \in (0{,}9; 1{,}1)$ при $n_0 \ge 10$.

637.

а) $y_n = \dfrac{2}{n}$. $y = 0$.



б) $y_n = \left(\dfrac{1}{3}\right)^n$. $y = 0$



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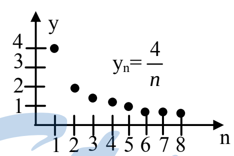
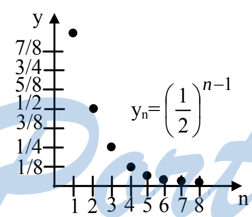
в) $y_n = \dfrac{4}{n}$. $y = 0$.

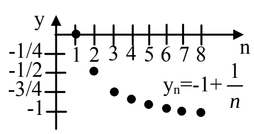


г) $y_n = \left(\dfrac{1}{2}\right)^{n-1}$. $y = 0$.

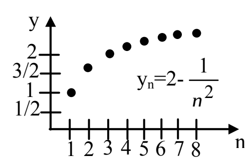


638.

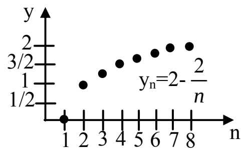
а) $y_n = -1 + \dfrac{1}{n}$. $y = -1$.

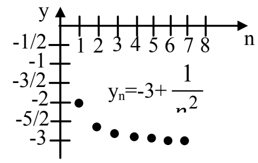


б) $y_n = 2 - \dfrac{1}{n^2}$. $y = 2$.



в) $y_n = 2-\frac{2}{n}$. $y = 2$. г) $y_n = -3+\frac{1}{n^2}$. $y = -3$.





639. а) $\lim\limits_{n\to\infty}\frac{5}{n^2}=0.$ б) $\lim\limits_{n\to\infty}\left(-\frac{17}{n^3}\right)=0.$

в) $\lim\limits_{n\to\infty}\left(-\frac{15}{n^2}\right)=0.$ г) $\lim\limits_{n\to\infty}\frac{3}{\sqrt{n}}=0.$

640. а) $\lim\limits_{n\to\infty}\left(\frac{7}{n}+\frac{8}{\sqrt{n}}+\frac{9}{n^3}\right)=0+0+0=0.$

б) $\lim\limits_{n\to\infty}\left(6-\frac{7}{n^2}-\frac{3}{n}-\frac{3}{\sqrt{n}}\right)=6-0-0-0=6.$

в) $\lim\limits_{n\to\infty}\left(\frac{3}{n}+\frac{7}{n^2}-\frac{5}{n^3}+\frac{13}{n^4}\right)=0+0-0+0=0.$

г) $\lim\limits_{n\to\infty}\left(\frac{1}{n}+\frac{3}{\sqrt{n}}-4+\frac{7}{n^2}\right)=0+0-4+0=-4.$

641. а) $\lim\limits_{n\to\infty}\frac{5n+3}{n+1}=\lim\limits_{n\to\infty}\frac{5+\frac{3}{n}}{1+\frac{1}{n}}=5.$ б) $\lim\limits_{n\to\infty}\frac{7n-5}{n+2}=\lim\limits_{n\to\infty}\frac{7-\frac{5}{n}}{1+\frac{2}{n}}=7.$

в) $\lim\limits_{n\to\infty}\frac{3n+1}{n+2}=\lim\limits_{n\to\infty}\frac{3+\frac{1}{n}}{1+\frac{2}{n}}=3.$ г) $\lim\limits_{n\to\infty}\frac{2n+1}{3n-1}=\lim\limits_{n\to\infty}\frac{2+\frac{1}{n}}{3-\frac{1}{n}}=\frac{2}{3}.$

642. а) $\lim\limits_{n\to\infty}\frac{5}{2^n}=0.$ б) $\lim\limits_{n\to\infty}\frac{1}{2}\cdot 5^{-n}=0.$

в) $\lim\limits_{n\to\infty}7\cdot 3^{-n}=0.$ г) $\lim\limits_{n\to\infty}\frac{4}{3^{n+1}}=0.$

643. а) $\lim\limits_{n\to\infty}\frac{2n^2-1}{n^2}=\lim\limits_{n\to\infty}\left(2-\frac{1}{n^2}\right)=2.$

б) $\lim\limits_{n\to\infty}\frac{\left(1+2n+n^2\right)}{n^2}=\lim\limits_{n\to\infty}\left(\frac{\frac{1}{n^2}+\frac{2}{n}+1}{1}\right)=1.$

в) $\lim\limits_{n\to\infty}\frac{3-n^2}{n^2}=\lim\limits_{n\to\infty}\left(\frac{3}{n^2}-1\right)=-1.$ г) $\lim\limits_{n\to\infty}\frac{3n-4-2n^2}{n^2}=\lim\limits_{n\to\infty}\left(\frac{\frac{3}{n}-\frac{4}{n^2}-2}{1}\right)=-2.$

644. а) $b_1 = 3$ $q=\frac{1}{3}.$ $S_n=\frac{3}{1-\frac{1}{3}}=4\frac{1}{2}.$

б) $b_1 = -5$ $q = -0{,}1.$ $S_n=\frac{-5}{1+0{,}1}=-\frac{50}{11}=-4\frac{6}{11}.$

в) $b_1 = -1$ $q = 0{,}2.$ $S_n=\frac{-1}{0{,}8}=-\frac{5}{4}=-1\frac{1}{4}.$

г) $b_1 = 2$ $q=-\frac{1}{3}.$ $S_n=\frac{2}{1+\frac{1}{3}}=\frac{6}{4}=\frac{3}{2}.$

645. а) 32, 16, 8, 4…

$b_1 = 32$ $q=\frac{1}{2}.$ $S_n=\frac{32}{1-\frac{1}{2}}=64.$

б) 24, -8, $\frac{8}{3}$, $-\frac{8}{9}$…

$b_1 = 24$ $q=-\frac{1}{3}.$ $S_n=\frac{24}{1+\frac{1}{3}}=36.$

в) 27, 9, 3, 1, $\frac{1}{3}$…

$b_1 = 27$ $q=\frac{1}{3}.$ $S_n=\frac{27}{\frac{2}{3}}=\frac{81}{2}=40{,}5.$

г) 18, -6, 2, $-\frac{1}{3}$…

$b_1 = 18$ $q=-\frac{1}{3}.$ $S_n=\frac{18}{1+\frac{1}{3}}=\frac{27}{2}.$

646. а) $2 + 1 + \frac{1}{2} + \frac{1}{4}$…

$b_1 = 2$ $q=\frac{1}{2}.$ $S_n=\frac{2}{1-\frac{1}{2}}=4.$

б) $49 + 7 + 1 + \frac{1}{7}...$

$b_1 = 49$ $\quad q = \frac{1}{7}.$ $\quad S_n = \frac{49}{1-\frac{1}{7}} = 57\frac{1}{6}.$

в) $\frac{3}{2} - 1 + \frac{2}{3} - \frac{4}{9}...$

$b_1 = \frac{3}{2}$ $\quad q = -\frac{2}{3}.$ $\quad S_n = \frac{\frac{3}{2}}{1+\frac{2}{3}} = \frac{3}{2}\cdot\frac{3}{5} = \frac{9}{10}.$

г) $125 + 25 + 5 + 1...$

$b_1 = 125$ $\quad q = \frac{1}{5}.$ $\quad S_n = \frac{125}{\frac{4}{5}} = \frac{625}{4} = 156{,}25.$

647. а) $-6 + \frac{2}{3} - \frac{2}{27} + \frac{2}{243}...$

$b_1 = -6$ $\quad q = -\frac{1}{9}.$ $\quad S_n = \frac{-6}{1+\frac{1}{9}} = -\frac{54}{10} = -\frac{27}{5}.$

б) $3 + \sqrt{3} + 1 + \frac{1}{\sqrt{3}}...$

$b_1 = 3$ $\quad q = \frac{1}{\sqrt{3}}.$ $\quad S_n = \frac{3}{1-\frac{1}{\sqrt{3}}} = \frac{3\sqrt{3}}{\sqrt{3}-1} = \frac{3\sqrt{3}(\sqrt{3}+1)}{2}.$

в) $49 - 14 + 4 - \frac{8}{7}...$

$b_1 = 49$ $\quad q = -\frac{2}{7}.$ $\quad S_n = \frac{49}{1+\frac{2}{7}} = \frac{343}{9} = 38\frac{1}{9}.$

г) $4 + 2\sqrt{2} + 2 + \sqrt{2}...$

$b_1 = 4$ $\quad q = \frac{1}{\sqrt{2}}.$ $\quad S_n = \frac{4}{1-\frac{1}{\sqrt{2}}} = \frac{4\sqrt{2}}{\sqrt{2}-1} = 4\sqrt{2}(\sqrt{2}+1).$

648. а) $b_1 = -2$ $\quad b_2 = 1.$ $\quad q = -\frac{1}{2}.$ $\quad S_n = \frac{-2}{1+\frac{1}{2}} = -\frac{4}{3} = -1\frac{1}{3}.$

б) $b_1 = 3$ $b_2 = \frac{1}{3}$. $q = \frac{1}{9}$. $S_n = \frac{3}{1-\frac{1}{9}} = \frac{27}{8} = 3\frac{3}{8}$.

в) $b_1 = 7$ $b_2 = -1$. $q = -\frac{1}{7}$. $S_n = \frac{7}{1+\frac{1}{7}} = \frac{49}{8} = 6\frac{1}{8}$.

г) $b_1 = -20$ $b_2 = 4$. $q = -\frac{1}{5}$. $S_n = \frac{-20}{1+\frac{1}{5}} = -\frac{100}{6} = -\frac{50}{3} = -16\frac{2}{3}$.

649. а) $S_n = 2$ $b_1 = 3$.

$S_n = \frac{b_1}{1-q}$ $1 - q = \frac{b_1}{S_n}$. $q = 1 - \frac{b_1}{S_n} = 1 - \frac{1}{2} = \frac{1}{2}$.

б) $S_n = -10$ $b_1 = -5$. $q = 1 - \frac{5}{10} = \frac{1}{2}$.

в) $S_n = -\frac{9}{4}$ $b_1 = -3$. $q = 1 - \frac{3}{9/4} = 1 - \frac{4}{3} = -\frac{1}{3}$.

г) $S_n = 1{,}5$ $b_1 = 2$. $q = 1 - \frac{2}{3/2} = 1 - \frac{4}{3} = -\frac{1}{3}$.

650.

а) $S = 10$, $q = \frac{1}{10}$, $S = \frac{b_1}{1-q}$, $b_1 = S(1-q) = 10 \cdot \frac{9}{10} = 9$.

б) $S = -3$, $q = -\frac{1}{3}$, $b_1 = -3 \cdot (1 + \frac{1}{3}) = -4$.

в) $S = 6$, $q = -\frac{1}{2}$, $b_1 = 6 \cdot \frac{3}{2} = 9$.

г) $S = -21, q = \frac{1}{7}$, $b_1 = -21 \cdot \left(1 - \frac{1}{7}\right) = -18$.

651.

а) $S = 15$, $q = -\frac{1}{3}$, $n = 3$. $b_1 = 15 \cdot \frac{4}{3} = 20$.

$b_1 = 20 \cdot \left(-\frac{1}{3}\right)^2 = \frac{20}{9} = 2\frac{2}{9}$.

б) $S = -20$, $b_1 = -16$, $n = 4$. $q = 1 - \frac{16}{20} = \frac{1}{5}$.

$b_4 = -16 \cdot \left(\frac{1}{5}\right)^3 = -\frac{16}{125}$.

в) S = 20, $b_1 = 22$, n = 4, $q = 1 - \frac{22}{20} = -\frac{1}{10}$.

$b_4 = 22 \cdot \left(-\frac{1}{10}\right)^3 = -\frac{11}{500}$.

г) S = 21, $q = \frac{2}{3}$, n = 3, $b_1 = 21 \cdot \left(1 - \frac{2}{3}\right) = 7$.

$b_3 = 7 \cdot \left(\frac{2}{3}\right)^2 = \frac{28}{9} = 3\frac{1}{9}$.

652. а) $x_n = \frac{1}{2n}$, a = 0 r = 0,1.

$\frac{1}{2n} < 0{,}1$, 2n > 10, n > 5 начиная с 6-ого.

б) $x_n = 3 + \frac{1}{n^2}$ a = 3 r = 0,2.

$3 + \frac{1}{n^2} < 3{,}2$, $n^2 > 5$, $n > \sqrt{5}$ начиная с 3-его.

в) $x_n = 1 + \frac{2}{n^2}$ a = 1 r = 0,01.

$1 + \frac{2}{n^2} < 1{,}01$, $\frac{2}{n^2} < 0{,}01$, $\frac{1}{n^2} < \frac{1}{200}$.

n > 14 начиная с 15-ого.

г) $x_n = -\frac{3}{n}$, a = 0, r = 0,1.

$-\frac{3}{n} > -\frac{1}{10}$. $\frac{3}{n} < \frac{1}{10}$. n > 30 начиная с 31-ого.

653. а) $x_n = \left(\frac{1}{3}\right)^n$, a = 0, $r = \frac{1}{27}$.

$\left(\frac{1}{3}\right)^n < \frac{1}{27}$. $\left(\frac{1}{3}\right)^n < \left(\frac{1}{3}\right)^3$. n > 3 начиная с 4-ого.

б) $x_n = (-1)^n \frac{1}{2^n}$ a = 0 $r = \frac{1}{64}$.

$\frac{1}{2^n} < \frac{1}{64}$. $\left(\frac{1}{2}\right)^n < \left(\frac{1}{2}\right)^6$.

n > 6 начиная с 7-ого.

в) $x_n = 2 + \left(\frac{1}{2}\right)^n$, $a = 2$, $r = \frac{1}{128}$.

$2 + \left(\frac{1}{2}\right)^n < 2\frac{1}{128}$. $\left(\frac{1}{2}\right)^n < \frac{1}{128} = \left(\frac{1}{2}\right)^7$.

$n > 7$ начиная с 8-ого.

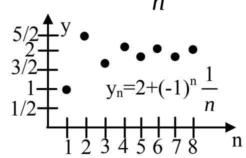
г) $x_n = 3 - \left(\frac{1}{3}\right)^n$, $a = 3$, $r = \frac{1}{81}$.

$3 - \left(\frac{1}{3}\right)^n > 2\frac{80}{81}$. $\left(\frac{1}{3}\right)^n < \frac{1}{81} = \left(\frac{1}{3}\right)^4$.

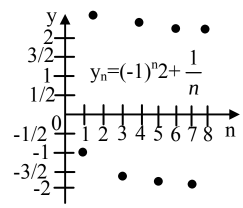
$n > 4$ начиная с 5-ого.

654.

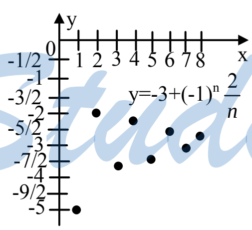
а) $y_n = 2 + (-1)^n \frac{1}{n}$. $y = 2$.



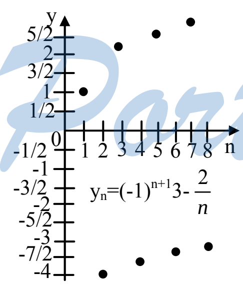
б) $y_n = (-1)^n 2 + \frac{1}{n}$.



в) $y_n = -3 + (-1)^n \frac{2}{n}$. $y = -3$.



г) $y_n = (-1)^{n+1} \cdot 3 - \frac{2}{n}$.



655. а) нет. $y_n = \left(-\frac{1}{n}\right)^n$. б) нет. $y_n = n$.

в) да. $y_n = \frac{1}{n}$. г) нет. $y_n = \left(-\frac{1}{n}\right)^n$.

656. а) $\lim\limits_{n\to\infty}\frac{(2n+1)(n-3)}{n^2}=\lim\limits_{n\to\infty}\frac{2n^2-5n-3}{n^2}=\lim\limits_{n\to\infty}\left(2-\frac{5}{n}-\frac{3}{n^2}\right)=2.$

б) $\lim\limits_{n\to\infty}\frac{(3n+1)(4n-1)}{(n-1)^2}=\lim\limits_{n\to\infty}\left(\frac{12n^2+n-1}{n^2-2n+1}\right)=\lim\limits_{n\to\infty}\frac{12+\frac{1}{n}-\frac{1}{n^2}}{1-\frac{2}{n}+\frac{1}{n^2}}=12.$

в) $\lim\limits_{n\to\infty}\frac{(3n-2)(2n+3)}{n^2}=\lim\limits_{n\to\infty}\frac{6n^2+5n-6}{n^2}=\lim\limits_{n\to\infty}\left(6+\frac{5}{n}-\frac{6}{n^2}\right)=6.$

г) $\lim\limits_{n\to\infty}\frac{(1-2n)(1+n)}{(n+2)^2}=\lim\limits_{n\to\infty}\frac{1-n-2n^2}{n^2+4n+4}=\lim\limits_{n\to\infty}\frac{\frac{1}{n^2}-\frac{1}{n}-2}{1+\frac{4}{n}+\frac{4}{n^2}}=-2.$

657. а) $\lim\limits_{n\to\infty}\frac{(2n+1)(3n-4)-6n^2+12n}{n+5}=\lim\limits_{n\to\infty}\frac{7n-4}{n+5}=7.$

б) $\lim\limits_{n\to\infty}\frac{n^2(2n+5)-2n^3+5n^2-13}{n(n+1)(n-7)+1-n}=\lim\limits_{n\to\infty}\frac{10n^2-13}{n^3-6n^2-8n+1}=$

$=\lim\limits_{n\to\infty}\frac{10/n-13/n^3}{1-6/n-8/n^2-1/n^3}=0.$

в) $\lim\limits_{n\to\infty}\frac{(1-n)(n^2+1)+n^3}{n^2+2n}=\lim\limits_{n\to\infty}\frac{n^2+1-n}{n^2+2n}=1.$

г) $\lim\limits_{n\to\infty}\frac{n(7-n^2)+n^3-3n-1}{(n+1)(n+2)+2n^2+1}=\lim\limits_{n\to\infty}\frac{7n-n^3+n^3-3n-1}{n^2+3n+2+2n^2+1}=$

$=\lim\limits_{n\to\infty}\frac{4n-1}{3n^2+3n+3}=\lim\limits_{n\to\infty}\frac{4/n-1/n^2}{3+3/n+3/n^2}=0.$

658. а) $b_n=\frac{25}{3^n}$ $b_1=\frac{25}{3}$ $b_2=\frac{25}{9}$ $q=\frac{1}{3}$. $S_n=\frac{25/3}{2/3}=12{,}5.$

б) $b_n=(-1)^n\frac{13}{2^{n-1}}$ $b_1=-13$ $b_2=\frac{13}{2}$ $q=-\frac{1}{2}$. $S_n=\frac{-13}{3/2}=-\frac{26}{3}=-8\frac{2}{3}.$

в) $b_n=\frac{45}{3^n}$ $b_1=\frac{45}{3}$ $b_2=\frac{45}{9}$ $q=\frac{1}{3}$. $S_n=\frac{45/3}{2/3}=\frac{45}{2}=22{,}5.$

г) $b_n=(-1)^n\frac{7}{6^{n-2}}$ $b_1=-42$ $b_2=7$ $q=-\frac{1}{6}$. $S_n=\frac{-42}{1+1/6}=-36.$

659. $\begin{cases}b_1+b_3=29\\ b_2+b_4=11{,}6\end{cases}$ $\begin{cases}b_1+b_1q^2=29\\ b_1q+b_1q^3=11{,}6\end{cases}$ $\begin{cases}b_1(1+q^2)=29\\ b_1q(1+q^2)=11{,}6\end{cases}$ $q=\frac{2}{5}.$

$$b_1 = \frac{29}{1+\frac{4}{25}} = 25. \quad S_n = \frac{25}{1-\frac{2}{5}} = \frac{25\cdot 5}{3} = 41\frac{2}{3}.$$

660. а) $S_n = 24$ $S_3 = 21$.

$$\begin{cases} \frac{b_1}{1-q} = 24 \\ \frac{b_1\left(q^3-1\right)}{q\text{-}1} = 21 \end{cases} \quad q^3 - 1 = -\frac{7}{8} \quad q^3 = \frac{1}{8} \quad q = \frac{1}{2} \quad \begin{cases} \frac{b_1}{1/2} = 24 \\ q = \frac{1}{2} \end{cases} \quad b_1 = 12$$

Ответ: $b_1 = 12$ $\quad q = \frac{1}{2}$.

б) $\begin{cases} \frac{b_1}{1-q} = 31,25 \\ \frac{b_1\left(q^3-1\right)}{q\text{-}1} = 31 \end{cases}$ $\quad -\left(q^3-1\right) = \frac{31\cdot 4}{125} = \frac{124}{125}$ $\quad q = \frac{1}{5}$

$$\frac{b_1}{1-1/5} = \frac{125}{4} \qquad \frac{5b_1}{4} = \frac{125}{4} \qquad b_1 = 25.$$

$$q = \frac{1}{5} \qquad b_1 = 25. \qquad b_7 = 25\cdot\left(\frac{1}{5}\right)^6 = \frac{1}{625}.$$

Ответ: $\frac{1}{625} = b_7$.

661. $\begin{cases} S_n = 18 \\ b_1^2 + b_1^2q^2 + b_1^2q^4 \ldots = 162 \end{cases}$

$$\begin{cases} \frac{b_1}{q\text{-}1} = -18 \\ \frac{b_1^2}{1\text{-}q^2} = 162 \end{cases} \quad \begin{cases} \frac{b_1}{1\text{-}q} = 18 \\ \frac{b_1^2}{1\text{-}q^2} = 162 \end{cases} \quad \begin{cases} b_1 = 18(1\text{-}q) \\ 324\left(1\text{-}2q + q^2\right) = 162 - 162q^2 \end{cases}$$

$2q^2 - 4q + 2 = 1 - q^2 \qquad 3q^2 - 4q + 1 = 0$

$q = \frac{2+1}{3} = 1 \qquad b_1 = 0$ не может быть.

$q = \frac{1}{3} \qquad b_1 = 12.$

662. а) $2 + 4 + 6 + \ldots + 20 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} \ldots =$

$$= 10\frac{20+2}{2} + \frac{1/2}{1-1/2} = 110 + 1 = 111.$$

б) $1 + 3 + 5 + \ldots + 99 + \frac{2}{5} + \frac{4}{25} + \frac{8}{125} + \ldots =$

$= \frac{1+99}{2} \cdot \frac{99-1+2}{2} + \frac{2/5}{1-2/5} = 2500 + \frac{2}{3} = 2500\frac{2}{3}$.

в) $21 + 24 + 27 + \ldots + 51 + \frac{1}{3} - \frac{1}{9} + \frac{1}{27} \ldots =$

$= \frac{21+51}{2} \cdot \frac{51-21+3}{3} + \frac{1/3}{1+1/3} = 12 \cdot 33 + \frac{1}{4} = 396\frac{1}{4}$.

г) $1 + 4 + 7 + \ldots + 100 + 0{,}1 + 0{,}01 + 0{,}001 + \ldots =$

$= \frac{1+100}{2} \cdot \frac{100-1+3}{3} + \frac{0{,}1}{1-0{,}1} = 1717 + \frac{1}{9} = 1717\frac{1}{9}$.

663. а) $\sin x + \sin^2 x + \sin^3 x + \ldots = \frac{\sin x}{1-\sin x}$.

б) $\cos x - \cos^2 x + \cos^3 x + \ldots = \frac{\cos x}{1+\cos x}$.

в) $\cos^2 x + \cos^4 x + \cos^6 x + \ldots = \frac{\cos^2 x}{1-\cos^2 x} = \mathrm{ctg}^2 x$.

г) $1 - \sin^3 x + \sin^6 x - \sin^9 x + \ldots = \frac{1}{1+\sin^3 x}$.

664. а) $x + x^2 + x^3 + \ldots = 4$.

$\frac{x}{1-x} = 4 \qquad x = 4 - 4x \qquad x = \frac{4}{5}$.

б) $2x - 4x^2 + 8x^3 - 16x^4 + \ldots = \frac{3}{8}$.

$\frac{2x}{1+2x} = \frac{3}{8} \qquad 2x = \frac{3}{8} + \frac{3}{4}x \qquad 10x = 3 \qquad x = \frac{3}{10}$.

665. а) $\frac{1}{x} + 1 + x + x^2 + \ldots = \frac{9}{2}$.

$\frac{\frac{1}{x}}{1-x} = \frac{9}{2}$. $2 = 9x - 9x^2$. $\quad 9x^2 - 9x + 2 = 0. \qquad x = \frac{1}{3}$ или $x = \frac{2}{3}$.

б) $2x + 1 + x^2 - x^3 + x^4 - \ldots = \frac{13}{6}. \qquad 2x + 1 + \frac{x^2}{1+x} = \frac{13}{6}$.

$2x+1+2x^2+x+x^2 - \frac{13}{6}x - \frac{13}{6} = 0$, $\;18x^2 + 18x - 13x - 7 = 0$, $\;18x^2 + 5x - 7 = 0$,

$x = \frac{-5+23}{36} = \frac{1}{2}. \qquad x = \frac{-5-23}{36} = -\frac{7}{9}$.

666. а) sinx + sin^{2}x + sin^{3}x + ...= 5.

$\frac{\sin x}{1-\sin x} = 5.$ 6sinx = 5. x = (-1)^{n}arcsin $\frac{5}{6}$ + πn.

б) cosx – cos^{2}x + cos^{3}x + ...= 2.

$\frac{\cos x}{1+\cos x} = 2.$ cosx = 2 + 2cosx. cosx = –2. решений нет.

в) 1 + sin^{2}x + sin^{4}x + ...= $\frac{4}{3}$

$\frac{1}{1-\sin^2 x} = \frac{4}{3}$. cos^{2}x = $\frac{3}{4}$.

$x = \pm\frac{\pi}{6} + 2\pi n.$ $x = \pm\frac{5\pi}{6} + 2\pi n.$

г) 7cos^{3}x + 7cos^{6}x + ...= 1.

$\frac{\cos^3 x}{1-\cos^3 x} = \frac{1}{7}.$ 7cos^{3}x = 1 – cos^{3}x.

cos^{3}x = $\frac{1}{8}$. cosx = $\frac{1}{2}$. $x = \pm\frac{\pi}{3} + 2\pi n.$

§ 31. Предел функции

667. а) при x → +∞ рис. 23, 25 учебника.
б) при x → –∞ рис. 24, 25 учебника.
в) при x → ∞ рис. 25 учебника.

668.

а) у = 3 – горизонт. асимптота на луче (–∞; 4] $\lim\limits_{x\to\infty} f(x) = 3,$

$\lim\limits_{x\to+\infty} f(x),$ $\lim\limits_{x\to-\infty} f(x)$ не существуют

б) у = –2 – горизонт. асимптота на луче [-6; +∞) $\lim\limits_{x\to\infty} f(x),$

$\lim\limits_{x\to-\infty} f(x)$ не существуют

$\lim\limits_{x\to+\infty} f(x) = -2.$

в) у = –5 – горизонт. асимптота на луче (–∞; 3] $\lim\limits_{x\to-\infty} f(x) = -5,$

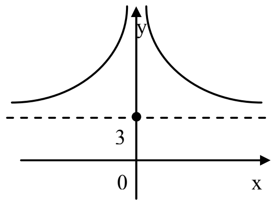
$\lim\limits_{x\to+\infty} f(x),$ $\lim\limits_{x\to\infty} f(x)$ не существуют

г) у = 5 – горизонт. асимптота на луче [4; +∞) $\lim\limits_{x\to\infty} f(x),$

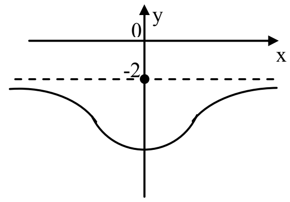
$\lim\limits_{x\to-\infty} f(x)$ не существуют

$\lim\limits_{x\to+\infty} f(x) = 5.$

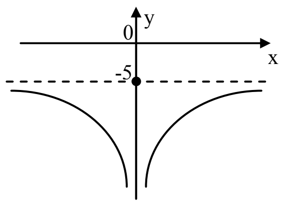
669. а) $\lim\limits_{x\to\infty} f(x)=3.$



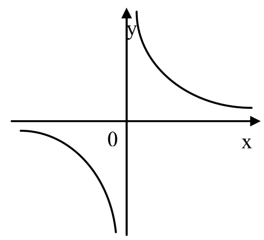
б) $\lim\limits_{x\to\infty} f(x)=-2.$



в) $\lim\limits_{x\to\infty} f(x)=-5.$



г) $\lim\limits_{x\to\infty} f(x)=0.$



670. $\lim\limits_{x\to+\infty} f(x)=-3.$

а) $\lim\limits_{x\to\infty} 6f(x)=-18.$ б) $\lim\limits_{x\to-\infty}\dfrac{f(x)}{3}=-1.$

в) $\lim\limits_{x\to-\infty} 8f(x)=-24.$ г) $\lim\limits_{x\to\infty} 0{,}4f(x)=-\dfrac{6}{5}.$

671. $\lim\limits_{x\to\infty} f(x)=2.$ $\lim\limits_{x\to\infty} g(x)=-3.$ $\lim\limits_{x\to\infty} h(x)=9.$

а) $\lim\limits_{x\to\infty}\left(f(x)+g(x)\right)=\lim\limits_{x\to\infty} f(x)+\lim\limits_{x\to\infty} g(x)=2-3=1.$

б) $\lim\limits_{x\to\infty}\left(f(x)-h(x)\right)=2-9=-7.$

в) $\lim\limits_{x\to\infty}\left(g(x)+h(x)\right)=-3+9=6.$

г) $\lim\limits_{x\to\infty}\left(f(x)+g(x)-h(x)\right)=2-3-9=-10.$

672. $\lim\limits_{x\to\infty} f(x)=-2.$ $\lim\limits_{x\to\infty} g(x)=7.$ $\lim\limits_{x\to\infty} h(x)=-2.$

а) $\lim\limits_{x\to\infty}\left(f(x)\cdot g(x)\right)=-14.$ б) $\lim\limits_{x\to\infty}\left(f(x)\right)^2=4.$

в) $\lim\limits_{x\to\infty}\left(g(x)\cdot(h(x))^2\right)=7\cdot 4=28.$ г) $\lim\limits_{x\to\infty}\left(f(x)\cdot g(x)\cdot h(x)\right)=7\cdot 4=28.$

673. $\lim\limits_{x\to\infty} f(x) = 6.$ $\lim\limits_{x\to\infty} g(x) = -10.$ $\lim\limits_{x\to\infty} L(x) = 25.$

а) $\lim\limits_{x\to\infty} \frac{f(x)}{g(x)} = \frac{6}{-10} = -\frac{3}{5}.$ б) $\lim\limits_{x\to\infty} \frac{(g(x))^2}{L(x)} = \frac{100}{25} = 4.$

в) $\lim\limits_{x\to\infty} \frac{f(x)g(x)}{h(x)} = \frac{6\cdot(-10)}{25} = -\frac{12}{5} = -2{,}4.$

г) $\lim\limits_{x\to\infty} \frac{2L(x)}{3g(x)} = \frac{50}{-30} = -\frac{5}{3}.$

674. а) $\lim\limits_{x\to\infty} \left(\frac{1}{x^2} + \frac{3}{x^3}\right) = 0.$ б) $\lim\limits_{x\to\infty} \left(\frac{7}{x^5} - \frac{2}{x^3}\right) = 0.$

в) $\lim\limits_{x\to\infty} \left(\frac{2}{x^2} + \frac{8}{x^3}\right) = 0.$ г) $\lim\limits_{x\to\infty} \left(\frac{9}{x^3} - \frac{5}{x^7}\right) = 0.$

675. а) $\lim\limits_{x\to\infty} \left(\frac{2}{x^9} + 1\right) = 1.$ б) $\lim\limits_{x\to\infty} \left(\frac{4}{x^3} - \frac{7}{x} - 21\right) = -21.$

в) $\lim\limits_{x\to\infty} \left(\frac{6}{x^5} + \frac{4}{x^2} + 9\right) = 9.$ г) $\lim\limits_{x\to\infty} \left(\frac{7}{x^2} - 7\right) = -7.$

676. а) $\lim\limits_{x\to\infty} \left(12 - \frac{1}{x^2}\right)\frac{16}{x^7} = \lim\limits_{x\to\infty} \left(\frac{12\cdot 16}{x^7} - \frac{16}{x^9}\right) = 0.$

б) $\lim\limits_{x\to\infty} \left(\frac{5}{x^3} + 1\right)\left(-\frac{8}{x^2} - 2\right) = 1\cdot(-2) = -2.$

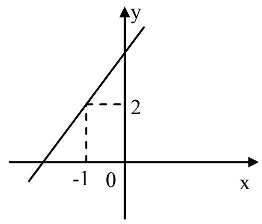
в) $\lim\limits_{x\to\infty} \left(4 + \frac{1}{x^3}\right)\frac{2}{x^5} = 0.$ г) $\lim\limits_{x\to\infty} \left(\frac{7}{x^6} - 2\right)\left(-\frac{6}{x^{10}} - 3\right) = 6.$

677. а) $\lim\limits_{x\to\infty} \frac{x+1}{x-2} = 1.$ б) $\lim\limits_{x\to\infty} \frac{3x-4}{2x+7} = \lim\limits_{x\to\infty} \frac{3 - 4/x}{2 + 7/x} = \frac{3}{2}.$

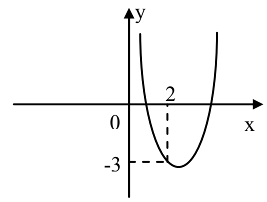
в) $\lim\limits_{x\to\infty} \frac{x-4}{x+3} = 1.$ г) $\lim\limits_{x\to\infty} \frac{7x+9}{6x-1} = \frac{7}{6} = 1\frac{1}{6}.$

678. при $x \to 3$
f(x) на рис. 28 имеет предел и он равен 4.
f(x) на рис. 26 имеет предел и он равен 3.
f(x) на рис. 27 имеет предел и он равен 4.
f(x) на рис. 33 имеет предел и он равен 0.

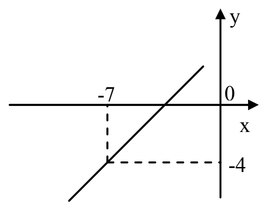
679. а) $\lim\limits_{x\to -1} g(x) = 2.$



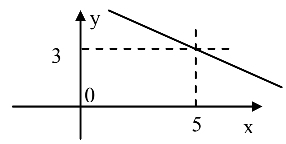
б) $\lim\limits_{x\to 2} g(x) = -3.$



в) $\lim\limits_{x\to -7} g(x) = -4.$



г) $\lim\limits_{x\to 5} g(x) = 3{,}5.$



680. а) $\lim\limits_{x\to -\infty} f(x) = 0.$ б) $\lim\limits_{x\to 0} f(x) = 4.$

в) $\lim\limits_{x\to 3} f(x) = 9.$ г) $\lim\limits_{x\to +\infty} f(x) = 4.$

681. а) $\lim\limits_{x\to 1}(x^2 - 3x + 5) = 1 - 3 + 5 = 3.$ б) $\lim\limits_{x\to \frac{1}{2}}\left(\frac{2x+3}{4x+2}\right) = \frac{1+3}{2+2} = 1.$

в) $\lim\limits_{x\to -1}(x^2 + 6x - 8) = 1 - 6 - 8 = -13.$ г) $\lim\limits_{x\to -\frac{1}{3}}\frac{7x-14}{21x+2} = \frac{-\frac{7}{3}-14}{-7+2} = \frac{49}{15}.$

682. а) $\lim\limits_{x\to 5}\sqrt{x+4} = \sqrt{9} = 3.$ б) $\lim\limits_{x\to 3{,}5}\sqrt{2x-6} = \sqrt{1} = 1.$

в) $\lim\limits_{x\to 6}\sqrt{x+3} = \sqrt{9} = 3.$ г) $\lim\limits_{x\to 4}\sqrt{3x-8} = \sqrt{4} = 2.$

683. а) $\lim\limits_{x\to 0}\frac{2x-1}{x^2+3x-4} = \frac{-1}{-4} = \frac{1}{4}.$ б) $\lim\limits_{x\to 1}\frac{3+4x}{2x^2+6x-3} = \frac{3+4}{2+6-3} = \frac{7}{5} = 1\frac{2}{5}.$

в) $\lim\limits_{x\to 0}\frac{4x+7}{x^2-5x+3} = \frac{7}{3} = 2\frac{1}{3}.$ г) $\lim\limits_{x\to -1}\frac{5\text{-}2x}{3x^2-2x+4} = \frac{5+2}{3+2+4} = \frac{7}{9}.$

684. а) $\lim\limits_{x\to 41} \dfrac{\sin \pi x}{x-1} = \dfrac{\sin 4\pi}{3} = 0.$ б) $\lim\limits_{x\to 2} \dfrac{\sin \frac{\pi}{x}}{2x+1} = \dfrac{\sin \frac{\pi}{2}}{5} = \dfrac{1}{5}.$

в) $\lim\limits_{x\to 0} \dfrac{\cos \pi x}{x+2} = \dfrac{\cos 0}{2} = \dfrac{1}{2}.$ г) $\lim\limits_{x\to 2} \dfrac{\cos \frac{2\pi}{x}}{3x-1} = \dfrac{\cos \pi}{5} = -\dfrac{1}{5}.$

685. а) $\lim\limits_{x\to 0} \dfrac{x^2}{x^2-x} = \lim\limits_{x\to 0} \dfrac{x}{x-1} = 0.$ б) $\lim\limits_{x\to -1} \dfrac{x+1}{x^2+x} = \lim\limits_{x\to -1} \dfrac{1}{x} = -1.$

в) $\lim\limits_{x\to 3} \dfrac{x^2-3x}{x-3} = \lim\limits_{x\to 3} x = 3.$ г) $\lim\limits_{x\to 5} \dfrac{x+5}{x^2+5x} = \lim\limits_{x\to 5} \dfrac{1}{x} = \dfrac{1}{5}.$

686. а) $\lim\limits_{x\to 1} \dfrac{x^2-1}{x-1} = \lim\limits_{x\to 1} (x-1) = 0.$ б) $\lim\limits_{x\to -2} \dfrac{x^2-4}{2+x} = \lim\limits_{x\to -2} (x-2) = -4.$

в) $\lim\limits_{x\to 5} \dfrac{x^2-25}{x-5} = \lim\limits_{x\to 5} (x+5) = 10.$ г) $\lim\limits_{x\to -3} \dfrac{3+x}{x^2-9} = \lim\limits_{x\to -3} \dfrac{1}{x-3} = -\dfrac{1}{6}.$

687. $y = 2x - 3$ $x_0 = 3.$ $y(x_0) = 3$

а) $x_1 = 3{,}2.$ $y(x_1) = 3{,}4.$ $\Delta y = 0{,}4.$

б) $x_1 = 2{,}9.$ $y(x_1) = 2{,}8.$ $\Delta y = -0{,}2.$

в) $x_1 = 3{,}5.$ $y(x_1) = 4.$ $\Delta y = 1.$

г) $x_1 = 2{,}5.$ $y(x_1) = 2.$ $\Delta y = -1.$

688. $y = x^2 + 2x$ $x_0 = -2.$ $y(x_0) = 0.$

а) $x_1 = -1{,}9.$ $y(x_1) = -0{,}19.$ $\Delta y = -0{,}19.$

б) $x_1 = -2{,}1.$ $y(x_1) = 0{,}21.$ $\Delta y = 0{,}21.$

в) $x_1 = -1{,}5.$ $y(x_1) = -0{,}75.$ $\Delta y = -0{,}75.$

г) $x_1 = -2{,}5.$ $y(x_1) = 1{,}25.$ $\Delta y = 1{,}25.$

689. $y = \sin x$ $x_0 = 0.$ $y(x_0) = 0.$

а) $x_1 = \dfrac{\pi}{6}.$ $y(x_1) = \dfrac{1}{2}.$ $\Delta y = \dfrac{1}{2}.$

б) $x_1 = -\dfrac{\pi}{6}.$ $y(x_1) = -\dfrac{1}{2}.$ $\Delta y = -\dfrac{1}{2}.$

в) $x_1 = \dfrac{\pi}{4}.$ $y(x_1) = \dfrac{\sqrt{2}}{2}.$ $\Delta y = \dfrac{\sqrt{2}}{2}.$

г) $x_1 = -\dfrac{\pi}{3}.$ $y(x_1) = -\dfrac{\sqrt{3}}{2}.$ $\Delta y = -\dfrac{\sqrt{3}}{2}.$

690. $y = 2\sin x\cos x = \sin 2x$ $x_0 = 0.$ $y(x_0) = 0.$

а) $x_1 = -\dfrac{\pi}{8}.$ $y(x_1) = -\dfrac{\sqrt{2}}{2}.$ $\Delta y = -\dfrac{\sqrt{2}}{2}.$

б) $x_1 = \dfrac{\pi}{12}.$ $y(x_1) = \dfrac{1}{2}.$ $\Delta y = \dfrac{1}{2}.$

в) $x_1 = \frac{\pi}{8}$. $y(x_1) = \frac{\sqrt{2}}{2}$. $\Delta y = \frac{\sqrt{2}}{2}$.

г) $x_1 = -\frac{\pi}{12}$. $y(x_1) = -\frac{1}{2}$. $\Delta y = -\frac{1}{2}$.

691. $y = \sqrt{x}$ $x_0 = 1$.

а) $\Delta x = 0,44$. $y(x + \Delta x) - y(x) = 1,2 - 1 = 0,2$.

б) $\Delta x = -0,19$. $y(x + \Delta x) - y(x) = 0,9 - 1 = -0,1$.

в) $\Delta x = 0,21$. $y(x + \Delta x) - y(x) = 1,1 - 1 = 0,1$.

г) $\Delta x = 0,1025$ $y(x + \Delta x) - y(x) = 1,05 - 1 = 0,05$.

692. а) $f(x_1) - f(x_0) = 1,4 - 2 = -0,6$.

б) $f(x_1) - f(x_0) = 1 - 6 = -5$.

693. $y = 4x^2 - x$.

а) $x = 0$ $\Delta x = 0,5$. $y(x + \Delta x) - y(x) = \frac{1}{2}$.

б) $x = 1$ $\Delta x = -0,1$. $y(x + \Delta x) - y(x) = 2,34 - 3 = -0,66$.

в) $x = 0$ $\Delta x = -\frac{1}{2}$. $y(x + \Delta x) - y(x) = \frac{3}{2}$.

г) $x = 1$ $\Delta x = 0,1$. $y(x + \Delta x) - y(x) = 3,74 - 3 = 0,74$.

694. а) $f(x) = 3x + 5$. $f(x + \Delta x) = 3x + 3\Delta x + 5$. $f(x + \Delta x) - f(x) = 3\Delta x$.

б) $f(x) = -x^2$. $f(x + \Delta x) = -x^2 - 2x\Delta x - (\Delta x)^2$.

$f(x + \Delta x) - f(x) = -2x\Delta x - (\Delta x)^2$.

в) $f(x) = 4 - 2x$. $f(x + \Delta x) = 4 - 2x - 2\Delta x$. $f(x + \Delta x) - f(x) = -2\Delta x$.

г) $f(x) = 2x^2$. $f(x + \Delta x) = 2x^2 + 4\Delta x x + 2\Delta x$. $f(x + \Delta x) - f(x) = 4\Delta x x + 2\Delta x$.

695. $y = x^2 - 4x + 1$. $x_0 = 2$. $y(x_0) = -3$.

а) $x = 2,1$. $y(x) = -2,29$. $y(x) - y(x_0) = 0,01$. $\frac{\Delta y}{\Delta x} = \frac{0,01}{0,1} = 0,1$.

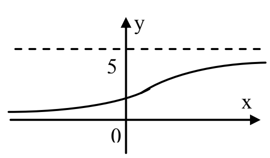
б) $x = 1,9$. $y(x) = -2,29$. $y(x) - y(x_0) = 0,1$. $\frac{\Delta y}{\Delta x} = -\frac{0,01}{0,1} = -0,1$.

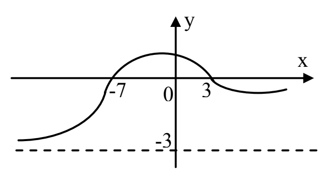
в) $x = 2,5$. $y(x) = -2,75$. $y(x) - y(x_0) = 0,25$. $\frac{\Delta y}{\Delta x} = \frac{0,25}{0,5} = 0,5$.

г) $x = 1,5$. $y(x) = -2,75$. $y(x) - y(x_0) = 0,25$. $\frac{\Delta y}{\Delta x} = -\frac{0,25}{0,5} = -0,5$.

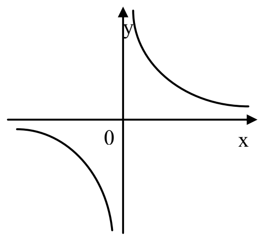
696.

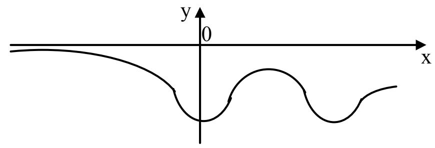
а) $\lim\limits_{x \to +\infty} f(x) = 5$. $f(x) > 0$. $x \in \Re$. б) $\lim\limits_{x \to -\infty} f(x) = -3$. $f(x) \geq 0$. $x \in [-7; 3]$.





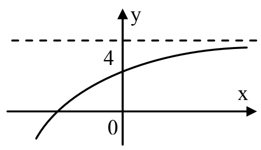
в) $\mathrm{Lim}_{x\to+\infty} f(x) = 0$. $f(x) > 0$. $x \in [0; +\infty)$. г) $\mathrm{Lim}_{x\to-\infty} f(x) = 0$. $f(x) < 0$. $x \in \Re$.

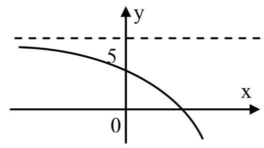




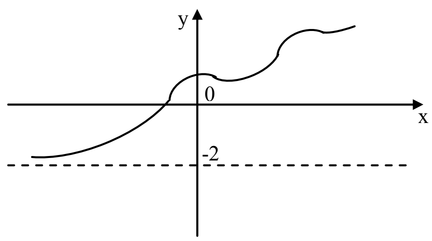
697.

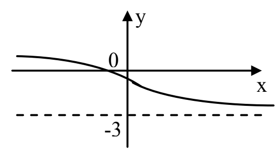
а) $\mathrm{Lim}_{x\to+\infty} h(x) = 4$ и функция возрастает. б) $\mathrm{Lim}_{x\to-\infty} h(x) = 5$ и функция убывает.



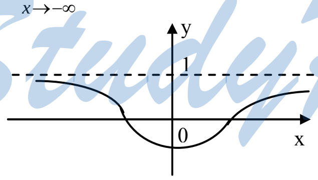


в) $\mathrm{Lim}_{x\to-\infty} h(x)=-2$ и функция возрастает. г) $\mathrm{Lim}_{x\to+\infty} h(x)=-3$ и функция убывает.

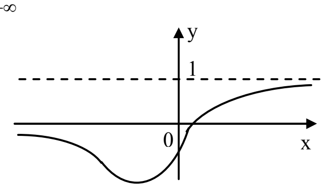




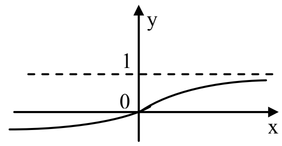
698. а) $\mathrm{Lim}_{x\to-\infty} h(x) = 1$ и функция ограничена сверху.



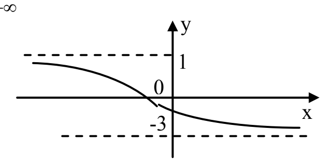
б) $\mathrm{Lim}_{x\to+\infty} h(x) = 1$ и функция ограничена снизу.



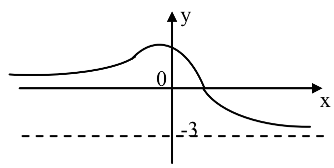
в) $\lim\limits_{x \to +\infty} h(x) = 1$ и функция ограничена сверху.



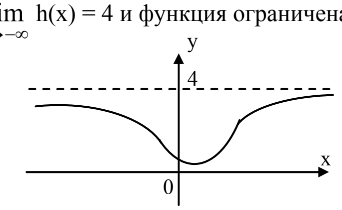
г) $\lim\limits_{x \to -\infty} h(x) = 1$ и функция ограничена снизу.



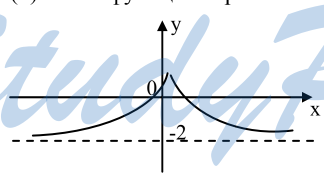
699. а) $\lim\limits_{x \to +\infty} h(x) = -3$ и функция ограничена.



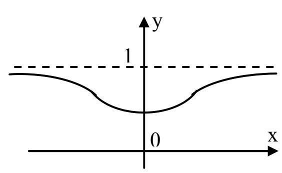
б) $\lim\limits_{x \to -\infty} h(x) = 4$ и функция ограничена.



в) $\lim\limits_{x \to \infty} h(x) = -2$ и функция ограничена.



г) $\lim\limits_{x \to \infty} h(x) = 1$ и функция ограничена.



700. а) $\lim\limits_{x\to\infty} \dfrac{4x^2+9}{x^2+2} = \lim\limits_{x\to\infty} \dfrac{4+\frac{9}{x^2}}{1+\frac{2}{x^2}} = 4.$

б) $\lim\limits_{x\to\infty} \dfrac{12x^2+5x+2}{6x^2+5x-3} = \lim\limits_{x\to\infty} \dfrac{12+\frac{5}{x}+\frac{2}{x^2}}{6+\frac{5}{x}-\frac{3}{x^2}} = 2.$

в) $\lim\limits_{x\to\infty} \dfrac{3x^2-8}{x^2-1} = \lim\limits_{x\to\infty} \dfrac{3-\frac{8}{x^2}}{1-\frac{1}{x^2}} = 3.$

г) $\lim\limits_{x\to\infty} \dfrac{10x^2+4x-3}{5x^2+2x+1} = \lim\limits_{x\to\infty} \dfrac{10+\frac{4}{x}-\frac{3}{x^2}}{5+\frac{2}{x}+\frac{1}{x^2}} = 2.$

701. а) $\lim\limits_{x\to\infty} \dfrac{3x-1}{x^2+7x+5} = \lim\limits_{x\to\infty} \dfrac{\frac{3}{x}-\frac{1}{x^2}}{1+\frac{7}{x}+\frac{5}{x^2}} = 0.$

б) $\lim\limits_{x\to\infty} \dfrac{5-5x}{2x^2-9x} = \lim\limits_{x\to\infty} \dfrac{\frac{5}{x^2}-\frac{5}{x}}{2-\frac{9}{x}} = 0.$

в) $\lim\limits_{x\to\infty} \dfrac{-2x-1}{3x^2-4x+1} = \lim\limits_{x\to\infty} \dfrac{-\frac{2}{x}-\frac{1}{x^2}}{3-\frac{4}{x}+\frac{1}{x^2}} = 0.$

г) $\lim\limits_{x\to\infty} \dfrac{4x+3}{12x^2-6x} = \lim\limits_{x\to\infty} \dfrac{\frac{4}{x}+\frac{3}{x^2}}{12-\frac{6}{x}} = 0.$

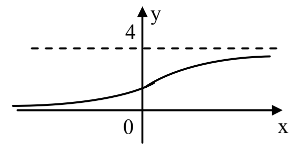
702. а) $\lim\limits_{x\to\infty} \dfrac{4x-x^2+1}{5x^2-2x} = \lim\limits_{x\to\infty} \dfrac{\frac{4}{x}-1+\frac{1}{x^2}}{5-\frac{2}{x}} = -\dfrac{1}{5}.$

б) $\lim\limits_{x\to\infty} \dfrac{x^3-8}{x^3+18} = \lim\limits_{x\to\infty} \dfrac{1-\frac{8}{x^3}}{1+\frac{18}{x^3}} = 1.$

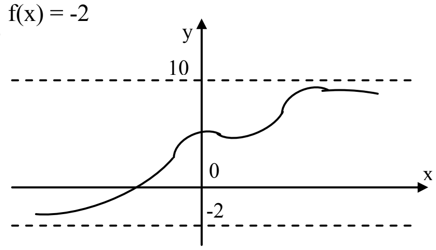
в) $\lim\limits_{x\to\infty} \dfrac{3x-2x^2+4}{3x^2+2x} = \lim\limits_{x\to\infty} \dfrac{\frac{3}{x}-2+\frac{4}{x^2}}{3+\frac{2}{x}} = -\dfrac{2}{3}.$

г) $\lim\limits_{x\to\infty} \dfrac{x^3-3x^2}{x^4+2x+1} = \lim\limits_{x\to\infty} \dfrac{\frac{1}{x}-\frac{3}{x^2}}{1+\frac{2}{x^3}+\frac{1}{x^4}} = 0.$

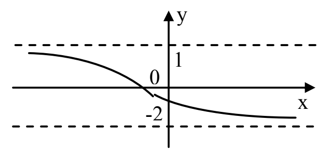
703. а) $\lim\limits_{x\to+\infty} f(x) = 4$ и $\lim\limits_{x\to-\infty} (x) = 0$



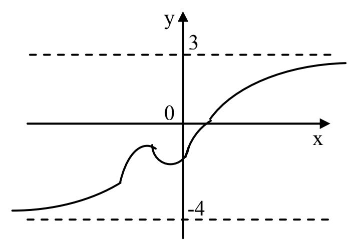
б) $\lim\limits_{x\to+\infty} f(x) = 10$ и $\lim\limits_{x\to-\infty} f(x) = -2$



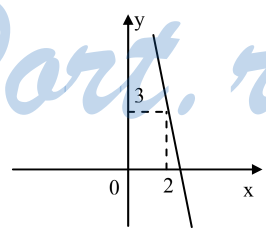
в) $\lim\limits_{x\to+\infty} f(x) = -2$ и $\lim\limits_{x\to-\infty} f(x) = 1$



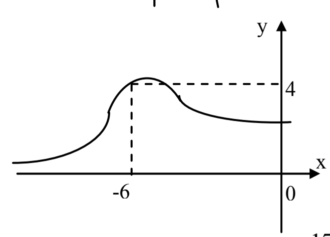
г) $\lim\limits_{x\to+\infty} f(x) = 3$ и $\lim\limits_{x\to-\infty} f(x) = -4$



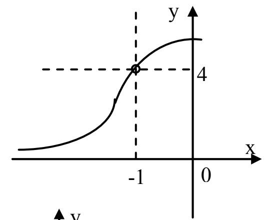
704. а) $\lim\limits_{x\to 2} f(x) = 3$ и $f(2) = -3$



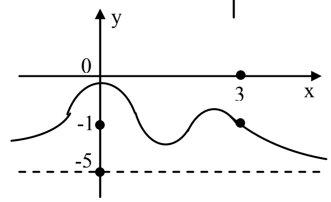
б) $\lim\limits_{x\to -6} f(x) = 4$ и $\lim\limits_{x\to-\infty} f(x) = 0$



в) $\lim\limits_{x\to -1} f(x) = 4$ и f(-1) не существует



г) $\lim\limits_{x\to 3} f(x) = -1$ и $\lim\limits_{x\to +\infty} f(x) = -5$



705. а) $\lim\limits_{x\to 1}\dfrac{x^2+2x-3}{x-1} = \lim\limits_{x\to 1}\dfrac{(x-1)(x+3)}{x-1} = 1+3 = 4.$

б) $\lim\limits_{x\to -2}\dfrac{x+2}{2x^2-x-6} = \lim\limits_{x\to -2}\dfrac{x+2}{(x+2)(2x-3)} = -\dfrac{1}{7}.$

в) $\lim\limits_{x\to -1}\dfrac{x+1}{x^2-2x-3} == \lim\limits_{x\to -1}\dfrac{x+1}{(x+1)(x-3)} = \dfrac{1}{-1-3} = -\dfrac{1}{4}.$

г) $\lim\limits_{x\to 9}\dfrac{x^2-11x+18}{x-9} = \lim\limits_{x\to 9}\dfrac{(x-9)(x-2)}{x-9} = 7.$

706. а) $\lim\limits_{x\to -2}\dfrac{x+2}{x^3+8} = \lim\limits_{x\to -2}\dfrac{1}{x^2-2x+4} = \dfrac{1}{4+4+4} = \dfrac{1}{12}.$

б) $\lim\limits_{x\to -1}\dfrac{1+x^3}{1-x^2} = \lim\limits_{x\to -1}\dfrac{1-x+x^2}{1-x} = \dfrac{3}{2}.$

в) $\lim\limits_{x\to 3}\dfrac{x-3}{x^3-27} = \lim\limits_{x\to 3}\dfrac{x-3}{(x-3)(x^2+3x+9)} = \dfrac{1}{9+9+9} = \dfrac{1}{27}.$

г) $\lim\limits_{x\to 4}\dfrac{16-x^2}{64-x^3} = \lim\limits_{x\to 4}\dfrac{(4-x)(4+x)}{(4-x)(16+4x+x^2)} = \dfrac{8}{16+16+16} = \dfrac{1}{6}.$

707. а) $\lim\limits_{x\to 0}\dfrac{\sin x}{\mathrm{tg}x} = \lim\limits_{x\to 0}\cos x = 1.$

б) $\lim\limits_{x\to \frac{\pi}{2}}\dfrac{\sin 3x-\sin x}{\cos 3x+\cos x} = \lim\limits_{x\to \frac{\pi}{2}}\dfrac{\sin x\cos 2x}{\cos 2x\cos x} = \lim\limits_{x\to \frac{\pi}{2}}\mathrm{tg}x.$ предела не существует.

в) $\lim\limits_{x\to \frac{\pi}{2}}\dfrac{\cos x}{\mathrm{ctg}x} = \lim\limits_{x\to \frac{\pi}{2}}\sin x = 1.$ г) $\lim\limits_{x\to 0}\dfrac{\cos 5x-\cos 3x}{\sin 5x+\sin 3x} = \lim\limits_{x\to 0}-\dfrac{\sin 4x\sin x}{\sin 4x\cos x} = 0.$

708. а) $f(x) = Kx + m$, $f(x + \Delta x) = Kx + m + K\Delta x$,
$f(x + \Delta x) - f(x) = K\Delta x$;
б) $f(x) = ax^2$, $f(x + \Delta x) = ax^2 + 2a\Delta x\cdot x + a\cdot\Delta x^2$,
$f(x + \Delta x) - f(x) = 2a\Delta x\cdot x + a\cdot\Delta x^2$;
в) $f(x) = \frac{1}{x}$, $f(x + \Delta x) = \frac{1}{x + \Delta x}$,

$$f(x + \Delta x) - f(x) = \frac{x - x - \Delta x}{(x + \Delta x)x} = -\frac{\Delta x}{x^2 + x\Delta x};$$

г) $f(x) = \sqrt{x}$, $f(x + \Delta x) = \sqrt{x + \Delta x}$,
$f(x + \Delta x) - f(x) = \sqrt{x + \Delta x} - \sqrt{x}$.

709. а) $f(x) = Kx + m$. $\Delta f = K\Delta x$. $\frac{\Delta f}{\Delta x} = K$.

б) $f(x) = ax^2$. $\Delta f = \Delta x(2ax + a\Delta x)$. $\frac{\Delta f}{\Delta x} = 2ax + a\Delta x$.

в) $f(x) = \frac{1}{x}$. $\Delta f = -\frac{\Delta x}{x^2 + x\Delta x}$. $\frac{\Delta f}{\Delta x} = -\frac{1}{x^2 + x\Delta x}$.

г) $f(x) = \sqrt{x}$. $\Delta f = \frac{\Delta x}{\sqrt{x + \Delta x} + \sqrt{x}}$. $\frac{\Delta f}{\Delta x} = \frac{1}{\sqrt{x + \Delta x} + \sqrt{x}}$.

710. а) $f(x) = Kx + m$. $\frac{\Delta f}{\Delta x} = K$. $\lim\limits_{\Delta x\to 0} \frac{\Delta f}{\Delta x} = K$.

б) $f(x) = ax^2$. $\lim\limits_{\Delta x\to 0} \frac{\Delta f}{\Delta x} = \lim\limits_{\Delta x\to 0} (2ax + a\Delta x) = 2ax$.

в) $f(x) = \frac{1}{x}$. $\frac{\Delta f}{\Delta x} = -\frac{1}{x^2 + x\Delta x}$. $\lim\limits_{\Delta x\to 0} \frac{\Delta f}{\Delta x} = -\frac{1}{x^2}$.

г) $f(x) = \sqrt{x}$. $\frac{\Delta f}{\Delta x} = \frac{1}{\sqrt{x + \Delta x} + \sqrt{x}}$. $\lim\limits_{\Delta x\to 0} \frac{1}{\sqrt{x + \Delta x} + \sqrt{x}} = \frac{1}{2\sqrt{x}}$.

711. а) $$\lim_{x\to 3} \frac{\sqrt{x+6} - 3}{x^2 - 3x} = \lim_{x\to 3} \frac{x + 6 - 9}{x(x-3)\left(\sqrt{x+6} + 3\right)} =$$

$$= \lim_{x\to 3} \frac{1}{x\left(\sqrt{x+6} + 3\right)} = \frac{1}{3(3+3)} = \frac{1}{18}.$$

б) $$\lim_{x\to\infty} \left(\sqrt{2x+3} - \sqrt{2x-7}\right) = \lim_{x\to\infty} \frac{10}{\sqrt{2x+3} + \sqrt{2x-7}} = 0$$

712. а) $$\lim_{x\to 0} \frac{1 - \cos x}{x^2} = \lim_{x\to 0} \frac{1 - \cos^2 x}{x^2(1 - \cos x)} =$$

$$= \lim_{x\to 0} \frac{\sin^2 x}{x^2(1 - \cos x)} = \lim_{x\to 0} \frac{1}{1 - \cos x} = \frac{1}{2}.$$

б) $\lim\limits_{x\to 0}\dfrac{\sin 7x-\sin 3x}{\sin 8x-\sin 2x}=\lim\limits_{x\to 0}\dfrac{\sin 2x\cos 5x}{\sin 3x\cos 5x}=$

$=\lim\limits_{x\to 0}\dfrac{\sin 2x}{\sin 3x}=\lim\limits_{x\to 0}\dfrac{2\sin x\cos x}{3\sin x-4\sin^3 x}=\lim\limits_{x\to 0}\dfrac{2\cos x}{3-4\sin^2 x}=\dfrac{2}{3}$.

§ 32. Определение производной

713. $s(t) = 2t + 1$. $t_1 = 2$.
а) $t_2 = 3$.

$S(3) – S(2) = 7 – 5 = 2 = \Delta S$. $\Delta t = 1$. $V_{ср} = \dfrac{\Delta S}{\Delta t} = 2$ (м/с).

б) $t_2 = 2,5$.

$S(2,5) – S(2) = 6 – 5 = 1$. $\Delta t = 0,5$. $\dfrac{\Delta S}{\Delta t} = V_{ср} = \dfrac{1}{0,5} = 2$ (м/с).

в) $t_2 = 2,1$.

$S(2,1) – S(2) = 0,2$. $\Delta t = 0,1$. $\dfrac{\Delta S}{\Delta t} = \dfrac{0,2}{0,1} = 2$ (м/с).

г) $t_2 = 2,05$.

$S(2,05) – S(2) = 0,1$. $\Delta t = 0,05$. $\dfrac{\Delta S}{\Delta t} = \dfrac{0,1}{0,05} = 2$ (м/с).

V мгновенная в $t = 2$ равна 2 (м/с).
714. $s(t) = t^2$. $t_1 = 0$.

а) $t_2 = 0,1$. $S(0,1) – S(0) = 0,01$. $\dfrac{\Delta S}{\Delta t} = 0,1$ (м/с).

б) $t_2 = 0,01$. $S(0,01) – S(0) = 0,0001$. $\dfrac{\Delta S}{\Delta t} = 0,01$ (м/с).

в) $t_2 = 0,2$. $S(0,2) – S(0) = 0,04$. $\dfrac{\Delta S}{\Delta t} = 0,2$ (м/с).

г) $t_2 = 0,02$. $S(0,02) – S(0) = 0,0004$. $\dfrac{\Delta S}{\Delta t} = 0,02$ (м/с).

мгновенная скорость в момент $t = 1$ равна 2 (м/с).
715. а) $S(t) = 4t + 1$. $V_{мгнов} = 4$ (м/с).
б) $S(t) = 6t – 2$. $V_{мгнов} = 6$ (м/с).
в) $S(t) = 3t + 2$. $V_{мгнов} = 3$ (м/с).
г) $S(t) = 5t – 1$. $V_{мгнов} = 5$ (м/с).
716. а) $y_1 = \sqrt{3}x + K$. $y_1'(x_1) = \sqrt{3} \Rightarrow f'(x_1) = \sqrt{3}$.
$y_2 = x + K_2$, $y_2'(x_2) = 1 \Rightarrow f'(x_2) = 1$.

б) $f'(x_1) = 0$, $y = \dfrac{\sqrt{3}}{3}x + K$. $y'(x_2) = \dfrac{\sqrt{3}}{3}$ $\Rightarrow f'(x_2) = \dfrac{\sqrt{3}}{3}$.

в) $f'(x_1) = 0$. $f'(x_2) = -\frac{\sqrt{3}}{3}$.

г) $f'(x_1) = 0$. $f'(x_2) = 0$.

717. а) $y = 9{,}5x - 3$. $V = 9{,}5$ м/с.

б) $y = -16x + 3$. $V = -16$ м/с.

в) $y = 6{,}7x - 13$. $V = 6{,}7$ м/с.

г) $y = -9x + 4$. $V = -9$ м/с.

718. а) $f(x) = x^2$. $x_0 = 2$. $f'(x_0) = 2x_0 = 4$.

б) $f(x) = x^2$. $x_0 = -1$. $f'(x_0) = -2$.

в) $f(x) = x^2$. $x_0 = -2$. $f'(x_0) = -4$.

г) $f(x) = x^2$. $x_0 = 9$. $f'(x_0) = 18$.

719. а) $f(x) = \frac{1}{x}$. $x_0 = 2$. $f'(x_0) = -\frac{1}{4}$.

б) $f(x) = \frac{1}{x}$. $x_0 = -1$. $f'(x_0) = 1$.

в) $f(x) = \frac{1}{x}$. $x_0 = 5$. $f'(x_0) = -\frac{1}{25}$.

г) $f(x) = \frac{1}{x}$. $x_0 = -0{,}5$. $f'(x_0) = -4$.

720. $S(t) = t^2$. $S'(t) = 2t$. $S''(t) = 2$.

а) $t = 1$. $V = 2$ (м/с). $a = 2$ (м/с^2).

б) $t = 2{,}1$. $V = 4{,}2$ (м/с). $a = 2$ (м/с^2).

в) $t = 2$. $V = 4$ (м/с). $a = 2$ (м/с^2).

г) $t = 3{,}5$. $V = 7$ (м/с). $a = 2$ (м/с^2).

721. $y = x^2$. $y' = 2x$.

а) $y' > 0$, при $x > 0$.

б) $y' < 0$, при $x < 0$.

722. $S(t) = 2t^2 + t$. $t_1 = 0$.

а) $t_2 = 0{,}6$. $S(t_2) - S(t_1) = 1{,}32$. $\frac{\Delta S}{\Delta t} = \frac{1{,}33}{0{,}6} = 2{,}2$. (м/с)

б) $t_2 = 0{,}2$. $S(t_2) - S(t_1) = 0{,}28$. $\frac{\Delta S}{\Delta t} = 1{,}4$. (м/с)

в) $t_2 = 0{,}5$. $S(t_2) - S(t_1) = 1$. $\frac{\Delta S}{\Delta t} = 2$. (м/с)

г) $t_2 = 0{,}1$. $S(t_2) - S(t_1) = 0{,}12$. $\frac{\Delta S}{\Delta t} = 1{,}2$. (м/с)

723. а) $S(t) = t^2 + 3$. $S'(t) = 2t$. $V_{мгнов} = 2t$. (м/с)

б) $S(t) = t^2 - t$. $S'(t) = 2t - 1$. $V_{мгнов} = 2t - 1$. (м/с)

в) $S(t) = t^2+4$. $S'(t) = 2t$. $V_{мгнов} = 2t$. (м/с)

г) $S(t) = t^2 - 2t$. $S'(t) = 2t - 2$. $V_{мгнов} = 2t - 2$. (м/с)

724. а) $f'(-7) < f'(-2)$. б) $f'(-4) < f'(2)$.
в) $f'(-9) < f'(0)$. г) $f'(-1) > f'(5)$.

725. а) $f'(x_1) > 0$, $f'(x_2) > 0$. $x_1 = 0$ $x_2 = 1$.
б) $f'(x_1) < 0$, $f'(x_2) > 0$. $x_1 = -6$ $x_2 = 0$.
в) $f'(x_1) < 0$, $f'(x_2) < 0$. $x_1 = -5$ $x_2 = -4$.
г) $f'(x_1) > 0$, $f'(x_2) < 0$. $x_1 = 2$ $x_2 = 4$.

726. а) $\varphi'(x) > 0$ $x = -7$, -6, -5.
б) $\varphi'(x) < 0$ и $x > 0$ $x = 4, 5$.
в) $\varphi'(x) < 0$ $x = -3$, -2.
г) $\varphi'(x) > 0$ и $x < 0$ $x = -5, -6$.

727. $S(t) = t^2 + 4t$. $S'(t) = 2t + 4$. $S''(t) = 2$.
а) $t = 1$. $V = 2 + 4 = 6$. (м/с) $a = 2$. (м/с^2)
б) $t = 2{,}1$. $V = 4{,}2 + 4 = 8{,}2$. (м/с) $a = 2$. (м/с^2)
в) $t = 2$. $V = 8$. (м/с) $a = 2$. (м/с^2)
г) $t = 3{,}5$. $V = 7 + 4 = 11$. (м/с) $a = 2$. (м/с^2)

§ 33. Вычисление производных

728. а) $y = 7x + 4$. $y' = 7$. б) $y = x^2$. $y' = 2x$.
в) $y = -6x + 1$. $y' = -6$. г) $y = \frac{1}{x}$. $y' = -\frac{1}{x^2}$.

729. а) $y = \sin x$. $y' = \cos x$. б) $y = \sqrt{x}$. $y' = \frac{1}{2\sqrt{x}}$.
в) $y = \cos x$. $y' = -\sin x$ г) $y = 10^{10}$. $y' = 0$.

730. а) $g(x) = \sqrt{x}$. $x_0 = 4$. $g'(x) = \frac{1}{2\sqrt{x}}$. $g'(x_0) = \frac{1}{4}$.
б) $g(x) = x^2$. $x_0 = -7$. $g'(x) = 2x$. $g'(x_0) = -14$.
в) $g(x) = -3x - 1$. $x_0 = -3$. $g'(x) = -3$. $g'(x_0) = -3$.
г) $g(x) = \frac{1}{x}$. $x_0 = 0{,}5$. $g'(x) = -\frac{1}{x^2}$. $g'(x_0) = -4$.

731. а) $g(x) = \sin x$. $x_0 = -\frac{\pi}{2}$. $g'(x) = \cos x$. $g'(x_0) = 0$.
б) $g(x) = \cos x$. $x_0 = \frac{\pi}{6}$. $g'(x) = -\sin x$. $g'(x_0) = -\frac{1}{2}$.
в) $g(x) = \cos x$. $x_0 = -3\pi$. $g'(x) = -\sin x$. $g'(x_0) = 0$.
г) $g(x) = \sin x$. $x_0 = 0$. $g'(x) = \cos x$. $g'(x_0) = 1$.

732. а) $h(x) = 7x - 19$. $x_0 = -2$. $h'(x_0) = 7$.
б) $h(x) = \sqrt{x}$. $x_0 = 16$. $h'(x) = \frac{1}{2\sqrt{x}}$. $h'(x_0) = \frac{1}{8}$.
в) $h(x) = -6x + 4$. $x_0 = 0{,}5$. $h'(x_0) = -6$.
г) $h(x) = \sqrt{x}$. $x_0 = 9$. $h'(x) = \frac{1}{2\sqrt{x}}$. $h'(x_0) = \frac{1}{6}$.

733. а) $h(x) = \frac{1}{x}$. $x_0 = -2$. $h'(x) = -\frac{1}{x^2}$. $h'(x_0) = -\frac{1}{4}$.

б) $h(x) = \sin x$. $x_0 = \frac{\pi}{2}$. $h'(x) = \cos x$. $h'(x_0) = 0$.

в) $h(x) = x^2$. $x_0 = -0{,}1$. $h'(x) = 2x$. $h'(x_0) = -\frac{1}{5}$.

г) $h(x) = \cos x$. $x_0 = \pi$. $h'(x) = -\sin x$. $h'(x_0) = 0$.

734. а) $f(x) = x^2$. $x_0 = -4$. $f'(x) = 2x$. $f'(x_0) = -8$.

б) $f(x) = \frac{1}{x}$. $x_0 = -\frac{1}{3}$. $f'(x) = -\frac{1}{x^2}$. $f'(x_0) = -9$.

в) $f(x) = \frac{1}{x}$. $x_0 = \frac{1}{2}$. $f'(x) = -\frac{1}{x^2}$. $f'(x_0) = -4$.

г) $f(x) = x^2$. $x_0 = 2$. $f'(x) = 2x$. $f'(x_0) = 4$.

735. а) $f(x) = \sin x$. $x_0 = \frac{\pi}{3}$. $f'(x) = \cos x$. $f'(x_0) = \frac{1}{2}$.

б) $f(x) = \cos x$. $x_0 = -\frac{\pi}{4}$. $f'(x) = -\sin x$. $f'(x_0) = \frac{\sqrt{2}}{2}$.

в) $f(x) = \cos x$. $x_0 = \frac{\pi}{3}$. $f'(x) = -\sin x$. $f'(x_0) = -\frac{\sqrt{3}}{2}$.

г) $f(x) = \sin x$. $x_0 = -\frac{\pi}{6}$. $f'(x) = \cos x$. $f'(x_0) = \frac{\sqrt{3}}{2}$.

736. а) $f'(x) = 2x$. $f(x) = x^2 + c$. б) $f'(x) = \cos x$. $f(x) = \sin x + c$.

в) $f'(x) = 3$. $f(x) = 3x + c$. г) $f'(x) = -\sin x$. $f(x) = \cos x + c$.

737. а) $y = x^2 - 7x$. $y' = 2x - 7$. б) $y = -3x^2 - 13x$. $y' = -6x - 13$.

в) $y = 7x^2 + 3x$. $y' = 14x + 3$. г) $y = -x^2 + 8x$. $y' = -2x + 8$.

738. а) $y = 12x + \sqrt{x}$. $y' = 12 + \frac{1}{2\sqrt{x}}$. б) $y = \sqrt{x} - 9x^2$. $y' = \frac{1}{2\sqrt{x}} - 18x$.

в) $y = 15x + \sqrt{x}$. $y' = 15 + \frac{1}{2\sqrt{x}}$. г) $y = \sqrt{x} - 5x^2$. $y' = \frac{1}{2\sqrt{x}} - 10x$.

739. а) $y = \frac{1}{x} + 4x$. $y' = \frac{-1}{x^2} + 4$. б) $y = -2x^2 - \frac{1}{x}$. $y' = -4x + \frac{1}{x^2}$.

в) $y = \frac{1}{x} - 6x$. $y' = -\frac{1}{x^2} - 6$. г) $y = 10x^2 + \frac{1}{x}$. $y' = 20x - \frac{1}{x^2}$.

740. а) $y = 6\sqrt{x} + \frac{3}{x}$. $y' = \frac{3}{\sqrt{x}} - \frac{3}{x^2}$.

б) $y = -2\sqrt{x} - \frac{1}{x}$. $y' = -\frac{1}{\sqrt{x}} + \frac{1}{x^2}$.

в) $y = 10\sqrt{x} + \frac{5}{x}$. $y' = \frac{5}{\sqrt{x}} - \frac{5}{x^2}$.

г) $y = -8\sqrt{x} - \frac{1}{x}$. $y' = -\frac{4}{\sqrt{x}} + \frac{1}{x^2}$.

741. а) $y = \sin x + 3$. $y' = \cos x$.
б) $y = 4\cos x$. $y' = -4\sin x$.
в) $y = \cos x - 6$. $y' = -\sin x$.
г) $y = -2\sin x$. $y' = -2\cos x$.

742. а) $y = \cos x + 2x$. $y' = -\sin x + 2$.
б) $y = 2\sin x - 6x$. $y' = 2\cos x - 6$.
в) $y = \sin x - 3x$. $y' = \cos x - 3$.
г) $y = 3\cos x + 15x$. $y' = -3\sin x + 15$.

743. а) $y = 5\sin x + \cos x$. $y' = 5\cos x - \sin x$.
б) $y = 3\sin x + \cos x$. $y' = 3\cos x - \sin x$.
в) $y = \sin x - \cos x$. $y' = \cos x + \sin x$.
г) $y = 2\cos x + \sin x$. $y' = -2\sin x + \cos x$.

744. а) $y = x^5$. $y' = 5x^4$.
б) $y = x^{10}$. $y' = 10x^9$.
в) $y = x^4$. $y' = 4x^3$.
г) $y = x^{201}$. $y' = 201x^{200}$.

745. а) $y = x^3 + 2x^5$. $y' = 3x^2 + 10x^4$.
б) $y = x^4 - x^9$. $y' = 4x^3 - 9x^8$.
в) $y = x^3 + 4x^{100}$. $y' = 3x^2 + 400x^{99}$.
г) $y = x^4 - 7x^9$. $y' = 4x^3 - 63x^8$.

746. а) $y = x^5 + 9x^{20} + 1$. $y' = 5x^4 + 180x^{19}$.
б) $y = x^7 - 4x^{16} - 3$. $y' = 7x^6 - 64x^{15}$.
в) $y = x^6 + 13x^{10} + 12$. $y' = 6x^5 + 130x^9$.
г) $y = x^9 - 6x^{21} - 36$. $y' = 9x^8 - 126x^{20}$.

747. а) $y = (x^2 - 1)(x^4 + 2)$. $y' = (x^4 + 2)(2x) + (x^2 - 1)(4x^3)$.
б) $y = (x^2 + 3)(x^6 - 1)$. $y' = (x^2 + 3)6x^5 + (x^6 - 1)2x$.
в) $y = (x^2 + 3)(x^4 - 1)$. $y' = 2x(x^4 - 1) + (x^2 + 3)4x^3$.
г) $y = (x^2 - 2)(x^7 + 4)$. $y' = 2x(x^7 + 4) + (x^2 - 2)(7x^6)$.

748. а) $y = \sqrt{x}\,(2x - 4)$. $y' = \frac{x-2}{\sqrt{x}} + 3\,2\sqrt{x} = \frac{3x-2}{\sqrt{x}}$.

б) $y = \sqrt{x}\,(x^3 + 1)$. $y' = \frac{x^3+1}{2\sqrt{x}} + 3x^2\sqrt{x} = \frac{7x^3+1}{2\sqrt{x}}$.

в) $y = \sqrt{x}\,(8x - 10)$. $y' = \frac{4x-5}{\sqrt{x}} + 8\sqrt{x} = \frac{12x-5}{\sqrt{x}}$.

г) $y = \sqrt{x}\,(x^4 + 2)$. $y' = \frac{x^4+2}{2\sqrt{x}} + 4x^3\sqrt{x} = \frac{5x^4+2}{2\sqrt{x}}$.

749. а) $y = x\sin x$. $y' = \sin x + x\cos x$.

б) $y = \sqrt{x}\cos x$. $y' = \frac{\cos x}{2\sqrt{x}} - \sqrt{x}\sin x = \frac{\cos x - 2x\sin x}{2\sqrt{x}}$.

в) $y = x\cos x$. $y' = \cos x - x\sin x$.

г) $y = \sqrt{x}\sin x$. $y' = \frac{\sin x}{2\sqrt{x}} + \sqrt{x}\cos x = \frac{\sin x + 2x\cos x}{2\sqrt{x}}$.

750. а) $y = \left(\frac{1}{x} + 1\right)(2x - 3)$. $y' = -\frac{1}{x^2}(2x - 3) + \frac{1}{x} + 2 = 2 + \frac{3}{x^2}$.

б) $y = \left(7 - \frac{1}{x}\right)(6x + 1)$. $y' = \frac{1}{x^2}(6x + 1) - \frac{6}{x} + 42 = 42 + \frac{1}{x^2}$.

в) $y = \left(\frac{1}{x} + 8\right)(5x - 2)$. $y' = -\frac{1}{x^2}(5x - 2) + \frac{5}{x} + 40 = 40 + \frac{2}{x^2}$.

г) $y = \left(9 - \frac{1}{x}\right)(3x + 2)$. $y' = \frac{1}{x^2}(3x + 2) - \frac{3}{x} + 27 = 27 + \frac{2}{x^2}$.

751. а) $y = \frac{x^3}{2x + 4}$.

$$y' = \frac{3x^2(2x+4) - 2x^3}{4x^2 + 16x + 16} = \frac{4x^3 + 12x^2}{4x^2 + 16x + 16} = \frac{x^3 + 3x^2}{x^2 + 4x + 4} = \frac{x^2(x+3)}{(x+2)^2}.$$

б) $y = \frac{x^2}{x^2 - 1}$. $y' = \frac{2x(x^2 - 1) - 2x^3}{(x^2 - 1)^2} = \frac{-2x}{(x^2 - 1)^2}$.

в) $y = \frac{x^2}{3 - 4x}$. $y' = \frac{2x(3 - 4x) + 4x^2}{(3 - 4x)^2} = \frac{6x - 4x^2}{(3 - 4x)^2} = \frac{2x(3 - 2x)}{(3 - 4x)^2}$.

г) $y = \frac{x}{x^2 + 1}$. $y' = \frac{x^2 + 1 - 2x^2}{(x^2 + 1)^2} = \frac{1 - x^2}{(x^2 + 1)}$.

752. а) $y = \frac{3\sqrt{x}}{2x + 9}$.

$$y' = \frac{\frac{3}{2\sqrt{x}}(2x + 9) - 6\sqrt{x}}{(2x + 9)^2} = \frac{6x + 27 - 12x}{2\sqrt{x}(2x + 9)^2} = \frac{27 - 6x}{2\sqrt{x}(2x + 9)^2}.$$

б) $y = \frac{\sin x}{x}$. $y' = \frac{x\cos x - \sin x}{x^2}$.

в) $y = \frac{-2\sqrt{x}}{8 - 3x}$. $y' = \frac{-\frac{1}{\sqrt{x}}(8 - 3x) + 3\left(-2\sqrt{x}\right)}{(8 - 3x)^2} = \frac{3x - 8 - 6x}{\sqrt{x}(8 - 3x)^2} = -\frac{8 + 3x}{\sqrt{x}(8 - 3x)^2}$.

г) $y = \frac{\cos x}{x}$. $y' = \frac{-x\sin x - \cos x}{x^2} = -\frac{\cos x + x\sin x}{x^2}$.

753. а) $y = tgx$. $y' = \dfrac{1}{\cos^2 x}$.

б) $y = ctgx$. $y' = -\dfrac{1}{\sin^2 x}$.

в) $y = tgx + 4$. $y' = \dfrac{1}{\cos^2 x}$.

г) $y = ctgx + 8$. $y' = -\dfrac{1}{\sin^2 x}$.

754. а) $y = 3sinx + ctgx$. $y' = 3cosx - \dfrac{1}{\sin^2 x}$.

б) $y = tgx - cosx$. $y' = \dfrac{1}{\cos^2 x} + sinx$.

в) $y = cosx + tgx$. $y' = -sinx + \dfrac{1}{\cos^2 x}$.

г) $y = 6tgx - sinx$. $y' = \dfrac{6}{\cos^2 x} - cosx$.

755. а) $y = xtgx$. $y' = tgx + \dfrac{x}{\cos x}$.

б) $y = sinxtgx$. $y' = sinx + \dfrac{\sin x}{\cos^2 x}$.

в) $y = xctgx$. $y' = ctgx - \dfrac{x}{\sin^2 x}$.

г) $y = cosxctgx$. $y' = -cosx - \dfrac{\cos x}{\sin^2 x}$.

756. а) $y = 6x - 9$. $x_0 = 3$. $y' = 6$. $y'(x_0) = 6$.
б) $y = -11x + 7$. $x_0 = 5$. $y' = -11$. $y'(x_0) = -11$.
в) $y = 5x - 8$. $x_0 = 2$. $y' = 5$. $y'(x_0) = 5$.
г) $y = -20x + 3$. $x_0 = 6$. $y' = -20$. $y'(x_0) = -20$.

757. а) $y = x^2 + 2x - 1$. $x_0 = 0$. $y' = 2x + 2$. $y'(x_0) = 2$.
б) $y = x^3 - 3x + 2$. $x_0 = -1$. $y' = 3x^2 - 3$. $y'(x_0) = 3 - 3 = 0$.
в) $y = x^2 + 3x - 4$. $x_0 = 1$. $y' = 2x + 3$. $y'(x_0) = 5$.
г) $y = x^3 - 9x^2 + 7$. $x_0 = 2$. $y' = 3x^2 - 18x$. $y'(x_0) = 12 - 36 = -24$.

758. а) $y = \dfrac{2}{x} - 1$. $x_0 = 4$. $y' = -\dfrac{2}{x^2}$. $y'(x_0) = -\dfrac{1}{8}$.

б) $y = \sqrt{x} + 4$. $x_0 = 9$. $y' = \dfrac{1}{2\sqrt{x}}$. $y'(x_0) = \dfrac{1}{6}$.

в) $y = \dfrac{8}{x} - 6$. $x_0 = 1$. $y' = -\dfrac{8}{x^2}$. $y'(x_0) = -8$.

г) $y = \sqrt{x} + 5$. $x_0 = 4$. $y' = \dfrac{1}{2\sqrt{x}}$. $y'(x_0) = \dfrac{1}{4}$.

759. а) $y = 2\sin x - 13$. $x_0 = \frac{\pi}{2}$. $y' = 2\cos x$. $y'(x_0) = 0$.

б) $y = -\cos x + 2$. $x_0 = \frac{\pi}{3}$. $y' = \sin x$. $y'(x_0)= \frac{\sqrt{3}}{2}$.

в) $y = -\sin x - 3$. $x_0 = \frac{\pi}{6}$. $y' = -\cos x$. $y'(x_0)= -\frac{\sqrt{3}}{2}$.

г) $y = 4\cos x + 1$. $x_0 = \frac{\pi}{4}$. $y' = -4\sin x$. $y'(x_0)= -2\sqrt{2}$.

760. а) $y = \operatorname{tg} x + 14$. $x_0 = -\frac{\pi}{4}$. $y' = \frac{1}{\cos^2 x}$. $y'(x_0)= 2$.

б) $y = 2\operatorname{ctg} x$. $x_0 = \frac{\pi}{3}$. $y' = \frac{-2}{\sin^2 x}$.

$y'(x_0)= -\frac{2}{\left(\frac{\sqrt{3}}{2}\right)^2} = -\frac{8}{3} = -2\frac{2}{3}$.

в) $y = \operatorname{ctg} x - 2$. $x_0 = -\frac{\pi}{6}$. $y' = -\frac{1}{\sin^2 x}$. $y'(x_0)= -4$.

г) $y = 4\operatorname{tg} x$. $x_0 = 0$. $y' = \frac{-4}{\cos^2 x}$. $y'(x_0)= -4$.

761. а) $y = \frac{\sin x}{x}$. $x_0 = \frac{\pi}{2}$. $y' = \frac{x\cos x - \sin x}{x^2}$. $y'(x_0) = -\frac{4}{\pi^2}$.

б) $y = \frac{x+1}{x-1}$. $x_0 = 2$. $y' = \frac{x-1-x-1}{(x-1)^2} = -\frac{2}{(x-1)^2}$. $y'(x_0) = -2$.

в) $y = \frac{\cos x}{x}$. $x_0 = \pi$. $y' = \frac{-x\sin x - \cos x}{x^2} = \frac{1}{\pi^2}$.

г) $y = \frac{2x}{x+1}$. $x_0 = 0$. $y' = \frac{2x+2-2x}{(x+1)^2} = \frac{2}{(x+1)^2}$. $y'(x_0) = 2$.

762. а) $g = x^3 + 2x$. $x_0 = 2$. $g'(x) = 3x^2 + 2$. $g'(x_0) = 14$.

б) $g = \left(\sqrt{x}+1\right)\sqrt{x}$. $x_0 = 1$. $g'(x) = \frac{\sqrt{x}}{2\sqrt{x}} + \frac{\sqrt{x}+1}{2\sqrt{x}} = \frac{1}{2} + \frac{1}{2} + \frac{1}{2\sqrt{x}}$.

$g'(x_0) = \frac{3}{2}$.

в) $g = x^2 - 3x$. $x_0 = 4$. $g'(x) = 2x - 3$. $g'(x_0) = 5$.

г) $g = \frac{1}{x}\left(\frac{4}{x} - 2\right)$. $x_0 = -\frac{1}{2}$. $g'(x) = -\frac{1}{x^2}\left(\frac{4}{x} - 2\right) + \frac{1}{x}\left(-\frac{4}{x^2}\right) = -\frac{4}{x^3} + \frac{2}{x^2} - \frac{4}{x^3}$.

$g'(x_0) = 4\cdot 8 + 2\cdot 4 + 4\cdot 8 = 72$.

763. а) $g = \frac{6}{x} - 2x$. $x_0 = -1$. $g'(x) = -\frac{6}{x^2} - 2$. $g'(x_0) = -6 - 2 = -8$.

б) $g = 2x^3 - 4x + 3$. $x_0 = 2$. $g'(x) = 6x^2 - 4$. $g'(x_0) = 20$.

в) $g = 4x^2 - \frac{12}{x}$. $x_0 = -2$. $g'(x) = 8x + \frac{12}{x^2}$. $g'(x_0) = -16 + 3 = -13$.

г) $g = -x^3 + 2x^2 + 1$. $x_0 = 2$. $g'(x) = -3x^2 + 4x$. $g'(x_0) = -4$.

764. а) $g = 2\sin x - 4x$. $x_0 = \frac{\pi}{2}$. $g'(x) = 2\cos x - 4$. $g'(x_0) = -4$.

б) $g = \frac{\operatorname{tg} x}{3}$. $x_0 = -\frac{\pi}{3}$. $g'(x) = \frac{1}{3\cos^2 x}$. $g'(x_0) = \frac{4}{3}$.

в) $g = -3\cos x + x$. $x_0 = -\frac{\pi}{6}$. $g'(x) = 3\sin x + 1$. $g'(x_0) = -\frac{1}{2}$.

г) $g = \frac{\operatorname{ctg} x}{5}$. $x_0 = \frac{\pi}{3}$. $g'(x) = -\frac{1}{5\sin^2 x}$. $g'(x_0) = -\frac{4}{15}$.

765. а) $h(x) = x^6 - 4x$. $x_0 = 1$.
$h'(x) = 6x^5 - 4$. $h'(x_0) = 2$. $\operatorname{tg}\alpha = 2$.

б) $h(x) = \sqrt{x} - 3$. $x_0 = \frac{1}{4}$.
$h'(x) = \frac{1}{2\sqrt{x}}$. $h'(x_0) = 1$. $\operatorname{tg}\alpha = 1$.

в) $h(x) = -x^5 - 2x^2 + 2$. $x_0 = -1$.
$h'(x) = -5x^4 - 4x$. $h'(x_0) = -5 + 4 = -1$. $\operatorname{tg}\alpha = -1$.

г) $h(x) = \frac{25}{x} + 2$. $x_0 = \frac{5}{4}$.
$h'(x) = -\frac{25}{x^2}$. $h'(x_0) = -16$. $\operatorname{tg}\alpha = -16$

766. а) $h(x) = 10 - \cos x$. $x_0 = \frac{3\pi}{2}$.
$h'(x) = \sin x$. $h'(x_0) = -1$. $\operatorname{tg}\alpha = -1$.

б) $h(x) = 2\operatorname{tg} x$. $x_0 = \frac{\pi}{4}$.
$h'(x) = \frac{2}{\cos^2 x}$. $h'(x_0) = 4$. $\operatorname{tg}\alpha = 4$.

в) $h(x) = 4 - \sin x$. $x_0 = 6\pi$.
$h'(x) = -\cos x$. $h'(x_0) = -1$. $\operatorname{tg}\alpha = -1$.

г) $h(x) = -4\operatorname{ctg} x$. $x_0 = -\frac{\pi}{4}$.
$h'(x) = \frac{4}{\sin^2 x}$. $h'(x_0) = 8$. $\operatorname{tg}\alpha = 8$.

767. а) $f(x) = x^2\sin x$.

$f'(x) = 2x\sin x + x^2\cos x$. $f'\left(\frac{\pi}{2}\right) = \pi$.

б) $f(x) = \sqrt{3}\sin x + \frac{x^2}{\pi} + x\sin\frac{\pi}{6}$.

$f'(x) = \sqrt{3}\cos x + \frac{2x}{\pi} + \frac{1}{2}$. $f'\left(\frac{\pi}{6}\right) = \frac{3}{2} + \frac{1}{3} + \frac{1}{2} = 2\frac{1}{3}$.

в) $f(x) = x(1 + \cos x)$.

$f'(x) = 1 + \cos x + x(-\sin x)$. $f'(\pi) = 1 - 1 + 0 = 0$.

г) $f(x) = \sqrt{3}\cos x - x\cos\frac{\pi}{6} + \frac{x^2}{\pi}$. $f'(x) = -\sqrt{3}\sin x - \frac{\sqrt{3}}{2} + \frac{2x}{\pi}$.

$f'\left(\frac{\pi}{3}\right) = -\frac{3}{2} - \frac{\sqrt{3}}{2} + \frac{2}{3} = \frac{4 - 9 - 3\sqrt{3}}{6} = -\frac{5 + 3\sqrt{3}}{6}$.

768. а) $f'(x) = 3x^2 + 2x$. $f(x) = x^3 + x^2 + c$.

б) $f'(x) = -\frac{7}{x^2}$. $f(x) = \frac{7}{x} + c$.

в) $f'(x) = 5x^4 - 1$. $f(x) = x^5 - x + c$.

г) $f'(x) = \frac{9}{2\sqrt{x}}$ $f(x) = 9\sqrt{x} + c$.

769. а) $f(x) = 2\sqrt{x} - 5x + 3$. $f'(x) = \frac{1}{\sqrt{x}} - 5 = 2$. $x = \frac{1}{49}$.

б) $f(x) = 3x - \sqrt{x} + 13$. $f'(x) = 3 - \frac{1}{2\sqrt{x}} = 1$. $x = \frac{1}{16}$.

770. а) $y = (4x - 9)^7$. $y' = 7(4x - 9)^6 \cdot 4 = 28(4x - 9)^6$.

б) $y = \left(\frac{x}{3} + 2\right)^{12}$. $y' = 4\left(\frac{x}{3} + 2\right)^{11}$.

в) $y = (5x + 1)^9$. $y' = 45(5x + 1)^8$.

г) $y = \left(\frac{x}{4} - 2\right)^{14}$. $y' = \frac{7}{2}\left(\frac{x}{4} - 3\right)^{13}$.

771. а) $y = (3 - x)^5$. $y' = -5(3 - x)^4$.

б) $y = (7 - 24x)^{10}$. $y' = -240(7 - 24x)^9$.

в) $y = \left(12 - \frac{x}{5}\right)^6$. $y' = -\frac{6}{5}\left(12 - \frac{x}{5}\right)^5$.

г) $y = (15 - 9x)^{13}$. $y' = -117(15 - 9x)^{12}$.

772. а) $y = \sin(3x - 9)$. $y' = 3\cos(3x - 9)$.

б) $y = \sin(7 - 2x)$. $y' = -2\cos(7 - 2x)$.

в) $y = \sin\left(\frac{x}{2} + 1\right)$. $y' = \frac{1}{2}\cos\left(\frac{x}{2} + 1\right)$.

г) $y = \sin(5 - 3x)$. $y' = -3\cos(5 - 3x)$.

773. а) $y = \cos(5x + 9)$. $y' = -5\sin(5x + 9)$.

б) $y = \cos\left(\frac{\pi}{3} - 4x\right)$. $y' = 4\sin\left(\frac{\pi}{3} - 4x\right)$.

в) $y = \cos(9x - 10)$. $y' = -9\sin(9x - 10)$.

г) $y = \cos\left(\frac{\pi}{4} - \frac{x}{2}\right)$. $y' = \frac{1}{2}\sin\left(\frac{\pi}{4} - \frac{x}{2}\right)$.

774. а) $y = \mathrm{tg}\left(5x - \frac{\pi}{4}\right)$. $y' = \frac{5}{\cos^2\left(5x - \frac{\pi}{4}\right)}$.

б) $y = \mathrm{ctg}\left(\frac{\pi}{6} - 4x\right)$. $y' = \frac{4}{\sin^2\left(\frac{\pi}{6} - 4x\right)}$.

в) $y = \mathrm{tg}\left(2x + \frac{\pi}{3}\right)$. $y' = \frac{2}{\cos^2\left(2x + \frac{\pi}{3}\right)}$.

г) $y = \mathrm{ctg}\left(\frac{\pi}{4} - 5x\right)$. $y' = \frac{5}{\sin^2\left(\frac{\pi}{4} - 5x\right)}$.

775. а) $y = \sqrt{15 - 7x}$. $y' = \frac{-7}{2\sqrt{15 - 7x}}$.

б) $y = \sqrt{42 + 0{,}5x}$. $y' = \frac{1}{4\sqrt{42 + 0{,}5x}}$.

в) $y = \sqrt{4 + 9x}$. $y' = \frac{9}{2\sqrt{4 + 9x}}$.

г) $y = \sqrt{50 - 0{,}2x}$. $y' = \frac{-1}{10\sqrt{50 - 0{,}2x}}$.

776. а) $y = (3x - 2)^7$. $x_0 = 3$. $y' = 21(3x - 2)^6$. $y'(3) = 3\cdot 7^7$.

б) $y = (4 - 5x)^7$. $x_0 = -2$. $y' = -35(4 - 5x)^6$. $y'(-2) = -35\cdot 14^6$.

в) $y = (2x + 3)^5$. $x_0 = 2$. $y' = 10(2x + 3)^4$. $y'(x_0) = 10\cdot 7^4$.

г) $y = (5 - 3x)^7$. $x_0 = -1$. $y' = -21(5 - 3x)^6$. $y'(x_0) = -21\cdot 8^6$.

777. а) $y = \sin\left(2x - \frac{\pi}{3}\right)$. $x_0 = \frac{\pi}{6}$. $y' = 2\cos\left(2x - \frac{\pi}{3}\right)$. $y'(x_0) = 2$.

б) $y = \sin\left(\frac{\pi}{6} - 2x\right)$. $x_0 = \frac{\pi}{12}$. $y' = -2\cos\left(\frac{\pi}{6} - 2x\right)$. $y'(x_0) = -2$.

в) $y = \cos\left(\frac{\pi}{3} - 4x\right)$. $x_0 = \frac{\pi}{8}$.

$y' = 4\sin\left(\frac{\pi}{3} - 4x\right)$. $y'(x_0) = 4\sin\left(\frac{\pi}{3} - \frac{\pi}{2}\right) = -4\sin\frac{\pi}{6} = -2$.

г) $y=\cos\left(6x-\frac{\pi}{4}\right)$. $x_0=-\frac{\pi}{12}$.

$y'=-6\sin\left(6x-\frac{\pi}{4}\right)$. $y'(x_0)=-6\sin\left(-\frac{\pi}{2}-\frac{\pi}{4}\right)==6\frac{\sqrt{2}}{2}=3\sqrt{2}$.

778. а) $y=\operatorname{tg}\left(2x+\frac{\pi}{8}\right)$. $x_0=\frac{\pi}{16}$.

$y'=\frac{2}{\cos^2\left(2x+\frac{\pi}{8}\right)}$. $y'(x_0)=\frac{2}{\cos^2\frac{\pi}{4}}=4$.

б) $y=\operatorname{ctg}\left(\frac{\pi}{6}-x\right)$. $x_0=\frac{\pi}{3}$.

$y'=\frac{1}{\sin^2\left(\frac{\pi}{6}-x\right)}$. $y'(x_0)=\frac{1}{\sin^2\left(-\frac{\pi}{6}\right)}=4$.

в) $y=\operatorname{tg}\left(3x-\frac{\pi}{4}\right)$. $x_0=\frac{\pi}{12}$.

$y'=\frac{3}{\cos^2\left(3x-\frac{\pi}{4}\right)}$. $y'(x_0)=\frac{3}{\cos^2 0^\circ}=3$.

г) $y=\operatorname{ctg}\left(\frac{\pi}{3}-x\right)$. $x_0=\frac{\pi}{6}$.

$y'=\frac{1}{\sin^2\left(\frac{\pi}{3}-x\right)}$. $y'(x_0)=\frac{1}{\sin^2\frac{\pi}{6}}=4$.

779. а) $y=\sqrt{6x-1}$. $x_0=5$.

$y'=\frac{3}{\sqrt{6x-1}}$. $y'(x_0)=\frac{3}{\sqrt{29}}=\frac{3\sqrt{29}}{29}$.

б) $y=\sqrt{4-8x}$. $x_0=0$.

$y'=-\frac{4}{\sqrt{4-8x}}$. $y'(x_0)=-2$.

в) $y=\sqrt{7x+4}$. $x_0=3$.

$y'=\frac{7}{2\sqrt{7x+4}}$. $y'(x_0)=\frac{7}{10}$.

г) $y=\sqrt{25-9x}$. $x_0=1$.

$y'=-\frac{9}{2\sqrt{25-9x}}$. $y'(x_0)=-\frac{9}{8}=-1\frac{1}{8}$.

780. а) $y = (2x + 1)^5$. $x_0 = -1$. $y' = 10(2x + 1)^4$. $y'(x_0) = 10$.

б) $y = \sqrt{7x-3}$. $x_0 = 1$. $y' = \frac{7}{2\sqrt{7x-3}}$. $y'(x_0) = \frac{7}{4} = 1\frac{3}{4}$.

в) $y = \frac{4}{12x-5}$. $x_0 = 2$. $y' = \frac{-12\cdot 4}{(12x-5)^2}$. $y'(x_0) = -\frac{48}{19^2} = -\frac{48}{361}$.

г) $y = \sqrt{11-5x}$. $x_0 = -1$. $y' = \frac{-5}{2\sqrt{11-5x}}$. $y'(x_0) = -\frac{5}{8}$.

781.

а) $y = \sin\left(3x - \frac{\pi}{4}\right)$. $x_0 = \frac{\pi}{4}$. $y' = 3\cos\left(3x - \frac{\pi}{4}\right)$. $y'(x_0) = 0$.

б) $y = \mathrm{tg}6x$. $x_0 = \frac{\pi}{24}$. $y' = \frac{6}{\cos^2 6x}$. $y'(x_0) = \frac{6}{\cos^2\frac{\pi}{4}} = 12$.

в) $y = \cos\left(\frac{\pi}{3} - 2x\right)$. $x_0 = \frac{\pi}{3}$. $y' = 2\sin\left(\frac{\pi}{3} - 2x\right)$. $y'(x_0) = 2\sin\left(-\frac{\pi}{3}\right) = -\sqrt{3}$.

г) $y = \mathrm{ctg}\frac{x}{3}$. $x_0 = \pi$. $y' = \frac{-1}{6\sin^2\frac{x}{3}}$. $y'(x_0) = -\frac{1}{6\sin^2\frac{\pi}{3}} = -\frac{4}{9}$.

782.

а) $h(x) = (0{,}5x + 3)^7$. $x_0 = -4$. $h'(x) = \frac{7}{2}(0{,}5x + 3)^6$. $h'(x_0) = \frac{7}{2} = 3{,}5$.

б) $h(x) = \sqrt{16x+21}$. $x_0 = \frac{1}{4}$. $h'(x) = \frac{8}{\sqrt{16x+21}}$. $h'(x_0) = \frac{8}{5} = 1{,}6$.

в) $h(x) = \frac{18}{4x+1}$. $x_0 = \frac{1}{2}$. $h'(x) = \frac{-18\cdot 4}{(4x+1)^2}$. $h'(x_0) = -\frac{72}{9} = -8$.

г) $h(x) = \sqrt{6-2x}$. $x_0 = 1$. $h'(x) = -\frac{1}{\sqrt{6-2x}}$. $h'(x_0) = -\frac{1}{2}$.

783. а) $y = (x - 1)(x^2 + x + 1) = x^3 - 1$. $y' = 3x^2$.

б) $y = (x^2 + 2x + 4)(x - 2) = x^3 - 8$. $y' = 3x^2$.

в) $y = (x + 1)(x^2 - x + 1) = x^3 + 1$. $y' = 3x^2$.

г) $y = (x^2 - 3x + 9)(x + 3) = x^3 + 27$. $y' = 3x^2$.

784.

а) $y = \frac{x^9-3}{x^3} = x^6 - \frac{3}{x^3}$. $y' = 6x^5 + \frac{9}{x^4}$.

б) $y = \frac{x^{15}}{x^{10}+1}$. $y' = \frac{15x^{14}(x^{10}+1) - 10x^9\cdot x^{15}}{(x^{10}+1)^2} = \frac{5x^{24}+15x^{14}}{(x^{10}+1)^2}$.

в) $y = \frac{x^5 + x}{x^5 - 1}$. $y' = \frac{(5x^4+1)(x^5-1) - 5x^4(x^5+x)}{(x^5-1)^2} = .$

$= \frac{5x^9 + x^5 - 5x^4 - 1 - 5x^9 + 5x^5}{(x^5-1)^2} = \frac{-4x^5 - 5x^4 - 1}{(x^5-1)^2}.$

г) $y = \frac{x^{13}}{x^4 - 2}$. $y' = \frac{13x^{12}(x^4-2) - 4x^3 \cdot x^{13}}{(x^4-2)^2} = \frac{9x^{16} - 26x^{12}}{(x^4-2)^2}$.

785. а) $y = \cos^2\frac{x}{2} - \sin^2\frac{x}{2} = \cos x$. $y' = -\sin x$.

б) $y = 2\sin\frac{x}{2}\cos\frac{x}{2} = \sin x$. $y' = \cos x$.

в) $y = \cos^2 3x + \sin^2 3x = 1$. $y' = 0$.

г) $y = -\sin\frac{x}{2}\cos\frac{x}{2} = -\frac{1}{2}\sin x$. $y' = -\frac{1}{2}\cos x$.

786. а) $y = \sin 2x\cos x - \cos 2x\sin x = \sin x$. $y' = \cos x$.

б) $y = \sin\frac{x}{3}\cos\frac{2x}{3} + \cos\frac{x}{3}\sin\frac{2x}{3} = \sin x$. $y' = \cos x$.

в) $y = \cos 3x\cos 2x + \sin 3x\sin 2x = \cos x$. $y' = -\sin x$.

г) $y = \cos\frac{x}{5}\cos\frac{4x}{5} - \sin\frac{x}{5}\sin\frac{4x}{5} = \cos x$. $y' = -\sin x$.

787. а) $f(x) = a\sin 2x + b\cos x$. $f'\left(\frac{\pi}{6}\right) = 2$.

$f'\left(\frac{9\pi}{2}\right) = -4$. $f'(x) = 2a\cos 2x - b\sin x$.

$\begin{cases} 2a\cos\frac{\pi}{3} - b\sin\frac{\pi}{6} = 2. \\ 2a\cos 9\pi - b\sin\frac{9\pi}{2} = -4. \end{cases}$ $\begin{cases} a - \frac{b}{2} = 2. \\ -2a - b = -4. \end{cases}$ $\begin{cases} 2a - b = 4. \\ 2a + b = 4. \end{cases}$ $a = 2$. $b = 0$.

б) $f(x) = a\cos 2x + b\sin 4x$. $f'\left(\frac{7\pi}{12}\right) = 4$.

$f'\left(\frac{3\pi}{4}\right) = 2$. $f'(x) = -2a\sin 2x + 4b\cos 4x$.

$\begin{cases} -2a\sin\frac{7\pi}{6} + 4b\cos\frac{7\pi}{3} = 4. \\ -2a\sin\frac{3\pi}{2} + 4b\cos 3\pi = 2. \end{cases}$ $\begin{cases} a + 2b = 4. \\ 2a - 4b = 2. \end{cases}$ $\begin{cases} a + 2b = 4. \\ a - 2b = 1. \end{cases}$ $a = \frac{5}{2}$. $b = \frac{3}{4}$.

788. а) $f(x) = \sqrt{x} - x$. $k = 1$. $f'(x) = \frac{1}{2\sqrt{x}} - 1 = 1$.

$1 - 4\sqrt{x} = 0$. $\sqrt{x} = \frac{1}{4}$. $x = \frac{1}{16}$.

б) $f(x) = \sqrt{x} + 3x$. $k = 4$. $f'(x) = \frac{1}{2\sqrt{x}} + 3 = 4$.

$1 - 2\sqrt{x} = 0$. $x = \frac{1}{4}$.

789. а) $f(x) = \sin x \cos x$. $k = -\frac{\sqrt{2}}{2}$.

$f'(x) = \cos 2x = -\frac{\sqrt{2}}{2}$. $x = \pm\frac{3\pi}{8} + \pi n$.

б) $f(x) = \cos^2 x$. $k = \frac{1}{2}$.

$f'(x) = -2\cos x \sin x = -\sin 2x = \frac{1}{2}$. $x = (-1)^{k+1}\frac{\pi}{12} + \frac{\pi k}{2}$.

790. а) $f(x) = x^3 - x^4$. $f'(x) = 3x^2 - 4x^3 < 0$.

$x^2(3 - 4x) < 0$. $x > \frac{3}{4}$.

б) $f(x) = \frac{1}{5}x^5 - \frac{5}{3}x^3 + 6x$. $f'(x) = x^4 - 5x^2 + 6 < 0$.

$(x^2 - 2)(x^2 - 3) < 0$ $\left(x - \sqrt{2}\right)\left(x + \sqrt{2}\right)\left(x - \sqrt{3}\right)\left(x + \sqrt{3}\right) < 0$.

$x \in \left(-\sqrt{3}; -\sqrt{2}\right) \cup \left(\sqrt{2}; \sqrt{3}\right)$.

791. а) $f(x) = \sin 2x$. $f'(x) = 2\cos 2x < 0$.

$2x \in \left(\frac{\pi}{2} + 2\pi n; \frac{3\pi}{2} + 2\pi n\right)$. $x \in \left(\frac{\pi}{4} + \pi n; \frac{3\pi}{4} + \pi n\right)$.

б) $f(x) = -4\cos x + 2x$. $f'(x) = 4\sin x + 2 < 0$.

$\sin x < -\frac{1}{2}$. $x \in \left(-\frac{5\pi}{6} + 2\pi n; -\frac{\pi}{6} + 2\pi n\right)$.

792. а) $g(x) = x^3 + x^4$. $g'(x) = 3x^2 + 4x^3 > 0$.

$x^2(3 + 4x) > 0$ $x \in \left(-\frac{3}{4}; 0\right) \cup (0; +\infty)$.

б) $g(x) = \frac{4}{2 - 5x}$. $g'(x) = \frac{20}{(2 - 5x)^2} > 0$. $x \in \Re$, но $x \neq \frac{2}{5}$.

793. а) $g(x) = \cos^2 x - \sin^2 x = \cos 2x$.

$g'(x) = -2\sin 2x > 0$. $\sin 2x < 0$.

$2x \in (-\pi + 2\pi n; 2\pi n)$. $x \in (-\frac{\pi}{2} + \pi n; \pi n)$.

б) $g(x) = \sin^2 x$. $g'(x) = 2\sin x \cos x = \sin 2x > 0$.

$2x \in (2\pi n; \pi + 2\pi n)$. $x \in (+\pi n; \frac{\pi}{2} + \pi n)$.

794. а) $f(x) = \cos^2x + \sin x + 1$. $x \in [0; 2]$.
$f'(x) = -2\cos x\sin x + \cos x = 0$.
$\cos x(1 - 2\sin x) = 0$. $\cos x = 0$.
$x = \frac{\pi}{2} + \pi n$. $x = \frac{\pi}{2}$. $\sin x = \frac{1}{2}$.
$x = (-1)^k\frac{\pi}{6} + \pi k$. $x = \frac{\pi}{6}$.
б) $f(x) = \sin^2x - \cos x - 1$. $x \in \left[\frac{\pi}{2}; \frac{3\pi}{2}\right]$. $f'(x) = 2\sin x\cos x + \sin x = 0$.
$\sin x(2\cos x + 1) = 0$. $\sin x = 0$. $x = \pi n$. $x = \pi$.
$\cos x = -\frac{1}{2}$. $x = \pm\frac{2\pi}{3} + 2\pi n$, $x = \frac{2\pi}{3}$, $x = \frac{4\pi}{3}$.
795. а) $h(x) = x^3 - 3x^2 + 1$. $h'(x) = 3x^2 - 6x > 0$.
$x(3x - 6) > 0$. $x \in (-\infty;0) \cup (2;+\infty)$.
б) $h(x) = 4\sqrt{x} - x$. $h'(x) = \frac{2}{\sqrt{x}} - 1 > 0$.
$\frac{2}{\sqrt{x}} > 1$. $\sqrt{x} < 2$. $x \in (0; 4)$.
в) $y = x^3 - x^4 - 19$.
$y'(x)=3x^2-4x^3 = x^2(3 - 4x)> 0$. $x \in (-\infty; 0) \cup (0; 3/4)$.
г) $h(x) = \mathrm{tg}x - 4x$. $h'(x) = \frac{1}{\cos^2 x} - 4 > 0$.
$\cos^2x < \frac{1}{4}$. $\cos x \in \left(-\frac{1}{2};\frac{1}{2}\right)$. $x \in \left(\frac{\pi}{3};\frac{2\pi}{3}\right)$.
796. а) $\varphi(x) = \sin x + 3$. $\varphi'(x) = \cos x < 0$. $x\in\left(\frac{\pi}{2} + 2\pi n;\frac{3\pi}{2} + 2\pi n\right)$.
б) $\varphi(x) = 0{,}2x^5 - 3\frac{1}{3}x^3 + 9x$. $\varphi'(x) = x^4 - 10x^2 + 9 < 0$.
$(x^2 - 9)(x^2 - 1) < 0$. $(x - 3)(x + 3)(x - 1)(x + 1) < 0$.
$x \in (-3; -1) \cup (1; 3)$.
в) $\varphi(x) = \mathrm{ctg}x + 9x$. $\varphi'(x) = -\frac{1}{\sin^2 x} + 9 < 0$.
$\sin^2x < \frac{1}{9}$. $\sin x \in \left(-\frac{1}{3};\frac{1}{3}\right)$.
$x \in \left(-\arcsin\frac{1}{3} + \pi n;\arcsin\frac{1}{3} + \pi n\right)$.
г) $\varphi(x) = \sqrt{4 - 2x}$. $\varphi'(x) = -\frac{1}{\sqrt{4 - 2x}} < 0$. $x < 2$.

797. а) $f(x) = \frac{1}{3}x^3 - x^2$. $g(x) = 7{,}5x^2 - 16x$.

$f'(x) = x^2 - 2x$. $g'(x) = 15x - 16$.
$x^2 - 2x = 15x - 16$. $x^2 - 17x + 16 = 0$.

$x = \frac{17+15}{2} = 16$. $x = 1$.

б) $f(x) = \sqrt{x}$. $g(x) = -\frac{1}{x}$.

$f'(x) = \frac{1}{2\sqrt{x}}$. $g'(x) = \frac{1}{x^2}$.

$\frac{1}{2\sqrt{x}} = \frac{1}{x^2}$. $x \neq 0$.

$x^2 = 2\sqrt{x}$. $x^4 = 4x$.

$x(x^3 - 4) = 0$. $x = 0$ не подходит. $x = \sqrt[3]{4}$.
798. а) $f(x) = \cos 2x$. $g(x) = \sin x$.
$f'(x) = -2\sin 2x$. $g'(x) = \cos x$.
$-2\sin 2x = \cos x$. $\cos x(4\sin x + 1) = 0$

$\cos x = 0$. $x = \frac{\pi}{2} + \pi n$.

$\sin x = -\frac{1}{4}$. $x = (-1)^{k+1}\arcsin\frac{1}{4} + \pi k$.

б) $f(x) = \mathrm{tg}x$. $g(x) = \mathrm{ctg}x$.

$f'(x) = \frac{1}{\cos^2 x}$. $g'(x) = -\frac{1}{\sin^2 x}$.

$\frac{1}{\cos^2 x} = -\frac{1}{\sin^2 x}$. $\cos^2 x + \sin^2 x = 0$. решений нет.

799. а) $g(x) = x^3 - 3x^2$. $h(x) = 1{,}5x^2 - 9$.
$g'(x) = 3x^2 - 6x$. $h'(x) = 3x$.
$3x^2 - 6x > 3x$. $x^2 - 3x > 0$. $x \in (-\infty; 0) \cup (3; +\infty)$.

б) $g(x) = \sin\left(3x - \frac{\pi}{6}\right)$. $h(x) = 6x - 12$. $g'(x) = 3\cos\left(3x - \frac{\pi}{6}\right)$. $h'(x) = 6$.

$3\cos\left(3x - \frac{\pi}{6}\right) > 6$. $\cos\left(3x - \frac{\pi}{6}\right) > 2$. таких значений нет.

в) $g(x) = \mathrm{tg}x$. $h(x) = 4x - 81$. $g'(x) = \frac{1}{\cos^2 x}$. $h'(x) = 4$.

$\frac{1}{\cos^2 x} > 4$. $\cos^2 x < \frac{1}{4}$, $\cos^2 x \neq 0$.

$\cos x \in \left(-\frac{1}{2}; \frac{1}{2}\right)$, $\cos x \neq 0$. $x \in \left(\frac{\pi}{3} + \pi n; \frac{\pi}{2} + \pi n\right) \cup \left(\frac{\pi}{2} + \pi n; \frac{2\pi}{3} + \pi n\right)$.

г) $g(x) = \cos\left(\frac{\pi}{4} - 2x\right)$. $h(x) = 3 - \sqrt{2}\,x$.

$g'(x) = 2\sin\left(\frac{\pi}{4} - 2x\right)$. $h'(x) = -\sqrt{2}$.

$2\sin\left(\frac{\pi}{4} - 2x\right) > -\sqrt{2}$. $\sin\left(-\frac{\pi}{4} + 2x\right) < \frac{\sqrt{2}}{2}$.

$2x - \frac{\pi}{4} \in \left(-\frac{5\pi}{4} + 2\pi n; \frac{\pi}{4} + 2\pi n\right)$. $x \in \left(-\frac{\pi}{2} + \pi n; \frac{\pi}{4} + \pi n\right)$.

800. а) $f(x) = \sin(2x - 3)$. $g(x) = \cos(2x - 3)$.

$f'(x) = 2\cos(2x - 3)$. $g'(x) = -2\sin(2x - 3)$. $\cos(2x-3)+\sin(2x - 3) = 0$.

$\sin\left(2x - 3 + \frac{\pi}{4}\right) = 0$. $x = \frac{3}{2} - \frac{\pi}{8} + \frac{\pi n}{2}$.

б) $f(x) = \frac{6}{5x - 9}$. $g(x) = \frac{3}{7 - 5x}$.

$g'(x) = \frac{15}{(7 - 5x)^2}$. $f'(x) = -\frac{30}{(5x - 9)^2}$.

$\frac{2}{(5x - 9)^2} = \frac{1}{(7 - 5x)^2}$. $(5x - 9)^2 + 2(5x - 7)^2 = 0$. решений нет.

в) $f(x) = \sqrt{3x - 10}$. $g(x) = \sqrt{6x + 14}$.

$f'(x) = \frac{3}{2\sqrt{3x - 10}}$. $g'(x) = \frac{3}{\sqrt{6x + 14}}$.

$\frac{1}{2\sqrt{3x - 10}} = \frac{1}{\sqrt{6x + 14}}$. $2\sqrt{3x - 10} = \sqrt{6x + 14}$.

$12x - 40 = 6x + 14$. $6x = 54$. $x = 9$. проверка: $2\sqrt{17} = \sqrt{68}$.

г) $f(x) = \operatorname{ctg}x$. $g(x) = 2x + 15$.

$f'(x) = -\frac{1}{\sin^2 x}$. $g'(x) = 2$. $\sin^2 x = -\frac{1}{2}$. решений нет.

801. а) $f(x) = \sin x\cos x = \frac{1}{2}\sin 2x$. $g(x) = \frac{1}{2}x + 61$.

$f'(x) = \cos 2x$. $g'(x) = \frac{1}{2}$. $\cos 2x \le \frac{1}{2}$.

$2x \in \left(\frac{\pi}{3} + 2\pi n; \frac{5\pi}{3} + 2\pi n\right)$. $x \in \left(\frac{\pi}{6} + \pi n; \frac{5\pi}{6} + \pi n\right)$.

б) $f(x) = \sin x\cos 2x + \sin 2x\cos x = \sin 3x$.

$g(x) = 35 - 3x$. $f'(x) = 3\cos 3x$. $g'(x) = -3$.

$3\cos 3x \le -3$. $\cos 3x \le -1$. $3x = \pi + 2\pi n$. $x = \frac{\pi}{3} + \frac{2\pi n}{3}$.

в) $f(x) = \sin^2 x - \cos^2 x = -\cos 2x$. $g(x) = -2x + 9$.
$f'(x) = 2\sin 2x$. $g'(x) = -2$. $2\sin 2x \le -2$. $\sin 2x \le -1$.

$2x = \frac{3\pi}{2} + 2\pi n$. $x = \frac{3\pi}{4} + \pi n$.

г) $f(x) = x\cos x$. $g(x) = \sin x$. $f'(x) = \cos x - x\sin x$. $g'(x) = \cos x$.
$\cos x - x\sin x \le \cos x$. $x\sin x \ge 0$.
1. $x \ge 0$. $\sin x \ge 0$. $x \in [2\pi n;\ \pi + 2\pi n]$. $n = 0, 1, 2 \ldots$
2. $x \le 0$. $\sin x \le 0$. $x \in [-\pi + 2\pi n;\ 2\pi n]$. $n = 0, -1, -2, -3 \ldots$

802. а) $h(x) = x^2 - 3x + 19$. $\alpha = 45°$ $h'(x) = 2x - 3 = \text{tg}45$.
$2x - 3 = 1$. $x = 2$.

б) $\frac{4}{x+2}$ $\alpha = 135$. $\frac{-4}{(x+2)^2} = \text{tg}135$. $4 = (x+2)^2$.

$x = 0$, $x = -4$.

в) $h(x) = 2\sqrt{2x-4}$. $\alpha = 60°$

$h'(x) = \frac{2}{\sqrt{2x-4}} = \text{tg}60$. $2 = \sqrt{3(2x-4)}$.

$4 = 6x - 12$. $6x = 16$. $x = \frac{8}{3} = 2\frac{2}{3}$.

г) $h(x) = \sin\left(4x - \frac{\pi}{3}\right)$. $\alpha = 0$. $h'(x) = \cos\left(4x - \frac{\pi}{3}\right) = 0$.

$4x - \frac{\pi}{3} = \frac{\pi}{2} + \pi n$. $x = \frac{5\pi}{24} + \frac{\pi n}{4}$.

803. а) $y_1 = x^7$, $y_2 = x^8$, $x = a$. $y_1' = 7x^6$, $y_2' = 8x^7$.

$7a^6 = 8a^7$. $a = \frac{7}{8}$.

б) $y = \sqrt{x}$, $y = 2\sqrt{x+8}$, $x = a$. $y' = \frac{1}{2\sqrt{x}}$, $y' = \frac{1}{\sqrt{x+8}}$.

$2\sqrt{a} = \sqrt{a+8}$. $4a = a + 8$. $a = \frac{8}{3} = 2\frac{2}{3}$.

804. а) $f'(x) = 6(2x-1)^2$. $f(x) = (2x-1)^3 + c$.
б) $f'(x) = -20(4-5x)^3$. $f(x) = (4-5x)^4 + c$.

805. а) $f'(x) = -\frac{2}{(2x+3)^2}$. $f(x) = \frac{1}{2x+3} + c$.

б) $f'(x) = \frac{5}{2\sqrt{5x-7}}$, $f(x) = \sqrt{5x-7} + c$.

806. а) $f'(x) = \sin\left(3x - \frac{\pi}{3}\right)$, $f(x) = -\frac{1}{3}\cos\left(3x - \frac{\pi}{3}\right) + c$;

б) $f'(x) = \frac{4}{\cos^2(5x-1)}$, $f(x) = \frac{4}{5}\text{tg}(5x-1) + c$.

807. $y = ax^2 + bx + c$.
по рисунку видно, что $b = 0$, $a < 0$.
$y = ax^2 + c$. $y'(x) = 2ax$.
A(–100; 0) B(100; 0).

$y'(100) = tg(-15)$. $200a = -tg15$. $a = -\frac{tg15}{200}x^2 + c$.

подставим точку (100; 0).
$0 = -50tg15 + c$. $c = 50tg15$.

Ответ: $y = -\frac{tg15}{200}x^2 + 50tg15$.

808. а) $y = 4x^2 - |a|x$. $x(4x - |a|) = 0$. $x_1 = 0$. $x_2 = \frac{|a|}{4}$.

т.к. оси параболы направлены вверх и т.к. $x_2 \geq x_1$
то
1) $y'(x_1) = -tg60°$ и $y'(x_2) = tg60°$.
2) $y'(x_1) = -tg30°$ и $y'(x_2) = tg30°$.

1) $y'(0) = -|a| = -tg60°$. $a = \pm\sqrt{3}$. $y'\frac{|a|}{4} = |a| = tg60°$. $a = \pm\sqrt{3}$.

2) $y'(0) = -|a| = -tg30°$. $a = \pm\frac{\sqrt{3}}{3}$.

Ответ: $a = \pm\frac{\sqrt{3}}{3}$, $a = \pm\sqrt{3}$.

б) $y = x^2 + |a|x$. $y' = 2x + |a|$. $x(4x - |a|) = 0$. $x_1 = 0$. $x_2 = -|a|$.
т.к. оси параболы направлены вверх и $x_1 \geq x_2$, то

1) $y'(x_2) = -tg\frac{3\pi}{8}$.

2) $y'(x_2) = -tg\frac{\pi}{8}$. $tg\frac{3\pi}{4} = \frac{2tg\frac{3\pi}{8}}{1 - tg\frac{3\pi}{8}}$. $tg^2\frac{3\pi}{8} - 2tg\frac{3\pi}{8} - 1 = 0$.

$tg\frac{3\pi}{8} = 1 \pm \sqrt{2}$, но т.к. $0 < \frac{3\pi}{8} < \frac{\pi}{2}$, то $tg\frac{3\pi}{8} = 1 + \sqrt{2}$.

$tg\frac{\pi}{4} = \frac{2tg\frac{\pi}{8}}{1 - tg^2\frac{\pi}{8}}$. $tg^2\frac{\pi}{8} + 2tg\frac{\pi}{8} - 1 = 0$.

$tg\frac{\pi}{8} = -1 \pm \sqrt{2}$, но т.к. $0 < \frac{\pi}{8} < \frac{\pi}{2}$, то $tg\frac{\pi}{8} = -1 + \sqrt{2}$.

$\Rightarrow$ 1) $y'(x_2) = -1 - \sqrt{2}$. $-|a| = -1 - \sqrt{2}$. $a = \pm(1 + \sqrt{2})$.
2) $y'(x_2) = 1 - \sqrt{2}$. $a = \pm(\sqrt{2} - 1)$.
Ответ: $a = \pm\sqrt{2} \pm 1$.

809. а) a: $tg\alpha = 0$. b: $tg\alpha < 0$. c: $tg\alpha > 0$.
б) a: $tg\alpha < 0$. b: $tg\alpha < 0$. c: $tg\alpha > 0$.

810. а) $y' = 0$ при $x = 0$, $x = 3{,}5$; y' не существует при $x = –1$.
б) $y' = 0$ при $x = –4$, $x = –1{,}5$; y' не существует при $x = 4$.
в) $y' = 0$ при $x = –4$; y' не существует при $x = –2$.
г) $y' \neq 0$ при $x \in R$.

811. а) $f(x) = 4 + x^2$, $a = 2$. $f'(x) = 2x$ $f'(a) = 4 \Rightarrow$ острый.
б) $f(x) = 1 - \frac{1}{x}$, $a = 3$. $f'(x) = \frac{1}{x^2}$ $f'(a) = \frac{1}{9} \Rightarrow$ острый.
в) $f(x) = (1 – x)^3$, $a = –3$. $f'(x)=–3(1–x)^2$ $f'(a) = – 48 \Rightarrow$ тупой.
г) $f(x) = 2x – x^2$, $a = 1$. $f'(x) = 2 – 2x$ $f'(a) = 0 \Rightarrow$ острый.

812. $y = 1 – x^2$ $y'(x) = –2x$.
а) $A(0; 1)$ $y'(0) = 0$ $\Rightarrow tg\alpha = 0$.
б) $B(2; –3)$ $y'(2) = –4$ $\Rightarrow tg\alpha = –4$.
в) $C\left(\frac{1}{2};\frac{3}{4}\right)$ $y'\left(\frac{1}{2}\right)=–1$ $\Rightarrow tg\alpha = –1$.
г) $D(–1; 0)$ $y'(–1) = 2$ $\Rightarrow tg\alpha = 2$.

813. а) $f(x) = \frac{1}{2}x^2$, $a = 1$. $f'(x) = x$ $f'(a) = 1$, $tg\alpha = 1$.
б) $f(x) = –2x^3$, $a = 2$. $f'(x) = –6x^2$ $f'(a) = –24$, $tg\alpha = –24$.
в) $f(x) = 0{,}25x^4$, $a = –1$. $f'(x) = x^3$ $f'(a) = –1$, $tg\alpha = –1$.
г) $f(x) = –x^5$, $a = 1$. $f'(x) = –5x^4$ $f'(a) = –5$, $tg\alpha = –5$.

814.
а) $f(x) = x^3 – 2x^2 +3$, $a = –1$. $f'(x) = 3x^2 – 4x$ $f'(a) = 7$, $tg\alpha = 7$.
б) $f(x) = \frac{x-1}{x+3}$, $a = 1$. $f'(x) = \frac{x+3-x+1}{(x+3)^2} = \frac{4}{(x+3)^2}$

$f'(a) = \frac{1}{4}$, $tg\alpha = \frac{1}{4}$.
в) $f(x) = x^4 – 7x^3 +12x – 45$, $a = 0$.
$f'(x) = 4x^3 – 21x^2 + 12$ $f'(a) = 12$, $tg\alpha = 12$.
г) $f(x) = \frac{2x-1}{x+1}$, $a = 1$.
$f'(x) = \frac{2x+2-2x+1}{(x+1)^2} = \frac{3}{(x+1)^2}$, $f'(a) = \frac{3}{4}$, $tg\alpha = \frac{3}{4}$.

815. а) $f(x) = \sqrt{x-7}$, $a = 8$.
$f'(x) = \frac{1}{2\sqrt{x-7}}$ $f'(a) = \frac{1}{2}$ $tg\alpha = \frac{1}{2}$.
б) $f(x) = \sqrt{4-5x}$, $a = 0$.

$f'(x) = -\frac{5}{2\sqrt{4-5x}}$ $\quad f'(a) = -\frac{5}{4}$ $\quad tg\alpha = -\frac{5}{4}$.

в) $f(x) = \sqrt{10+x}$, $\quad a = -5$.

$f'(x) = \frac{1}{\sqrt{x+10}}$ $\quad f'(a) = \frac{1}{\sqrt{5}}$ $\quad tg\alpha = \frac{\sqrt{5}}{5}$.

г) $f(x) = \sqrt{3,5-0,5x}$, $\quad a = -1$.

$f'(x) = -\frac{1}{4\sqrt{3,5-0,5x}}$ $\quad f'(a) = -\frac{1}{8}$ $\quad tg\alpha = -\frac{1}{8}$.

816. а) $f(x) = \sin x$, $\quad a = 0$.

$f'(x) = \cos x$ $\quad f'(a) = 1$ $\quad tg\alpha = 1$.

б) $f(x) = tg2x$, $\quad a = \frac{\pi}{8}$.

$f'(x) = \frac{2}{\cos^2 2x}$ $\quad f'(a) = 4$ $\quad tg\alpha = 4$.

в) $f(x) = \cos 3x$, $\quad a = \frac{\pi}{2}$.

$f'(x) = -3\sin 3x$ $\quad f'(a) = 3$ $\quad tg\alpha = 3$.

г) $f(x) = ctgx$, $\quad a = \frac{\pi}{3}$.

$f'(x) = -\frac{1}{\sin^2 x}$ $\quad f'(a) = -\frac{4}{3}$ $\quad tg\alpha = -\frac{4}{3}$.

817. а) $f(x) = x^2$, $\quad a = 0,5$.

$f'(x) = 2x$ $\quad f'(a) = 1$ $\quad \alpha = \frac{\pi}{4}$.

б) $f(x) = -3x^3$, $\quad a = \frac{1}{3}$.

$f'(x) = -9x^2$ $\quad f'(a) = -1$ $\quad \alpha = \frac{3\pi}{4}$.

в) $f(x) = 0,2x^5$, $\quad a = -1$.

$f'(x) = x^4$ $\quad f'(a) = 1$ $\quad \alpha = \frac{\pi}{4}$.

г) $f(x) = -0,25x^4$, $\quad a = 0$.

$f'(x) = -x^3$ $\quad f'(a) = 0$ $\quad \alpha = 0$.

818. а) $f(x) = x^3 - 3x^2 + 2x - 7$, $\quad a = 1$.

$f'(x) = 3x^2 - 6x + 2$, $\quad f'(a) = -1$, $\quad \alpha = \frac{3\pi}{4}$.

б) $f(x) = -7x^3 + 10x^2 + x - 12$, $\quad a = 0$.

$f'(x) = -21x^2 + 20x + 1$ $\quad f'(a) = 1$ $\quad \alpha = \frac{\pi}{4}$.

819. а) $f(x)=\frac{2x-1}{3-2x}$, $a=-\frac{1}{2}$. $f'(x)=\frac{6-4x+4x-2}{(3-2x)^2}=\frac{4}{(3-2x)^2}$

$f'(a)=\frac{1}{4}$ $\alpha=\operatorname{arctg}\frac{1}{4}$.

б) $f(x)=\frac{x-1}{x-2}$, $a=1$. $f'(x)=\frac{x-2-x+1}{(x-2)^2}=-\frac{1}{(x-2)^2}$

$f'(a)=-1$ $\alpha=\frac{3\pi}{4}$.

820. а) $f(x)=\sqrt{6x+7}$, $a=3\frac{1}{3}$. $f'(x)=\frac{3}{\sqrt{6x+7}}$

$f'(a)=\frac{1}{\sqrt{3}}$ $\alpha=\frac{\pi}{4}$.

б) $f(x)=\sqrt{5-2x}$, $a=2$. $f'(x)=-\frac{1}{\sqrt{5-2x}}$

$f'(a)=-1$ $\alpha=\frac{3\pi}{4}$.

821. а) $f(x)=\sqrt{3}\cos\frac{x}{3}$, $a=\frac{3\pi}{2}$. $f'(x)=-\frac{\sqrt{3}}{3}\sin\frac{x}{3}$

$f'(a)=-\frac{\sqrt{3}}{3}$ $\alpha=\frac{5\pi}{6}$.

б) $f(x)=\frac{1}{2}\sin 2x$, $a=\frac{\pi}{2}$. $f'(x)=\cos 2x$

$f'(a)=-1$ $\alpha=\frac{3\pi}{4}$.

822.

а) $f(x)=-\frac{2}{3}\operatorname{tg}x+\sin\frac{x}{3}$, $a=3\pi$. $f'(x)=-\frac{2}{3\cos^2 x}+\frac{1}{3}\cos\frac{x}{3}$

$f'(a)=-\frac{2}{3}-\frac{1}{3}=-1$ $\alpha=\frac{3\pi}{4}$.

б) $f(x)=\cos x+\frac{\sqrt{3}}{4}\operatorname{ctg}\frac{x}{2}$, $a=\frac{\pi}{3}$. $f'(x)=-\sin x-\frac{\sqrt{3}}{8\sin^2 x/2}$

$f'(a)=-\frac{\sqrt{3}}{2}-\frac{\sqrt{3}}{2}=-\sqrt{3}$, $\alpha=\frac{2\pi}{3}$.

823. а) $f(x)=x^2$, $a=3$, $f'(x)=2x$
$f'(a)=6$, $f(a)=9$, $y=9+6(x-3)=6x-9$.
б) $f(x)=2-x-x^3$, $a=0$, $f'(x)=-1-3x^2$
$f'(a)=-1$, $f(a)=2$, $y=2-x$.

в) $f(x) = x^3$, $a = 1$, $f'(x) = 3x^2$

$f'(a) = 3$, $f(a) = 1$, $y = 1 + 3(x - 1) = 3x - 2$.

г) $f(x) = x^2 - 3x + 5$, $a = -1$, $f(a) = 1 + 3 + 5 = 9$.

$f'(x) = 2x - 3$, $f'(a) = -5$, $y = 9 - 5(x + 1) = -5x + 4$.

824. а) $f(x) = \frac{3x-2}{3-x}$, $a = 2$. $f(a) = 4$.

$f'(x) = \frac{9-3x+3x-2}{(3-x)^2} = \frac{7}{(x-3)^2}$ $f'(a) = 7$.

$y = 4 + 7(x - 2) = 7x - 10$.

б) $f(x) = \frac{2x-5}{5-x}$, $a = 4$. $f(a) = 3$.

$f'(x) = \frac{10-2x+2x-5}{(5-x)^2} = \frac{5}{(5-x)^2}$ $f'(a) = 5$.

$y = 3 + 5(x - 4) = 5x - 17$.

825. а) $f(x) = \frac{1}{(x+2)^3}$, $a = -3$. $f(a) = -1$.

$f'(x) = -\frac{3}{(x+2)^4}$ $f'(a) = -3$. $y = -1 - 3(x + 3) = -3x - 10$.

б) $f(x) = \frac{1}{4(2x-1)^2}$, $a = 1$. $f'(x) = \frac{-4}{4(2x-1)^3} = -\frac{1}{(2x-1)^3}$

$f(a) = \frac{1}{4}$. $f'(a) = -1$. $y = \frac{1}{4} - x + 1 = -x + \frac{5}{4}$.

826. а) $f(x) = 2\sqrt{3x-5}$, $a = 2$, $f(a) = 2$;

$f'(x) = \frac{3}{\sqrt{3x-5}}$. $f'(a) = 3$. $y = 2 + 3(x - 2) = 3x - 4$.

б) $f(x) = \sqrt{7-2x}$, $a = 3$, $f(a) = 1$;

$f'(x) = -\frac{1}{\sqrt{7-2x}}$. $f'(a) = -1$. $y = 1 - x + 3 = -x + 4$.

827. а) $f(x) = \cos\frac{x}{3}$, $a = 0$, $f(a) = 1$;

$f'(x) = -\frac{1}{3}\sin\frac{x}{3}$, $f'(a) = 0$. $y = 1$.

б) $f(x) = \text{ctg}2x$, $a = \frac{\pi}{4}$, $f(a) = 0$;

$f'(x) = -\frac{2}{\sin^2 2x}$. $f'(a) = -2$. $y = -2\left(x - \frac{\pi}{4}\right) = \frac{\pi}{2} - 2x$.

в) $f(x) = \sin 2x$, $a = \frac{\pi}{4}$, $f(a) = 1$;

$f'(x) = 2\cos 2x$ $f'(a) = 0$; $y = 1$.

г) $f(x) = 2\text{tg}\frac{x}{3}$, $a = 0$, $f(a) = 0$;

$f'(x) = \frac{2}{3\cos^2\frac{x}{3}}$, $f'(a) = \frac{3}{2}$; $y = \frac{2}{3}x$.

828.
$y = 9 - x^2$; $9 - x^2 = 0$, $x = \pm 3$. $y' = -2x$.
$y'(3) = -6$, $y'(-3) = 6$. $y = 6(x + 3) = 6x + 18$.
$y = -6(x - 3) = 18 - 6x$.

829. $y = x^2 - 3x$; $x^2 - 3x - 4 = 0$, $x = 4$, $x = -1$.
$y' = 2x - 3$, $y'(4) = 8 - 3 = 5$, $y'(-1) = -5$;
$y = 4 + 5(x - 4) = 5x - 16$;
$y = 4 - 5(x + 1) = -5x - 1$.

830. $y = 3x^3 - 4x^2 + 1$, $y' = 9x^2 - 8x = 1$, $9x^2 - 8x - 1 = 0$.

$x = \frac{4 \pm 5}{9}$, $x = 1$, $x = -\frac{1}{9}$.

$y(1) = 0$, $y\left(-\frac{1}{9}\right) = -\frac{1}{243} - \frac{4}{81} + 1 = \frac{230}{243}$. $y = x - 1$.

$y = \frac{230}{243} + \left(x + \frac{1}{9}\right) = x + \frac{257}{243}$.

831. $y = x^2$, $y' = 2x$.
$y = x_0^2 + 2x_0\left(x - x_o\right) = 2x_0x - x_0^2$.

а) $y = 2x + 1$, $x_0 = 1$, $y_0 = 1$;

б) $y = -\frac{1}{2}x + 5$, $x_0 = -\frac{1}{4}$, $y_0 = \frac{1}{16}$;

в) $\frac{3}{4}x - 2 = y$, $x_0 = \frac{3}{8}$, $y_0 = \frac{9}{64}$

г) $y = -x + 5$, $x_0 = -\frac{1}{2}$, $y_0 = \frac{1}{4}$.

832. а) $f(x) = \frac{x^3}{3} - 3x^2 + 10x - 4$, $y = x + 3$. $f'(x) = x^2 - 6x + 10$.

$y = \frac{x_0^3}{3} - 3x_0^2 + 10x_0 - 4 + \left(x_0^2 - 6x_0 + 10\right)\left(x - x_0\right)$.

$x_0^2 - 6x_0 + 10 = 1$. $x_0^2 - 6x_0 + 9 = 0$. $x_0 = 3$.

б) $f(x) = \frac{x^4}{4} - x^2 + 8$, $y = 0$. $f'(x) = x^3 - 2x$,

$y = \frac{x_0^4}{4} - x_0^2 + 8 + \left(x_0^2 - 2x_0\right)\left(x - x_0\right)$;

$x_0^3 - 2x_0^2 = 0$, $x_0 = 0$, $x_0^2 = 2$, $x_0 = \pm\sqrt{2}$;

в) $f(x) = \frac{x^3}{3} - x^2 + 2x - 7$, $y = x - 3$, $f'(x) = x^2 - 2x + 2$,

$y = \frac{x_0^3}{3} - x_0^2 + 2x_0 - 7 + \left(x_0^2 - 2x_0 + 2\right)\left(x - x_0\right)$,

$x_0^2 - 2x_0 + 2 = 1$, $x_0 = 1$;

г) $f(x) = \frac{5x^4}{4} - x^3 + 6$, $y = 2$, $f'(x) = 5x^3 - 3x^2$,

$x_0 = 0$, $5x_0^3 - 3 = 0$, $x_0 = \frac{3}{5}$.

833.

а) $f(x) = \sin x$, $y = -x$, $f'(x) = \cos x$,
$\cos x_0 = -1$, $x_0 = \pi + 2\pi n$;
б) $f(x) = \cos 3x$, $y = 0$. $f'(x) = -3\sin 3x$.

$\sin 3x_0 = 0$, $x_0 = \frac{\pi n}{3}$;

в) $f(x) = \mathrm{tg}\,x$. $y = xv$ $f'(x) = \frac{1}{\cos^2 x}$,

$\frac{1}{\cos^2 x} = 1$, $\cos x = \pm 1$. $x = \pi n$;

г) $f(x) = \sin\frac{x}{2}$, $y = -1$, $f'(x) = \frac{1}{2}\cos\frac{x}{2}$,

$\cos\frac{x}{2} = 0$, $x_0 = \pi + 2\pi n$.

834. $y = \frac{x^3}{3} - 2$. $y' = x^2$. $y = \frac{x_0^3}{3} - 2 + x_0^2\left(x - x_o\right) = x_0 x - \frac{2}{3}x_0^3 - 2$.

а) $y = x - 3$, $x_0^2 = 1$, $x_0 = \pm 1$;

$y = x - \frac{2}{3} - 2 = x - \frac{8}{3}$. $y = x + \frac{2}{3} - 2 = x - 1\frac{1}{3}$.

б) $y = 9x - 5$. $x_0^2 = \pm 3$. $y = 9x - 18 - 2 = 9x - 20$. $y = 9x + 18 - 2 = 9x + 16$.

835. $y = 2 - x$.

а) $y = \frac{x^3}{3} + \frac{5}{2}x^2 + 8$, $y' = x^2 + 5x$. $x_0^2 + 5x_0 = -1$, $x_0^2 + 5x_0 + 1 = 0$.

$x_0 = \frac{-5 \pm \sqrt{21}}{2}$.

$$y=\left(\frac{-5\pm\sqrt{21}}{2}\right)^3\cdot\frac{1}{3}+\frac{5}{2}\left(\frac{-5\pm\sqrt{21}}{2}\right)^2+8+$$

$$+\left(\left(\frac{-5\pm\sqrt{21}}{2}\right)^2+5\left(\frac{-5\pm\sqrt{21}}{2}\right)\right)\left(x-\frac{\pm\sqrt{21}-5}{2}\right).$$

б) $y=\frac{x^3}{3}+x^2-x$. $y=2-x$, $y'=x^2+2x-1$; $x_0^2+2x_0=0$, $x_0=0$,

$x_0=-2$; $y(0)=0$, $y(-2)=\frac{10}{3}$

$y=-x'$. $y=-\frac{8}{3}+4+2-(x+2)=-x+\frac{4}{3}$; $y=0-(x-0)=-x$.

836. а) $y=\frac{3x+1}{x-3}$, $y=2-x$; $y'=\frac{3x-9-3x-1}{(x-3)^2}=-\frac{10}{(x-3)^2}$.

$-\frac{10}{(x_0-3)^2}=-1$, $(x_0-3)^2=10$, $x_0=\pm\sqrt{10}+3$;

$y(x_0)=\frac{3(\pm\sqrt{10}+3)+1}{\pm\sqrt{10}+3-3}=\frac{\pm3\sqrt{10}+10}{\pm10}=3\pm\sqrt{10}$

$y=3\pm\sqrt{10}-(x\mp\sqrt{10}-3)$; $y_1=3+\sqrt{10}-x+\sqrt{10}+3=6+2\sqrt{10}-x$;

$y_2=3-\sqrt{10}-x-\sqrt{10}+3=6-2\sqrt{10}-x$.

б) $y=\frac{x+9}{x+8}$, $y=2-x$, $y'=-\frac{1}{(x+8)^2}$;

$-\frac{1}{(x+8)^2}=-1$. $x_0+8=\pm1$. $x_0=-9$, $x_0=-7$;

$y(-9)=0$; $y(-7)=2$; $y=2-(x+7)=-x-5$, $y=-x-9$.

837. а) $y=-4\sqrt{x+7}$, $y=2-x$, $y'=-\frac{2}{\sqrt{x+7}}$;

$-\frac{2}{\sqrt{x+7}}=-1$, $x_0=-3$, $y(-3)=-8$; $y=-8-(x+3)=-x-11$.

б) $y=\sqrt{1-2x}$, $y=2-x$, $y'=-\frac{1}{\sqrt{1-2x}}$;

$-\frac{1}{\sqrt{1-2x}}=-1$. $x_0=0$. $y(0)=1$. $y=1-x$.

838. а) $0{,}998^5$.

$y=x^5=f(x)$, $x=0{,}998$; $a=1$. $f(a)=1$.

$f'(x)=5x^4$. $f'(a)=f'(1)=5$. $0{,}998^5\approx1-5\cdot0{,}002=0{,}99$.

б) $1{,}03^7\approx1+7\cdot0{,}03=1{,}21$.

839. а) $\sqrt{1,05} \approx 1 + \frac{1}{2} \cdot 0,05 = 1,025.$ б) $\sqrt{3,99} \approx 2 - \frac{1}{2\sqrt{4}} \cdot 0,01 = 1,9975.$

840. а) $f(x) = \frac{1}{\sqrt{3}}x^3 - 3\sqrt{3}x$, $\alpha = 60°$;

$f'(x) = \sqrt{3}x^2 - 3\sqrt{3} = \sqrt{3}$. $x^2 = 4$. $x = \pm 2$.

$f(2) = \frac{8}{\sqrt{3}} - 6\sqrt{3} = -\frac{10}{\sqrt{3}}$. $f(-2) = \frac{10}{\sqrt{3}}$. $y = \pm\frac{10}{\sqrt{3}} + \sqrt{3}(x \pm 2)$.

$y_1 = \sqrt{3}x - 2\sqrt{3} + \frac{10}{\sqrt{3}} = \sqrt{3x} + \frac{4}{\sqrt{3}}$. $y_2 = \sqrt{3}x + 2\sqrt{3} - \frac{10}{\sqrt{3}} = \sqrt{3x} - \frac{4}{\sqrt{3}}$.

б) $f(x) = \frac{4}{\sqrt{3}}x - \frac{\sqrt{3}}{3}x^3$, $\alpha = 30°$;

$f'(x) = \frac{4}{\sqrt{3}} - \sqrt{3}x^2 = \frac{\sqrt{3}}{3}$. $x^2 = \frac{4}{3} - \frac{1}{3}$. $x = \pm 1$.

$f(1) = \sqrt{3}$. $f(-1) = -\sqrt{3}$. $y = \sqrt{3} + \frac{\sqrt{3}}{3}(x-1) = \frac{\sqrt{3}}{3}x + \frac{2\sqrt{3}}{3}$.

$y = -\sqrt{3} + \frac{\sqrt{3}}{3}(x+1) = \frac{\sqrt{3}}{3}x - \frac{2\sqrt{3}}{3}$.

841. а) $y = \frac{3x-1}{x+8}$, $\alpha = 45°$; $y' = \frac{3x+24-3x+1}{(x+8)^2} = \frac{25}{(x+8)^2}$.

$25 = (x + 8)^2$. $x + 8 = \pm 5$. $x = -3$. $x = -13$.

$y(-3) = \frac{-9-1}{5} = -2$. $y(-13) = \frac{-39-1}{-5} = 8$.

$y_1 = -2 + (x + 3) = x + 1$. $y_2 = 8 + (x + 13) = x + 21$. $(-1; 0)$; $(-21; 0)$.

б) $y = \frac{x+4}{x-5}$, $\alpha = 135°$; $y' = -\frac{9}{(x-5)^2}$.

$x - 5 = \pm 3$. $x = 8$. $x = 2$.

$y(8) = \frac{12}{3} = 4$. $y(2) = \frac{6}{-3} = -2$.

$y_1 = 4 - (x - 8) = -x + 12$. $y_2 = -2 - (x - 2) = -x$. $(0; 0)$; $(12; 0)$.

842. а) не до конца написано условие.

843. $y = x^2 + 1$, $y' = 2x$. $y = x_0^2 + 1 + 2x_0(x - x_0) = 2x_0x - x_0^2 + 1$.

а) A $(-1; -2)$. $-2 = -2x_0 - x_0^2 + 1$ $x_0^2 + 2x_0 - 3 = 0$; $x_0 = -3$, $x_0 = 1$;
$y_1 = -6x - 8$; $y_2 = 2x$.

б) A $(0; 0)$. $0 = -x_0^2 + 1$ $x_0 = \pm 1$. $y = 2x$. $y = -2x$.

в) A $(0; -3)$. $-3 = -x_0^2 + 1$ $x_0 = \pm 2$. $y = 4x - 3$. $y = -4x - 3$.

г) A $(-1; 1)$. $1 = -2x_0 - x_0^2 + 1$. $x_0 = 0$. $x_0 = -2$. $y = 1$. $y = -4x - 3$.

844. а) $f(x) = -x^2 - 7x + 8$, B (1; 1); $f'(x) = -2x - 7$.

$y = -x_0^2 - 7x_0 + 8 + (-2x_0 - 7)(x - x_0) = (-2x_0 - 7)x + x_0^2 + 8$;

$x_0 = 0$. $x_0 = 2$. $f(0) = 8$. $f(2) = -10$;

$y = -10 - 11(x - 2) = -11x + 12$; $y = 8 - 7x$

б) $f(x) = -x^2 - 7x + 8$, B (0; 9); $9 = x_0^2 + 8$. $x = \pm 1$.

$f(1) = -1 - 7 + 8 = 0$. $f(-1) = -1 + 7 + 8 = 14$.

$y = 9(x - 1) = -9x + 9$. $y = 14 - 5(x + 1) = -5x + 9$.

845. а) $f(x) = \sqrt{3 - x}$, B (–2; 3),

$f'(x) = -\frac{1}{2\sqrt{3 - x}}$. $3 = \sqrt{3 - x_0} - \frac{1}{2\sqrt{3 - x_0}}(-2 - x_0)$.

$3 = \frac{6 - 2x_0 + 2 + x_0}{2\sqrt{3 - x_0}}$. $6\sqrt{3 - x_0} = 8 - x_0$. $108 - 36x_0 = 64 - 16x_0 + x_0^2$.

$x_0^2 + 20x_0 - 44 = 0$. $x_0 = -22$. $x_0 = 2$. $f(-22) = 5$. $f(2) = 1$.

$y = 5 - \frac{1}{10}(x + 22) = -\frac{x}{10} - \frac{11}{5} + 5 = -\frac{x}{10} + \frac{14}{5}$. $y = 1 - \frac{1}{2}(x - 2) = -\frac{1}{2}x + 2$.

б) $f(x) = \sqrt{3 - x}$, B (4; 0); $0 = \sqrt{3 - x_0} - \frac{1}{2\sqrt{3 - x_0}}(4 - x_0)$;

$6 - 2x_0 - 4 + x_0 = 0$; $x_0 = 2$; $f(2) = 1$; $y = 1 - \frac{1}{2}(x - 2) = -\frac{1}{2}x + 2$.

846. а) $f(x) = \sqrt{4x - 3}$, B (2; 3); $f'(x) = \frac{1}{2\sqrt{4x - 3}}$.

$y = \frac{2}{\sqrt{4x_0 - 3}}(x - x_0) + \sqrt{4x_0 - 3}$. $3\sqrt{4x_0 - 3} = 2(2 - x_0) + 4x_0 - 3$.

$36x_0 - 27 = 4x_0^2 + 4x_0 + 1$. $4x_0^2 - 32x_0 + 28 = 0$. $x_0^2 - 8x_0 + 7 = 0$.

$x_0 = 7$. $x_0 = 1$. $y = \frac{2}{5}(x - 7) + 5 = \frac{2}{5}x + \frac{11}{5}$. $y = 2x - 2 + 1 = 2x - 1$.

б) $f(x) = \sqrt{2x + 1}$, B (1; 2); $f'(x) = \frac{1}{\sqrt{2x + 1}}$.

$y = \frac{x - x_0}{\sqrt{2x_0 + 1}} + \sqrt{2x_0 + 1}$. $2\sqrt{2x_0 + 1} = 1 - x_0 + 2x_0 + 1$.

$8x_0 + 4 = x_0^2 + 4x_0 + 4$. $x_0^2 - 4x_0 = 0$. $x_0 = 0$, $x_0 = 4$. $y = x + 1$.

$y = \frac{x - 4}{3} + 3 = \frac{x}{3} + \frac{5}{3}$.

847. а) $y = \cos 7x + 7\cos x$. $y' = -7\sin 7x - 7\sin x$.

$y\left(\frac{\pi}{6}\right) = -\frac{7\sqrt{3}}{2} + \frac{7\sqrt{3}}{2} = 0$. $y'\left(\frac{\pi}{6}\right) = \frac{7}{2} - \frac{7}{2} = 0$.

$y = (-7\sin7a - 7\sin a)(x - a) + \cos7a + 7\cos a.$

$-7\sin7a - 7\sin a = 0.$ $\sin4a\cos3a = 0.$ $\sin4a = 0.$

$a = \frac{\pi n}{4}.$ $\cos3a = 0.$ $3a = \frac{\pi}{2} + \pi n,$ $a = \frac{\pi}{6} + \frac{\pi n}{3}.$

б) $y_1 = 2 - 14\sin3x.$ $y_2 = 6\sin7x.$

$y_1' = -42\cos3x.$ $y_2' = 42\cos7x.$

$y = 2 - 14\sin3a - 42\cos3a(x - a).$ $y = 6\sin7a + 42\cos7a(x - a).$

$42\cos7a = -42\cos3a.$ $\cos7a + \cos3a = 0.$ $\cos5a\cos2a = 0.$

$5a = \frac{\pi}{2} + \pi n.$ $a = \frac{\pi}{10} + \frac{\pi n}{5}.$ $a = \frac{\pi}{4} + \frac{\pi n}{2}.$

848. а) $y = \frac{1}{x^2},$ $\left|\frac{1}{2}xy\right| = 2{,}25.$ $x > 0.$

$y' = -\frac{2}{x^3}.$ $y = \frac{2}{x_0^3}(x_0 - x) + \frac{1}{x_0^2} = \frac{3}{x_0^2} - \frac{2x}{x_0^3}.$ $\begin{cases} y = \frac{3}{x_0^2} - \frac{2x}{x_0^3} \\ xy = \frac{9}{2} \end{cases}$

при $x = 0$ $y = \frac{3}{x_0^2}.$

при $y = 0$ $x = \frac{3x_0}{2}.$

$\frac{3}{x_0^2} \cdot \frac{3x_0}{2} = \frac{9}{2}.$ $\frac{1}{x_0} = 1.$ $x_0 = 1.$ $y = 3 - 2x.$

б) $y = \frac{1}{x^2},$ $x < 0,$ $-\frac{1}{2}xy = \frac{9}{8};$ $y = \frac{3}{x_0^2} - \frac{2x}{x_0^3}.$

при $x = 0$ $y = \frac{3}{x_0^2}.$

при $y = 0$ $x = \frac{3x_0}{2}.$

$\frac{3 \cdot 3}{2} \cdot \frac{1}{x_0} = -\frac{9}{4}.$ $\frac{1}{x_0} = -\frac{1}{2}.$ $x_0 = -2.$ $y = \frac{3}{4} + \frac{2x}{8} = \frac{3 + x}{4}.$

849. а) $y = x^3,$ $x > 0,$ $\left|\frac{1}{2}xy\right| = \frac{2}{3},$

но т.к. это кубическая парабола, то $xy = -\frac{4}{3}.$

$y = x_0^3 + 3x_0^2(x - x_0) = -2x_0^3 + 3x_0^2.$

при $x = 0$ $y = -2x_0^3.$

при $y = 0$ $x = \frac{2x_0^3}{3x_0^3} = \frac{2}{3}x_0$

$xy = -\frac{4}{3}x_0^4 = -\frac{4}{3}$. $x_0^4 = 1$, $x_0 = \pm 1$, но $x_0 > 0 \Rightarrow x_0 = 1$.

$y = 1 + 3(x - 1) = 3x - 2$.

б) $y = x^3$, $x < 0 \Rightarrow xy = -\frac{27}{4}$.

$y = -x_0^3 + 3x_0^2$. $xy = -\frac{4}{3}x_0^4 = -\frac{27}{4}$.

$x_0^4 = \left(\frac{3}{2}\right)^4$, $x_0 = \pm\frac{3}{2}$, но $x_0 < 0 \Rightarrow x_0 = -\frac{3}{2}$. $y = \frac{4}{3}\cdot\frac{81}{16} + 3x\cdot\frac{9}{4} = \frac{27}{4} + \frac{27}{4}x$.

850. а) $y = 3 - \frac{1}{2}x^2$,   B (0; z); $y' = -x$.

$y = 3 - \frac{1}{2}x_0^2 - x_0(x - x_0) = -x_0x + \frac{1}{2}x_0^2 + 3$. $z = \frac{1}{2}x_0^2 + 3$.

Т.к. график симметричен относительно Oy $\Rightarrow$ tg угла наклона касательной = ±1.

$y' = -x_0$. $x_0 = \pm 1$. $z = \frac{1}{2}x_0^2 + 3 = \frac{1}{2} + 3 = 3{,}5$. B (0; 3,5).

б) $y = 0{,}5x^2 - 2{,}5$, $\alpha = 90°$; $y' = x$.

$y = \frac{1}{2}x_0^2 - 2{,}5 + x_0(x - x_0) = x_0x - \frac{1}{2}x_0^2 - \frac{5}{2}$.

Рассуждения аналогичны пред. задаче.

$y' = x_0 = \pm 1$. $y = -x - 3$. $y = x - 3$.

851. а) $y = \frac{\sqrt{3}}{2}x^2 + \frac{\sqrt{3}}{2}$, $\alpha = 60°$; $y' = \sqrt{3}\,x$.

$y = \frac{\sqrt{3}}{2}\left(x_0^2 + 1\right) + \sqrt{3}x_0(x - x_0)$.

Рассуждения аналогичны пред. задаче.

1. $y' = \sqrt{3}\,x_0 = \pm\mathrm{tg}60°$. $\sqrt{3}\,x_0 = \pm\sqrt{3}$. $x_0 = \pm 1$. $y = \frac{\sqrt{3}}{2}\cdot 2 - \sqrt{3} = 0$. B (0; 0).

2. $y' = \sqrt{3}\,x_0 = \pm\mathrm{tg}\,30°$. $\sqrt{3}\,x_0 = \pm\frac{\sqrt{3}}{3}$; $x_0 = \pm\frac{1}{3}$; $y = \frac{\sqrt{3}}{2}\left(1 - \frac{1}{9}\right) = \frac{4\sqrt{3}}{9}$;

B(0, $\frac{4\sqrt{3}}{9}$).

б) $y = \frac{\sqrt{3}}{6}\left(1 - x^2\right)$, $\alpha = 120°$; $y' = -\frac{\sqrt{3}}{3}x$.

$y = -\frac{\sqrt{3}}{3}x_0x + \frac{\sqrt{3}}{3}x_0^2 - \frac{\sqrt{3}x_0^2}{6} + \frac{\sqrt{3}}{6} = -\frac{\sqrt{3}}{3}x_0x + \frac{\sqrt{3}}{6}x_0^2 + \frac{\sqrt{3}}{6}$.

1. $y' = -\frac{\sqrt{3}}{3}x_0 = \pm\mathrm{tg}60°$. $\frac{\sqrt{3}}{3}x_0 = \pm\sqrt{3}$. $x_0 = \pm 3$.

$y = -\sqrt{3}x + \frac{3\sqrt{3}}{2} + \frac{\sqrt{3}}{6} = -\sqrt{3}x + \frac{5\sqrt{3}}{3}$. $y = \sqrt{3}x + \frac{5\sqrt{3}}{3}$.

2. $y' = -\frac{\sqrt{3}}{3}$ $x_0 = \pm tg30°$. $x_0 = \pm 1$. $y = \frac{\sqrt{3}}{3}x + \frac{\sqrt{3}}{3}$. $y = -\frac{\sqrt{3}}{3}x + \frac{\sqrt{3}}{3}$.

Ответ: $y = \pm\frac{\sqrt{3}}{3}(x \pm 1)$. $y = \pm\frac{\sqrt{3}}{3}\left(x \pm \frac{5}{3}\right)$.

852. а) $y = x^2 - (2x - 6)$, $x = 5$, $x = -5$.
1. $y = x^2 - 2x + 6$, $x = 5$.
$y' = 2x - 2$. $y = 25 - 10 + 6 + (10 - 2)(x - 5) = 8x - 10$.
2. $y = x^2 + 2x - 6$, $x = -5$.
$y' = 2x + 2$. $y = 25 - 10 - 6 + (-10 + 2)(x + 5) = -8x - 31$.

$8x - 10 = -8x - 31$; $x = -\frac{3}{4}$. $\left(-\frac{3}{4};-25\right)$.

б) $y = x^3 + (x - 1)$, $x = 2$, $x = -2$.
1. $y = x^3 + x - 1$, $x = 2$. $y' = 3x^2 + 1$,
$y = 9 + 13(x - 2) = 13x - 17$.
2. $y = x^3 - x + 1$, $x = -2$. $y' = 3x^2 - 1$.
$y = -5 + 11(x + 2) = 11x + 17$.
$13x - 17 = 11x + 17$; $x = 17$.
(17, 204).

853. а) $y = x^3 - px$, $x = 1$, A (2; 3);
$y' = 3x^2 - p$. $y(1) = 1 - p$. $y'(1) = 3 - p$.
$y = 1 - p + (3 - p)(x - 1) = (3 - p)x + p - 3 - p + 1 = (3 - p)x - 2$.

$3 = 2(3 - p) - 2$. $5 = -2p + 6$. $p = \frac{1}{2}$.

б) $y = x^3 + px^2$, $x = 1$, A (3; 2)
$y' = 3x^2 + 2px$. $y(1) = 1 + p$. $y'(1) = 3 + 2p$. $y = 1 + p + (3 + 2p)(x - 1)$.
$2 = 1 + p + (3 + 2p)(3 - 1)$, $5p = -5$; $p = -1$.

§ 35. Применение производной для исследования функций на монотонность и экстремумы

854. а) $f'(a) > 0$. $f'(b) > 0$. $f'(c) < 0$. $f'(d) > 0$.
б) $f'(a) > 0$. $f'(b) = 0$. $f'(c) = 0$. $f'(d) > 0$.

855. а) ф-ция возрастает: $\left(-\infty;\frac{c+b}{2}\right) \cup \left[d - \frac{c-b}{2};+\infty\right)$.

убывает: $\left(\frac{c+b}{2};d - \frac{c-b}{2}\right]$

б) возрастает: $(-\infty;b] \cup [c;+\infty)$; убывает: $[b;c]$

856. а) убывает: $[-2;2]$; возрастает: $(+\infty;-2] \cup [2;+\infty)$

б) убывает: $[-4;0] \cup [3;+\infty)$; возрастает: $(-\infty;-4] \cup [0;3]$

в) убывает: $[-4,5;+\infty)$; возрастает: $(-\infty;-4,5]$

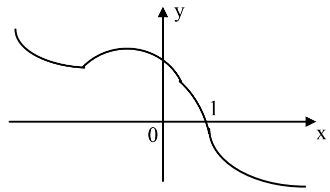
г) убывает: $(-\infty;-2,5]\cup[2,5;+\infty)$; возрастает: $[-2,5;2,5]$

857. в)

858. для ф-ции g(x).

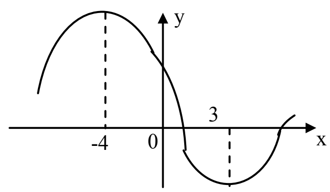
859. а) f(x); б) h(x)

860.

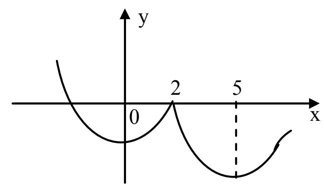


861.

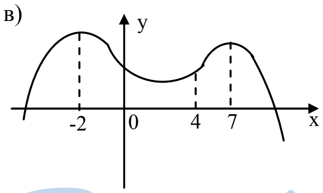
а)



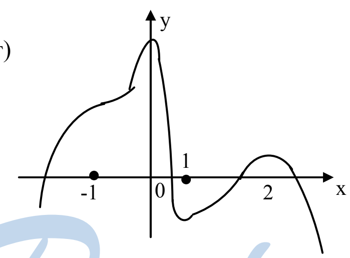
б)



в)



г)



862. а) $y = \cos x + 2x$.

$y' = -\sin x + 2$ $\qquad$ $y' > 0$ при любых x.

б) $y = x^5 + 3x^3 + 7x + 4$.

$y' = 5x^4 + 3x^2 + 7$. $\qquad$ $y' > 0$ при любых x.

в) $y = \sin x + x^3 + x$.

$y' = \cos x + 3x^2 + 1$. $\qquad$ $y' > 0$ при любых x.

г) $y = x^5 + 4x^3 + 8x - 8$.

$y' = 5x^4 + 12x^2 + 8$. $\qquad$ $y' > 0$ при любых x.

863. а) $y = \sin 2x - 3x$.

$y' = 2\cos 2x - 3$. $\qquad$ $y < 0$ при любых x.

б) $y = \cos 3x + 4x$. $\qquad$ $y' = -3\cos 3x + 4$.

$y' > 0$ при $\forall\, x \Rightarrow$ утверждение неверно.

864. а) $y = x^5 + 6x^3 - 7$. $y' = 5x^4 + 18x^2 \ge 0$, функция не убывает
б) $y = \sin x - 2x - 15$. $y' = \cos x - 2 < 0$, монотонно убывает.
в) $y = x - \cos x + 8$. $y' = 1 + \sin x \ge 0$, функция не убывает.
г) $y = 11 - 5x - x^3$. $y' = -5 - 3x^2 \le 0$, функция не возрастает.
865. а) $y = x^2 - 5x + 4$. $y' = 2x - 5$.
при $x \ge \frac{5}{2}$ ф-ция возрастает.
при $x \le \frac{5}{2}$ ф-ция убывает.
б) $y = 5x^2 + 15x - 1$. $y' = 10x + 15$.
при $x \ge -\frac{3}{2}$ ф-ция возрастает.
при $x \le -\frac{3}{2}$ ф-ция убывает.
в) $y = -x^2 + 8x - 7$. $y' = -2x + 8$.
при $x \ge 4$ ф-ция возрастает.
при $x \le 4$ ф-ция убывает.
г) $y = x^2 - x$. $y' = 2x - 1$.
при $x \ge \frac{1}{2}$ ф-ция возрастает.
при $x \le \frac{1}{2}$ ф-ция убывает.
866. а) $y = x^3 + 2x$. $y' = 3x^2 + 2$.
возрастает при любых x.
б) $y = 60 + 45x - 3x^2 - x^3$. $y' = 45 - 6x - 3x^2$. $-3(x^2 + 2x - 15) = 0$.
$x \in [-5; 3]$ возрастает. $x \in (-\infty; -5] \cup [3; +\infty)$ убывает.
в) $y = 2x^3 - 3x^2 - 36x + 40$. $y' = 6x^2 - 6x - 36 = 6(x^2 - x - 6) = 0$.
$x \in [-2; 3]$ убывает. $x \in (-\infty; -2] \cup [3; +\infty)$ возрастает.
г) $y = -x^5 + 5x$. $y' = 5x^4 + 5 = -5(x^4 - 1)$.
$x \in [-1; 1]$ возрастает. $x \in (-\infty; -1] \cup [1; +\infty)$ убывает.
867. а) $y = x^4 - 2x^2 - 3$. $y' = 4x^3 - 4x = 4x(x^2 - 1) = 4x(x - 1)(x + 1)$.
$x \in (-\infty; -1] \cup [0; 1]$ убывает. $x \in [-1; 0] \cup [1; +\infty)$ возрастает.
б) $y = -x^5 - x$. $y' = -5x^4 - 1 = -(5x^4 + 1) < 0$. убывает при всех x.
в) $y = -3x^4 + 4x^3 - 15$. $y' = -12x^3 + 12x^2 = -12x^2(x - 1)$.
$x \in (-\infty; 1]$ возрастает. $x \ge 1$ убывает.
г) $y = 5x^5 - 1$. $y' = 25x^4$. возрастает.
868. а) $y = \frac{1}{x+3}$; $y' = -\frac{1}{(x+3)^2}$. убывает на всей ОДЗ ($x \ne -3$).
б) $y = \frac{3x-1}{3x+1}$. $y' = \frac{9x+3-9x+3}{(3x+1)^2} = \frac{2}{(3x+1)^2}$, возрастает на всей ОДЗ ($x \ne -\frac{1}{3}$).

в) $y = \frac{2}{x} + 1$, $y' = -\frac{2}{x^2}$, убывает на всей ОДЗ ($x \neq 0$).

г) $y = \frac{1-2x}{3+2x}$, $y' = \frac{-6-4x-2+4x}{(3+2x)^2} = -\frac{8}{(3+2x)^2}$

убывает на всей ОДЗ ($x \neq -\frac{3}{2}$).

869. а) $y = \sqrt{3x-1}$, $y' = \frac{3}{2\sqrt{3x-1}}$, возрастает на всей ОДЗ ($x \geq \frac{1}{3}$).

б) $y = \sqrt{1-x} + 2x$, $y' = -\frac{1}{2\sqrt{1-x}} + 2 = 0$, $\frac{4\sqrt{1-x} - 1}{2\sqrt{1-x}} = 0$, $x = \frac{15}{16}$.

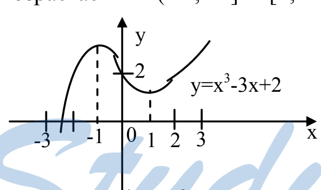
$x \in \left(-\infty; \frac{15}{16}\right]$ возрастает. $x \in \left[\frac{15}{16}; 1\right]$ убывает.

в) $y = \sqrt{1-2x}$, $y' = -\frac{1}{\sqrt{1-2x}}$, $x \in \left(-\infty; \frac{1}{2}\right]$ убывает.

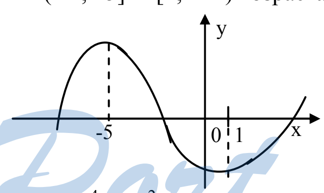
г) $y = \sqrt{2x-1} - x$, $y' = \frac{1}{\sqrt{2x-1}} - 1 = 0$, $\frac{1-\sqrt{2x-1}}{\sqrt{2x-1}} = 0$;

$x \in \left[\frac{1}{2}; 1\right]$ возрастает. $x \in [1; +\infty)$ убывает.

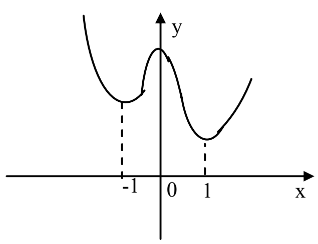
870. а) $y = x^3 - 3x + 2$.
$y' = 3x^2 - 3$.
убывает $x \in [-1; 1]$.
возрастает $x \in (-\infty; -1] \cup [1; +\infty)$



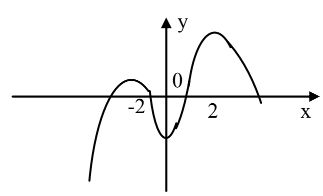
б) $y = x^3 + 6x^2 - 15x + 8$.
$y' = 3x^2 + 12x - 15$.
$x \in [-5; 1]$ убывает.
$x \in (-\infty; -5] \cup [1; +\infty)$ возрастает.



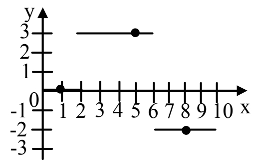
871. а) $y = x^4 - 2x^2 + 1$.
$y' = 4x^3 - 4x = 4x(x^2-1) = 4x(x-1)(x+1)$.
$(-\infty; -1] \cup [0; 1]$ убывает.
$[-1; 0] \cup [1; +\infty)$ возрастает.



б) $y = -x^4 + 8x^2 - 7$.
$y' = -4x^3 + 16x = -4x(x^2-4) = -4x(x-2)(x+2)$.
$[-2; 0] \cup [2; +\infty)$ убывает.
$(-\infty; -2] \cup [0; 2]$ возрастает.



872.



873. а) $f'(b) = f'(d) = 0$. б) $f'(c) = 0$. в) $f'(a) = f'(0) = 0$.
г) нет точек, в которых производная равна нулю.

874. а) $f'(e)$ б) $f'(a)$, $f'(b)$ в) $f'(b)$, $f'(c)$ г) $f'(a)$, $f'(b)$, $f'(c)$, $f'(d)$, $f'(e)$

875. а) 1 б) 2 в) 2 г) 2

876. а) 2 б) 1 в) 2 г) 2

877. а) $(-\infty; -5] \cup [-2; +\infty)$ б) $[-5; -2]$ в) -5. г) -2.

878. а) Да. б) Да. в) Да. г) Да.

879. а) Да. б) Нет. в) Нет. г) Да.

880. а) $y = 7 + 12x - x^3$; $y' = 12 - 3x^2$; $3x^2 = 12$; $x = \pm 2$.

б) $y = 3x^3 + 2x^2 - 7$; $y' = 9x^2 + 4x$; $x(9x + 4) = 0$; $x = 0$, $x = -\frac{4}{9}$.

в) $y = 8 + 2x^2 - x^4$; $y' = 4x - 4x^3$; $4x(1 - x^2) = 0$; $x = 0, x = \pm 1$.

г) $y = x^4 - 8x^2$; $y' = 4x^3 - 16x$; $4x(x^2 - 4) = 0$; $x = 0, x = \pm 2$.

881. а) $y = 2x + \frac{8}{x}$; $y' = 2 - \frac{8}{x^2} = 0$; $x = \pm 2$

б) $\sqrt{2x - 1} = y$; $y' = \frac{1}{\sqrt{2x - 1}}$ $x = \frac{1}{2}$ - критическая

в) $y = \frac{x}{5} + \frac{5}{x}$; $y' = \frac{1}{5} - \frac{5}{x^2} = 0$; $x^2 - 25 = 0$; $x = \pm 5$.

г) $y = (x - 3)^4$; $y' = 4(x - 3)^3 = 0$ $x = 3$.

882. а) $y = 2\sin 2x - \sin 4x$; $y' = 4\cos 2x - 4\cos 4x = 0$;

$\sin 3x \sin x = 0$; $x = \frac{\pi n}{3}$.

б) $y = \cos 2x - x$; $y' = -2\sin 2x - 1 = 0$; $\sin 2x = -\frac{1}{2}$;

$2x = (-1)^{k+1}\frac{\pi}{6} + \pi k$; $x = (-1)^{k+1}\frac{\pi}{12} + \frac{\pi k}{2}$;

в) $y = \cos 3x - \frac{3}{5}\cos 5x$; $y' = -3\sin 3x + 3\sin 5x$;

$\sin 3x - \sin 5x = 0$;

$\sin x \cos 4x = 0$;

$x = \pi n$, $x = \frac{\pi}{8} + \frac{\pi n}{4}$.

г) $y = \sin\frac{x}{2} - \frac{x}{4}$; $y' = \frac{1}{2}\cos\frac{x}{2} - \frac{1}{4}$; $\cos\frac{x}{2} = \frac{1}{2}$; $x = \pm\frac{2\pi}{3} + 4\pi n$.

В задачах 883-888 чтобы определить характер экстремума, необходимо проверить знак производной справа и слева от точки, где она равна нулю. Если слева $y' > 0$, а справа $y' < 0$, то это максимум, если слева $y' < 0$, а справа $y' > 0$, то это минимум.

883. а) $y = 2x^2 - 7x + 1$; $y' = 4x - 7 = 0$; $x = \frac{7}{4}$ – точка min.

б) $y = 3 - 5x - x^2$; $y' = -5 - 2x = 0$; $x = -2{,}5$ – max.

в) $y = 4x^2 - 6x - 7$; $y' = 8x - 6 = 0$; $x = \frac{3}{4}$ – min.

г) $y = -3x^2 - 12x + 50$; $y = -6x - 12 = 0$; $x = -2$ – max.

884. а) $y = \frac{x^3}{3} - \frac{5}{2}x^2 + 6x - 1$;

$y' = x^2 - 5x + 6 = 0$; $x = 2$, $x = 3$. $x = 2$ – max, $x = 3$ – min.

б) $y = x^3 - 27x + 26$;

$y' = 3x^2 - 27 = 0$; $x = \pm 3$; $x = -3$ – max, $x = 3$ – min.

в) $y = x^3 - 7x^2 - 5x + 11$;

$y' = 3x^2 - 14x - 5 = 0$; $x = \frac{7+8}{3} = 5$; $x = -\frac{1}{3}$; $x = 5$ – min, $x = -\frac{1}{3}$ – max.

г) $y = -2x^3 + 21x^2 + 19$; $y' = -6x^2 + 42x = 0$; $x^2 - 7x = 0$; $x(x-7) = 0$;
$x = 0$, $x = 7$; $x = 7$ – max, $x = 0$ – min.

885. а) $y = -5x^5 + 3x^3$; $y' = -25x^4 + 9x^2$; $x^2(9 - 25x^2) = 0$

$x = 0$, $x^2 = \frac{9}{25}$, $x = \pm\frac{3}{5}$; $x = \frac{3}{5}$ – max, $x = -\frac{3}{5}$ – min.

б) $y = x^4 - 4x^3 - 8x^2 + 13$; $y' = 4x^3 - 12x^2 - 16x$; $4x(x^2 - 3x - 4) = 0$;
$x = 0$, $x = 4$, $x = -1$; $x = 4$, $x = -1$ – min, $x = 0$ – max.

в) $y = x^4 - 50x^2$; $y' = 4x^3 - 100x$ $4x(x^2 - 25) = 0$ $x = 0$ $x = \pm 5$

$x = -5$, $x = 5$ – min, $x = 0$ – max.

г) $y = 2x^5 + 5x^4 - 10x^3 + 3$; $y' = 10x^4 + 20x^3 - 30x^2$; $10x^2(x^2 + 2x - 3) = 0$;

$x = 0$, $x = -3$, $x = 1$, $x = -3$ – max, $x = 1$ – min.

886. а) $y = x + \frac{4}{x}$; $y' = 1 - \frac{4}{x^2} = 0$;

$x = \pm 2$,
$x = 2$ – min,
$x = -2$ – max.

б) $y = \frac{x^2+9}{x} = x + \frac{9}{x}$; $y' = 1 - \frac{9}{x^2} = 0$;

$x = \pm 3$,

$x = 3$ – min,

$x = -3$ – max.

887. а) $y = x - 2\sqrt{x-2}$; $y' = 1 - \frac{1}{\sqrt{x-2}} = 0$; $\sqrt{x-2} = 1$;

$x = 3$, $x = 3 - \min$.

б) $y = 4\sqrt{2x-1} - x$; $y' = \frac{4}{\sqrt{2x-1}} - 1 = 0$; $\sqrt{2x-1} = 4$;

$x = 8{,}5$ – max.

888. а) $y = x = -2\cos x$, $x \in [-\pi; \pi]$;

$y' = 1 + 2\sin x$; $\sin x = -\frac{1}{2}$; $x = (-1)^{k+1}\frac{\pi}{6} + \pi k$;

$x = -\frac{5\pi}{6}$ - max; $x = -\frac{\pi}{6}$ - min.

б) $y = 2\sin x - x$, $x \in [\pi; 3\pi]$; $y' = 2\cos x - 1$;

$\cos x = \frac{1}{2}$; $x = \pm\frac{\pi}{3} + 2\pi n$; $x = \frac{5\pi}{3}$ – min; $x = \frac{7\pi}{3}$ – max.

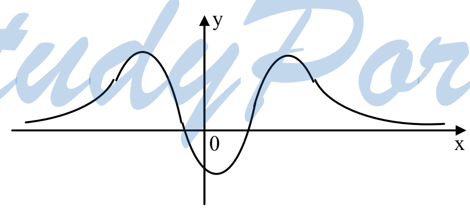
889. а) $y = x^3 - 3ax^2 + 27x - 5$; $y' = 3x^2 - 6ax + 27$;

$x^2 - 2ax + 9 = 0$; $\frac{D}{4} = a^2 - 9 = 0$; $a = \pm 3$.

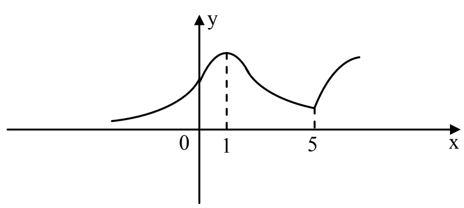
б) $y = x^3 - 3ax^2 + 75x - 10$; $y' = 3x^2 - 6ax + 75$; $x^2 - 2ax + 25 = 0$;

$\frac{D}{4} = a^2 - 25 = 0$; $a = \pm 5$.

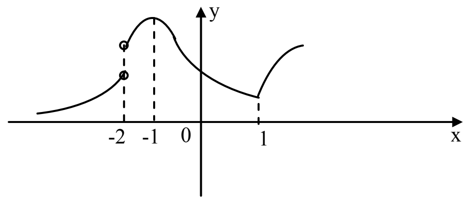
890. а)



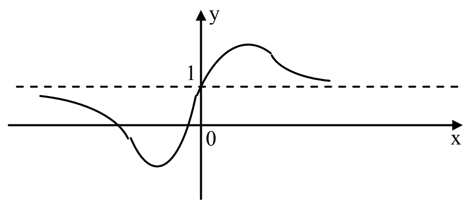
б)



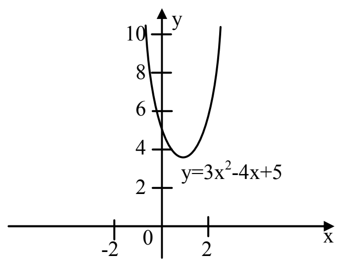
в)



г)



891. а)



$y = 3x^2 - 4x + 5$; $y' = 6x - 4$; $x = \frac{2}{3} - \min$

при $x \ge \frac{2}{3}$ функция возрастает

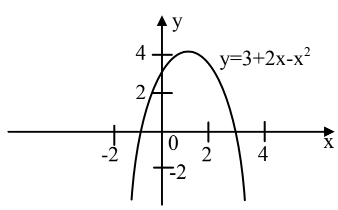
при $x \le \frac{2}{3}$ функция убывает

пересечение с Oy : (0;5)

с Ox : нет

$y > 0$ при $x \in R$

б)



$y = 3 + 2x - x^2$; $y' = 2 - 2x$; $x = 1 - \min$

при $x \ge 1$ функция убывает

при $x \le 1$ функция возрастает

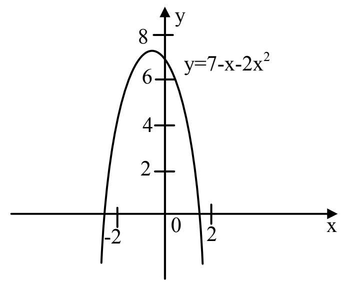
пересечение с Оу: $(0;3)$

с Ох: $(3;0), (-1;0)$

$y > 0$ при $x \in (-1; 3)$

$y < 0$ при $x < -1$, $x > 3$

в)



$y = 7 - x - 2x^2$; $y' = -1 - 4x$; $x = -\frac{1}{4} - \max$

при $x \ge -\frac{1}{4}$ функция убывает

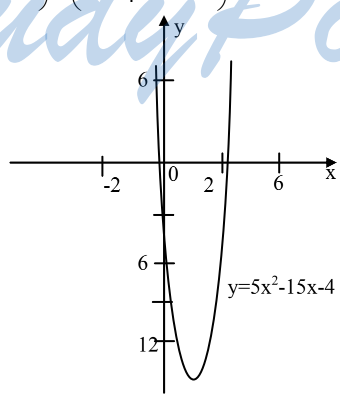
при $x \le -\frac{1}{4}$ функция возрастает

пересечение с Оу: $(0;7)$

с Ох: $\left(\frac{-1-\sqrt{57}}{4};0\right)$, $\left(\frac{-1+\sqrt{57}}{4};0\right)$; $x \in \left(\frac{-1-\sqrt{57}}{4};\frac{-1+\sqrt{57}}{4}\right)$

$x \in \left(-\infty;\frac{-1-\sqrt{57}}{4}\right)$, $\left(\frac{-1+\sqrt{57}}{4};+\infty\right)$

г)



$y = 5x^2 - 15x - 4$; $y' = 10x - 15$; $x = \frac{3}{2} - \min$

при $x \ge \frac{3}{2}$ функция возрастает, при $x \le \frac{3}{2}$ функция убывает

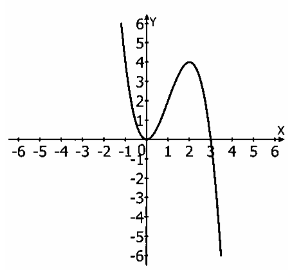
пересечение с Oy:(0;-4)

с Ox: $\left(\frac{15\text{-}\sqrt{305}}{10};0\right), \left(\frac{15+\sqrt{305}}{10};0\right)$

функция $y > 0$ при $x < \frac{15\text{-}\sqrt{305}}{10}, x > \frac{15+\sqrt{305}}{10}$

$y > 0$ при $x < \frac{15-\sqrt{305}}{10}$, $x > \frac{15-\sqrt{305}}{10}$

892. а)

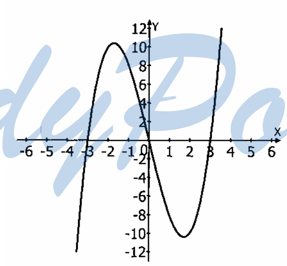


$y = 3x^2 - x^3$; $y' = 6x - 3x^2$; $x = 0$, $x = 2$

$y(0) = 0$ $y(2) = 12\text{-}8 = 4$, $x = 0 - \min$, $x = 2 - \max$,

при $x \in [0;2]$ функция возрастает, при $x \le 0, x \ge 2$ функция убывает.

б)



$y = -9x + x^3$; $y' = -9 + 3x^2$; $x^2 = 3$; $x = \pm\sqrt{3}$;

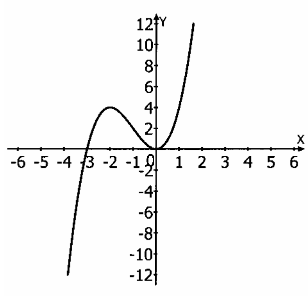
$y(\sqrt{3}) = -9\sqrt{3} + 3\sqrt{3}$; $y(-\sqrt{3}) = 9\sqrt{3} - 3\sqrt{3}$;

$x = \sqrt{3} - \min$, $x = -\sqrt{3} - \max$,

при $x \le -\sqrt{3}, x \ge \sqrt{3}$ функция возрастает,

при $x \in [\text{-}\sqrt{3};\sqrt{3}]$ функция убывает.

в)



$y = x^3 + 3x^2$; $y' = 3x^2 + 6x$; $x = 0, x = -2,$

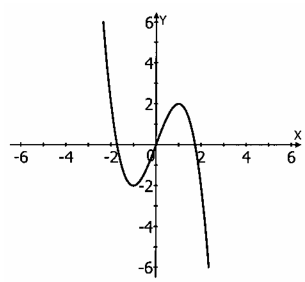
$y(0) = 0$

$y(-2) = -8 + 12 = 4$

$x = 0 - \min$, $x = -2 - \max$;

при $x \in [-2;0]$ функция убывает , при $x \le -2, x \ge 0$ функция возрастает

г)



$y = 3x - x^3$; $y' = 3 - 3x^2$; $x = \pm 1$,

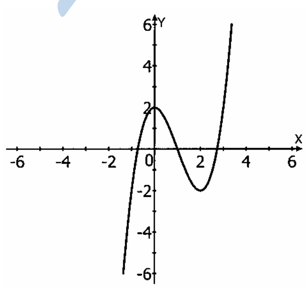
$y(1) = 3 - 1 = 2$

$y(-1) = -3 + 1 = -2$

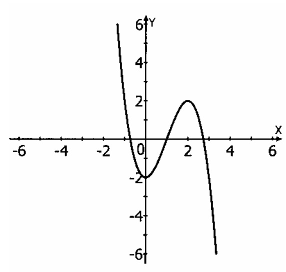
$x = 1 - \max$, $x = -1 - \min$;

при $x \in [-1;1]$ функция возрастает , при $x \le -1, x \ge 1$ функция убывает

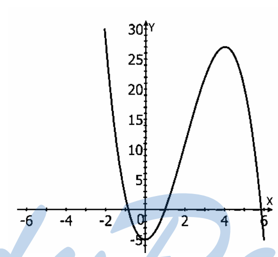
893. а)



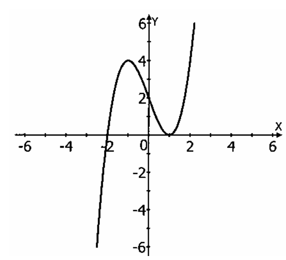
$y = x^3 - 3x^2 + 2$; $y' = 3x^2 - 6x$ $x = 0$, $x = 2$;
$y(0) = 2$
$y(2) = 8\text{-}12 + 2 = -2$
$x = 0 - \max$, $x = -2 - \min$,
при $x \in [0;2]$ функция убывает , при $x \le 0, x \ge 2$ функция возрастает
б)



$y = -x^3 + 3x - 2$; $y' = -3x^2 + 3$; $x = \pm 1$
$y(1) = -1 + 3 - 2 = 0$
$y(\text{-}1) = 1\text{-}3\text{-}2 = -4$
$x = 1 - \max$, $x = -1 - \min$
при $x \in [\text{-}1;1]$ функция возрастает , при $x \le -1, x \ge 1$ функция убывает
в)



$y = -x^3 + 6x^2 - 5$; $y' = -3x^2 + 12x$ $= \text{-}x^2 + 4x$, $x = 0$, $x = 4$,
$x = 0 - \min$, $x = 4 - \max$,
при $x \in [0;4]$ функция возрастает,   при $x \le 0$, $x \ge 4$ функция убывает
г)

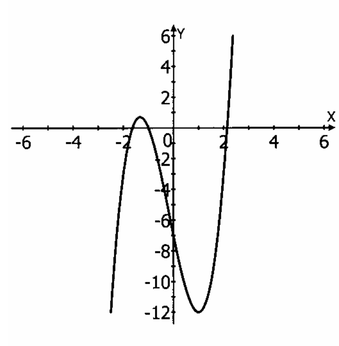


$y = x^3 - 3x + 2$; $y' = 3x^2 - 3$; $x = \pm 1$;

$x = 1 - \min$, $x = -1 - \max$

при $x \in [-1;1]$ функция убывает , при $x \le -1, x \ge 1$ функция возрастает

894. а)



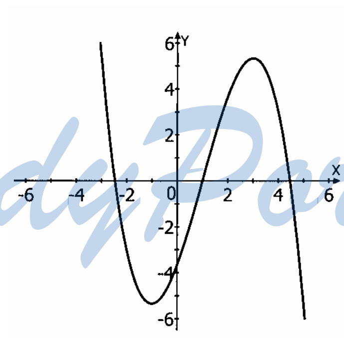
$y = 2x^3 + x^2 - 8x - 7$; $y' = 6x^2 + 2x - 8$;

$x = \dfrac{-1-7}{6} = -\dfrac{4}{3}$, $x = 1$, $x = -\dfrac{4}{3} - \max$, $x = 1 - \min$,

при $x \in \left[-\dfrac{4}{3};1\right]$ функция убывает ,

при $x \le -\dfrac{4}{3}, x \ge 1$ функция возрастает .

б)



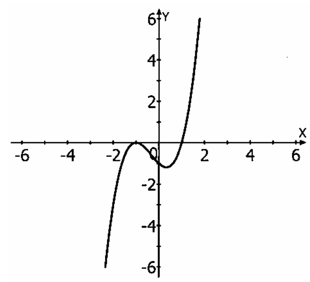
$y = -\dfrac{x^3}{3} + x^2 + 3x - \dfrac{11}{3}$; $y' = -x^2 + 2x + 3$; $x = 3$, $x = -1$;

$x = -1 - \min$, $x = 3 - \max$

при $x \in [-1;3]$ функция возрастает ,

при $x \le -1, x \ge 3$ функция убывает .

в)

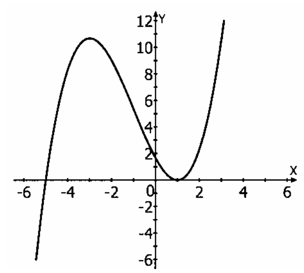


$y = x^3 + x^2 - x - 1$; $y' = 3x^2 + 2x - 1$; $x = \frac{-1-2}{3} = -1$, $x = \frac{1}{3}$;

$x = \frac{1}{3} - \min$, $x = -1 - \max$;

при $x \in \left[-1; \frac{1}{3}\right]$ функция убывает, при $x \le -1, x \ge \frac{1}{3}$ функция возрастает.

г)

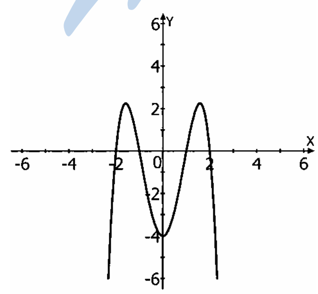


$y = \frac{x^3}{3} + x^2 - 3x + \frac{5}{3}$, $y' = x^2 + 2x - 3$, $x = -3$, $x = 1$

$x = 1 - \min$, $x = -3 - \max$

при $x \in [-3;1]$ функция убывает, при $x \le -3, x \ge 1$ функция возрастает.

895. а)



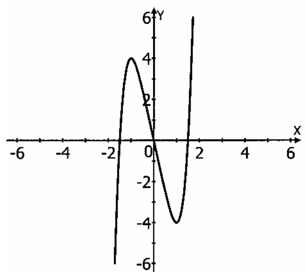
$y = -x^4 + 5x^2 - 4$, $y' = -4x^3 + 10x$, $2x(-2x^2 + 5) = 0$,

$x = 0$, $x = \sqrt{\frac{5}{2}}$, $x = -\sqrt{\frac{5}{2}}$; $x = 0 - \min$, $x = \pm\sqrt{\frac{5}{2}} - \max$

при $x \in \left(-\infty; -\frac{\sqrt{5}}{\sqrt{2}}\right] \cup \left[0; \frac{\sqrt{5}}{\sqrt{2}}\right]$ функция возрастает

при $x \in \left[-\frac{\sqrt{5}}{\sqrt{2}}; 0\right] \cup \left[\frac{\sqrt{5}}{\sqrt{2}}; +\infty\right)$ функция убывает

б)



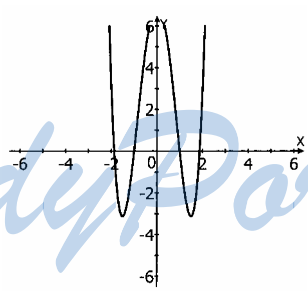
$y = x^5 - 5x$; $y' = 5x^4 - 5$; $x^4 - 1 = 0$; $x = \pm 1$;

$x = 1 - \min$, $x = -1 - \max$;

при $x \in [-1;1]$ функция убывает,

при $x \le -1, x \ge 1$ функция возрастает.

в)



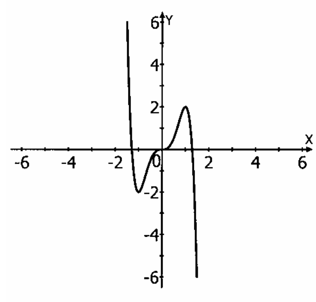
$y = 2x^4 - 9x^2 + 7$; $y' = 8x^3 - 18x$; $2x(4x^2 - 9) = 0$;

$x = 0$, $x = \pm\frac{3}{2}$; $x = 0 - \max$, $x = \pm\frac{3}{2}$ - min;

при $x \in \left(-\infty; -\frac{3}{2}\right] \cup \left[0; \frac{3}{2}\right]$ функция убывает,

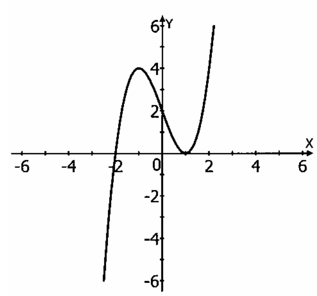
при $x \in \left[-\frac{3}{2}; 0\right] \cup \left[\frac{3}{2}; +\infty\right)$ функция возрастает.

г)



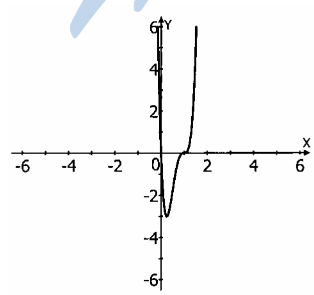
$y = 5x^3 - 3x^5$, $y' = 15x^2 - 15x^4$, $15x^2(1 - x^2) = 0$,
$x = 0$, $x = \pm 1$, $x = 1 - \max$, $x = -1 - \min$,
при $x \in [-1;1]$ функция возрастает,
при $x \in (-\infty;-1] \cup [1;+\infty)$ функция убывает.

896. а)



$y = (x - 1)^2(x + 2)$,
$y' = 2(x - 1)(x + 2) + (x - 1)^2 = (x - 1)(2x + 4 + x - 1) = (x - 1)(3x + 3) = 0$
$(x - 1)(3x + 1) = 0$, $x = 1$, $x = -1$, $x = -1 - \max$, $x = 1 - \min$,
при $x \in [-1; 1]$ функция убывает, при $x \le -1$, $x \ge 1$ функция возрастает

б)



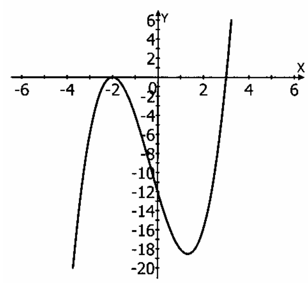
$y = \frac{256}{9}x(x-1)^3$

$y' = \frac{256}{9}\left((x-1)^3 + 3(x-1)^2 x\right) = \frac{256}{9}(x-1)^2(4x-1)$;

$\frac{256}{9}(x-1)^2(4x-1) = 0$; $x = 1$, $x = \frac{1}{4}$, $x = \frac{1}{4} - \min$,

при $x \geq \frac{1}{4}$ функция возрастает, при $x \leq \frac{1}{4}$ функция убывает

в)

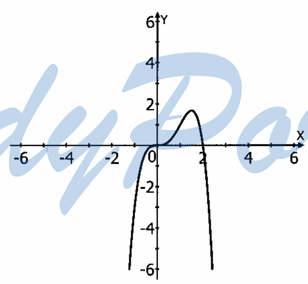


$y = (x+2)^2(x-3)$, $y' = 2(x+2)(x-3) + (x+2)^2 = (x+2)(3x-4)$,

$(x+2)(3x-4) = 0$, $x = -2$, $x = \frac{4}{3}$, $x = -2 - \max$, $x = \frac{4}{3} - \min$,

при $x \in \left[-2; \frac{4}{3}\right]$ функция убывает, при $x \leq -2, x \geq \frac{4}{3}$ функция возрастает.

г)

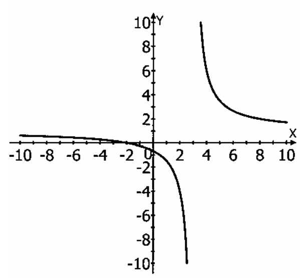


$y = x^3(2-x)$, $y' = \left(3x^2(2-x) - x^3\right) = x^2(6-4x)$, $x^2(6-4x) = 0$,

$x = 0$, $x = \frac{3}{2}$, $x = \frac{3}{2} - \max$,

при $x \geq \frac{3}{2}$ функция убывает, при $x \leq \frac{3}{2}$ функция возрастает.

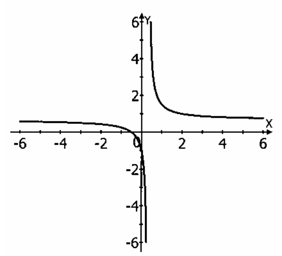
897. а)



$y=\frac{x+2}{x-3}$; $y'=\frac{x-3-x-2}{(x-3)^2}=-\frac{5}{(x-3)^2}$; $-\frac{5}{(x-3)^2}=0$

везде убывает, $x\neq 3$; $x=3$ – ассимптота .

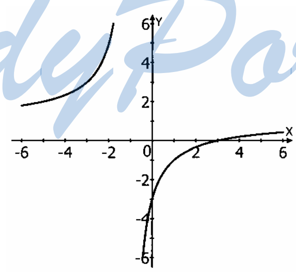
б)



$y=\frac{2x+1}{3x-1}$; $y'=\frac{6x-2-6x-3}{(3x-1)^2}=-\frac{5}{(3x-1)^2}$; $-\frac{5}{(3x-1)^2}=0$;

функция убывает везде, $x\neq\frac{1}{3}$; $x=\frac{1}{3}$ – ассимптота

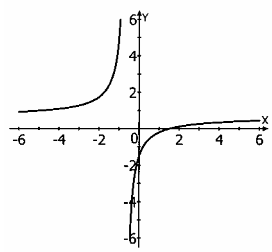
в)



$y=\frac{x-3}{x+1}$; $y'=\frac{x+1-x+3}{(x+1)^2}=\frac{4}{(x+1)^2}$; $\frac{4}{(x+1)^2}=0$;

возрастает везде, $x\neq -1$, $x=-1$ – ассимптота .

г)

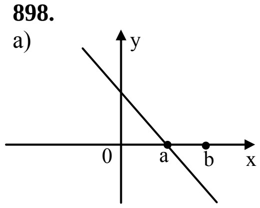


$$y=\frac{2x-3}{3x+2};\quad y'=\frac{6x+4-6x+9}{(3x+2)^2}=\frac{13}{(3x+2)^2};\quad \frac{13}{(3x+2)^2}=0;$$

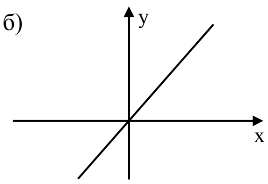
функция возрастает везде, $x\neq-\frac{2}{3}$; $x=-\frac{2}{3}$ – ассимптота .

898.

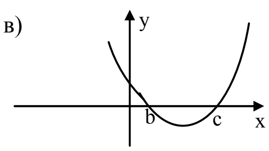
а)



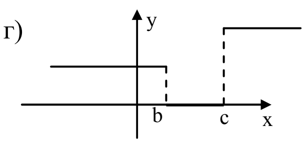
б)



в)

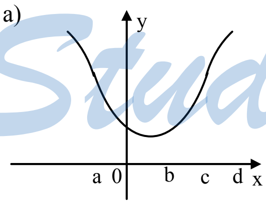


г)

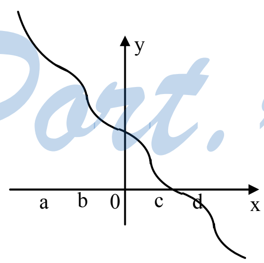


899.

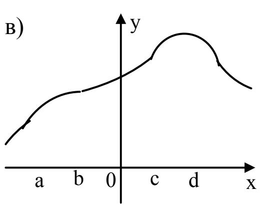
а)



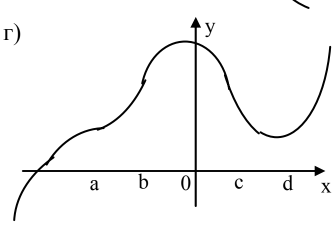
б)



в)

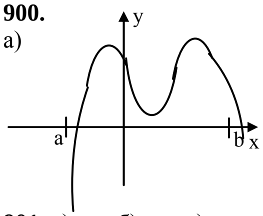


г)

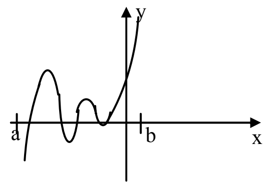


900.

а)



б)



901. а) да; б) да; в) нет; г) нет.

902. а) $y = \sin x - \frac{1}{2}x$, $y' = \cos x - \frac{1}{2}$, $\cos x = \frac{1}{2}$, $x = \pm\frac{\pi}{3} + 2\pi n$;

$x = -\frac{\pi}{3} + 2\pi n - \min$; $x = \frac{\pi}{3} + 2\pi n - \max$;

$\text{возрастает}: x \in \left[-\frac{\pi}{3} + 2\pi n; \frac{\pi}{3} + 2\pi n\right]$,

$\text{убывает}: x \in \left[\frac{\pi}{3} + 2\pi n; \frac{5\pi}{3} + 2\pi n\right]$.

б) $y = \frac{x}{2} - \cos x$; $y' = \frac{1}{2} + \sin x$; $x = (-1)^{k+1}\frac{\pi}{6} + \pi k$;

$x = -\frac{\pi}{6} + 2\pi n - \max$; $x = \frac{7\pi}{6} + 2\pi n - \min$;

$\text{возрастает}: x \in \left[-\frac{\pi}{6} + 2\pi n; \frac{7\pi}{6} + 2\pi n\right]$,

$\text{убывает}: x \in \left[\frac{7\pi}{6} + 2\pi n; \frac{11\pi}{6} + 2\pi n\right]$.

в) $y = \frac{1}{\sqrt{2}}x + \cos x$; $y' = \frac{\sqrt{2}}{2} - \sin x$; $x = (-1)^k\frac{\pi}{4} + \pi k$;

$x = \frac{\pi}{4} + 2\pi n - \max$; $x = \frac{3\pi}{4} + 2\pi n - \min$;

$\text{возрастает}: x \in \left[\frac{3\pi}{4} + 2\pi n; \frac{9\pi}{4} + 2\pi n\right]$,

$\text{убывает}: x \in \left[\frac{\pi}{4} + 2\pi n; \frac{3\pi}{4} + 2\pi n\right]$.

г) $y = x - \sin x$; $y' = 1 - \cos x$; $x = 2\pi n$, возрастает на R.

903. а) $y = 2 + \cos\frac{x}{2}$; $y' = -\frac{1}{2}\sin\frac{x}{2}$; $\sin\frac{x}{2} = 0$; $x = 2\pi n$;

возрастает: $x \in [-2\pi + 4\pi n; 4\pi n]$, $\text{убывает}: x \in [4\pi n; 2\pi + 4\pi n]$,

$x = 4\pi n - \max$, $x = 2\pi + 4\pi n - \min$.

б) $y=\frac{1}{6}-\sin\frac{x}{3}$, $y'=-\frac{1}{3}\cos\frac{x}{3}$, $x=\frac{3\pi}{2}+6\pi n$;

убывает: $x\in\left[6\pi n-\frac{3\pi}{2};\frac{3\pi}{2}+6\pi n\right]$,

возрастает: $x\in\left[\frac{3\pi}{2}+6\pi n;\frac{9\pi}{2}+6\pi n\right]$;

$x=\frac{3\pi}{2}+6\pi n-\min$, $x=-\frac{3\pi}{2}+6\pi n-\max$

904. а) $y=x-\sin 2x$, $y'=1-2\cos 2x$, $\cos 2x=-\frac{1}{2}$;

$2x=\pm\frac{\pi}{3}+2\pi n$, $x=\pm\frac{\pi}{6}+\pi n$.

убывает: $x\in\left[\pi n-\frac{\pi}{6};\frac{\pi}{6}+\pi n\right]$, возрастает: $x\in\left[\frac{\pi}{6}+\pi n;\frac{5\pi}{6}+\pi n\right]$,

$x=\frac{\pi}{6}+\pi n-\min$, $x=-\frac{\pi}{6}+\pi n-\max$.

б) $y=x+4\cos\frac{x}{2}$, $y'=1-2\sin\frac{x}{2}$, $\sin\frac{x}{2}=\frac{1}{2}$, $x=(-1)^k\frac{\pi}{3}+2\pi k$

убывает: $x\in\left[\frac{\pi}{3}+4\pi n;\frac{5\pi}{3}+4\pi n\right]$,

возрастает: $x\in\left[-\frac{7\pi}{3}+4\pi n;\frac{\pi}{3}+4\pi n\right]$,

$x=\frac{\pi}{3}+4\pi n-\max$, $x=\frac{5\pi}{3}+4\pi n-\min$

905. а) $y=|x-3|-2$;

1) $x\geq 3$; $y=x-5$; $y'=1$; возрастает $x\geq 3$;

2) $x\leq 3$; $y=1-x$; $y'=-1$; убывает $x\leq 3$.

Ответ: $x\in(-\infty;3]$ – убывает; $x\in[3;+\infty)$ – возрастает. $x=3-\min$.

б) $y=\left|\frac{1}{x}-1\right|$;

1) $x\geq 1, x<0$; $y=-\frac{1}{x}+1$; $y'=\frac{1}{x^2}$; везде возрастает;

2) $x\in(0;1]$; $y=-1+\frac{1}{x}$; $y'=-\frac{1}{x^2}$; везде убывает.

Ответ: $x\in(0;1]$ – убывает; $x\in(-\infty;0)\cup[1;+\infty)$ – возрастает. $x=1-\min$.

в) $y=|(x-2)(x+3)|$;

1) $x \ge 2, x \le -3$; $y=x^2+x-6$; $y'=2x+1$; $x=-\frac{1}{2}$;

возрастает $x \ge -\frac{1}{2}$, убывает $x \le -\frac{1}{2}$.

2) $x \in [-3;2]$; $y=-x^2-x+6$; $y'=-2x-1$; $x=-\frac{1}{2}$;

возрастает $x \le -\frac{1}{2}$, убывает $x \ge -\frac{1}{2}$.

Ответ: $x \in (-\infty;-3] \cup \left[-\frac{1}{2};2\right]$ – убывает; $x \in \left[-3;\frac{1}{2}\right] \cup [2;+\infty]$ – возрастает.

$x=-3, x=2-\min$, $x=-\frac{1}{2}-\max$.

г) $y=(|x|-2)|x|$;

1) $x \ge 0$; $y=x^2-2x$; $y'=2x-2$; $x=1$;

возрастает $x \ge 1$

убывает $x \le 1$

2) $x \le 0$; $y=x^2+2x$; $y'=2x+2$; $x=-1$;

возрастает $x \ge -1$

убывает $x \le -1$

Ответ: $x \in (-\infty;-3] \cup \left[-\frac{1}{2};2\right]$ – убывает; $x \in \left[-3;\frac{1}{2}\right] \cup [2;+\infty]$ – возрастает.

$x=\pm 1-\min$, $x=0-\max$.

906. а) $y=|x^3-3x|$, $y=|x(x-\sqrt{3})(x+\sqrt{3})|$

1) $x \in \left[-\sqrt{3};0\right] \cup \left[\sqrt{3};+\infty\right)$, $y'=3x^2-3$, $x=\pm 1$,

возрастает: $(-\infty;-1] \cup [1;+\infty)$

убывает: $[-1;1]$

2) $x \in \left(-\infty;-\sqrt{3}\right] \cup \left[0;\sqrt{3}\right]$, $y'=3-3x^2$, $x=\pm 1$,

возрастает: $[-1;1]$

убывает: $(-\infty;-1] \cup [1;+\infty)$

Ответ: $x \in \left(-\infty;-\sqrt{3}\right] \cup [-1;0] \cup \left[1;\sqrt{3}\right]$ – убывает
$x \in \left[-\sqrt{3};-1\right] \cup [0;1] \cup \left[\sqrt{3};+\infty\right)$ – возрастает

$x=\pm\sqrt{3}, x=0-\min$, $x=\pm 1-\max$.

б) $y = |x - x^3|$, $y = |x(\sqrt{3} - x)(\sqrt{3} + x)|$;

1) $x \in \left(-\infty; -\sqrt{3}\right] \cup \left[0; \sqrt{3}\right]$, $y' = 1 - 3x^2$, $x = \pm\frac{1}{\sqrt{3}}$,

убывает: $x \in \left(-\infty; -\frac{1}{\sqrt{3}}\right] \cup \left[\frac{1}{\sqrt{3}}; +\infty\right)$

возрастает : $x \in \left[-\frac{1}{\sqrt{3}}; \frac{1}{\sqrt{3}}\right]$

2) $x \in \left[-\sqrt{3}; 0\right] \cup \left[\sqrt{3}; +\infty\right)$, $y' = 3x^2 - 1$, $x = \pm\frac{\sqrt{3}}{3}$,

возрастает: $x \in \left(-\infty; -\frac{1}{\sqrt{3}}\right] \cup \left[\frac{1}{\sqrt{3}}; +\infty\right)$

убывает : $x \in \left[-\frac{\sqrt{3}}{3}; \frac{\sqrt{3}}{3}\right]$

Ответ:
$x \in (-\infty; -1] \cup \left[-\frac{1}{\sqrt{3}}; 0\right] \cup \left[\frac{1}{\sqrt{3}}; 1\right]$ – убывает
$x \in \left[-1; -\frac{1}{\sqrt{3}}\right] \cup \left[0; \frac{1}{\sqrt{3}}\right] \cup [1; +\infty)$ – возрастает

$x = \pm 1, x = 0 - \min$, $x = \pm\frac{\sqrt{3}}{3} - \max$.

907. а) $y = x^5 + 3x - 6$, $y' = 5x^4 + 3 > 0$ $\forall x \in (-\infty; +\infty)$;

б) $y = 15 - \frac{2}{x} - \frac{1}{x^3}$, $y' = \frac{2}{x^2} + \frac{3}{x^4} > 0$ $\forall x \in (-\infty; +\infty)$;

в) $y = x^7 + 7x^3 + 2x - 42$, $y' = x^6 + 21x^2 + 2$;

г) $y = 21x - \frac{1}{x^5}$, $y' = 21 + \frac{5}{x^6} > 0$ $\forall$ $x > 0$.

908. а) $y = 7x - \cos 2x$, $y' = 7 + 2\sin 2x > 0$ $\forall x \in (-\infty; +\infty)$;

б) $y = 3\mathrm{tg}x$, $y' = \frac{3}{\cos^2 x} > 0$ $\forall x \in \left(-\frac{\pi}{2}; \frac{\pi}{2}\right)$;

в) $y = -\mathrm{ctg}x$, $y' = \frac{1}{\sin^2 x} > 0$ $\forall x \in (0; \pi)$;

г) $y = 10x + \sin 3x$, $y' = 10 + 3\cos 3x > 0$ $\forall x \in (-\infty; +\infty)$.

909. а) $y = 2x^3 + 2x^2 + 11x - 35$, $y' = 6x^2 + 4x + 11$,
$D = 16 - 4 \cdot 6 \cdot 11 < 0$, следовательно, $6x^2 + 4x + 11 > 0$ $\forall x \in (-\infty; +\infty)$;

б) $y = 3x^3 - 6x^2 + 41x - 137$, $y' = 9x^2 - 12x + 41$,

$\frac{D}{4} = 36 - 41 \cdot 9 < 0$, следовательно, $9x^2 - 12x + 41 > 0$ $\forall x \in (-\infty; +\infty)$.

910. а) $y=\frac{4x}{4x+1}$, $y'=\frac{16x+4-16x}{(4x+1)^2}=\frac{4}{(4x+1)^2}>0\ \forall x\in\left(-\frac{1}{4};+\infty\right)$;

б) $y=\frac{2x-13}{x-5}$, $y'=\frac{2x-10-2x+13}{(x-5)^2}=\frac{3}{(x-5)^2}>0\ \forall x\in(-\infty;5)$.

911. а) $y=-x^3-5x+3$, $y'=-3x^2-5<0\ \forall x\in(-\infty;+\infty)$;

б) $y=-2x^5-7x^3-x+8$, $y'=-10x^4-21x^2-1$, $0\ \forall x\in(-\infty;+\infty)$;

в) $y=-x^3+3x^2-6x+1$,

$y'=-3x^2+6x-6=-3(x^2-2x+1)=-3(x-1)^2<0\ \forall x\in(-\infty;+\infty)$;

г) $y=-4x^3+4x^2-2x+9$, $y'=-12x^2+8x-2$,

$\frac{D}{4}=16-24<0\Rightarrow -12x^2+8x-2<0\ \forall x\in(-\infty;+\infty)$.

912. а) $y=\frac{3x+7}{x+2}$, $y'=\frac{3x+6-3x-7}{(x+2)^2}=-\frac{1}{(x+2)^2}<0\ \forall x\in(-\infty;+\infty)$;

б) $y=\frac{-4x+1}{2x+1}$, $y'=\frac{-8x-4+8x-2}{(2x+1)^2}=-\frac{6}{(2x+1)^2}<0\ \forall x\in\left(-\infty;-\frac{1}{2}\right)$.

913. а) $y=7\cos x-5\sin 3x-22x$,

$y'=-7\sin x-15\cos 3x-22<0\ \forall x\in(-\infty;+\infty)$, убывает на R;

б) $y=3\cos 7x-8\sin\frac{x}{2}-25x+1$,

$y'=-21\sin 7x-4\cos\frac{x}{2}-25<0\ \forall x\in(-\infty;+\infty)$, убывает на R.

914. а) $y=x^3+ax$, $y'=3x^2+a$, при $a\ge 0$ возрастает на R;

б) $y=\frac{x^3}{3}-ax^2+5x-3$, $y'=x^2-2ax+5$, $\frac{D}{4}=a^2-5$,

при $a\in\left[-\sqrt{5};\sqrt{5}\right]$ возрастает на R.

915. а) $y=ax-\cos x$, $y'=a+\sin x$, при $a\ge 1$ возрастает на R;

б) $y=2\sin 2x-ax$, $y'=4\cos 2x-a$, при $a\le -4$ возрастает на R.

916. а) $y=7+bx-x^2-x^3$, $y'=b-2x-3x^2$,

$D=4-4\cdot(-3)\cdot b=4+12b<0$, при $b<-\frac{1}{3}$ убывает на R;

б) $y=-2\sqrt{x+3}+bx$,

$y'=-\frac{1}{\sqrt{x+3}}+b<0,\ b<\frac{1}{\sqrt{x+3}}$ - при $b<0$ убывает при $\forall x\in(-3;+\infty)$;

в) $y=x^3+bx^2+3x+21$, $y'=3x^2+2bx+3$.

При x = 0 y' = 3, следовательно, ни при каких b функция не может убывать на всей области определения.

г) $y = -2bx + \sqrt{1-x}$, $y' = -2b - \frac{1}{2\sqrt{1-x}} < 0$, при $b \ge 0$

917. а) $y = 2x^3 - 3x^2 + 7$, $y' = 6x^2 - 6x = 6x(x-1)$,

возрастает $x \le 0, x \ge 1$ при $a \le -1, a \ge 2$,

следовательно, $a - 1 \ge 1$, $a + 1 \le 0$; $a \ge 2$, $a \le -1$.

б) $y = -x^3 + 3x + 5$, $y' = -3x^2 + 3 = -3(x^2 - 1)$,

возрастает $x \in [-1;1]$, убывает $x \le -1$, $x \ge 1$

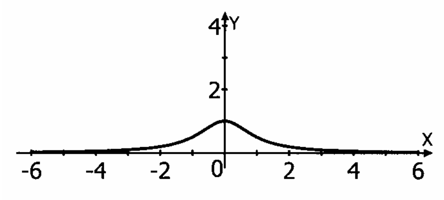
при $a \in \left(-\infty; -\frac{3}{2}\right] \cup [1; +\infty)$,

следовательно $a > 1$, $a + \frac{1}{2} < -1$; $a \ge 2$, $a \le -\frac{3}{2}$

при $a \in \left(-\infty; -\frac{3}{2}\right] \cup [1; +\infty)$

В задачах 918-925 для иследования функции на возрастание и убывание будем исследовать ее производную. На промежутках, где y' > 0, функция возрастает, а где y' < 0 функция убывает.

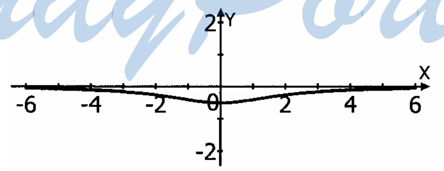
918. а)



$y = \frac{1}{x^2+1}$, $y' = -\frac{1}{(x^2+1)^2}$,

y' < 0 при x > 0, y' > 0 при x < 0, асимптота – y = 0, x = 0 - max

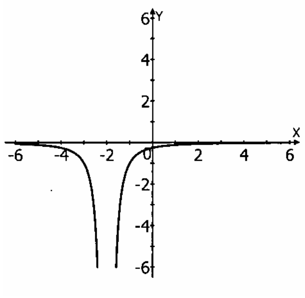
б)



$y = -\frac{2}{x^2+4}$, $y' = -\frac{2}{(x^2+4)^2}$,

y' < 0 при x < 0, y' > 0 при x > 0, асимптота – y = 0, x = 0 – min.

919. а)



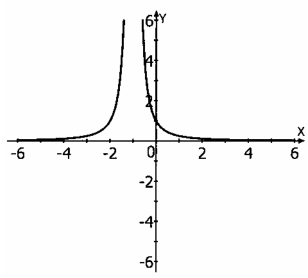
$y = -\frac{1}{(x^2+4x+4)} = -\frac{1}{(x+2)^2}$, $y = \frac{2}{(x+2)^3}$,

при $x > -2$ – возрастает

$x < -2$ – убывает

асимптота – $x = -2$;

б)



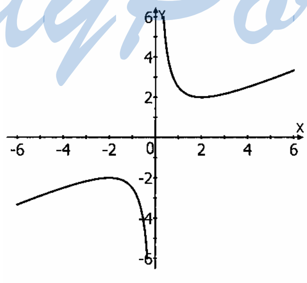
$y = \frac{1}{x^2+2x+1} = \frac{1}{(x+1)^2}$, $y' = -\frac{2(x+1)}{(x+1)^4}$,

при $x > -1$ – убывает

$x < -1$ – возрастает

асимптота – $x = -1$.

920. а)



$y = \frac{x}{2} + \frac{2}{x}$, $y' = \frac{1}{2} - \frac{2}{x^2}$, $y' > 0$: $\frac{1}{2} - \frac{2}{x^2} > 0$, $x^2 > 4$,

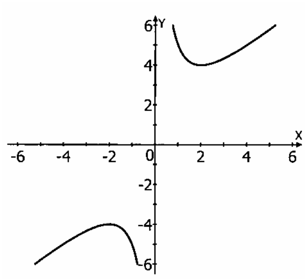
$y' > 0$ при $x \in (-\infty;-2) \cup (2;+\infty)$, $y' < 0$ при $x \in (-2;0) \cup (0;2)$,

при $x \in (-\infty;-2] \cup [2;+\infty)$ возрастает

$x \in [-2;0) \cup (0;2]$ убывает

$x = 2 - \min$, $x = -2 - \max$, асимптоты: $y = \frac{x}{2}$, $x = 0$;

б)



$y = \frac{x^2 + 4}{x} = x + \frac{4}{x}$, $y' = 1 - \frac{4}{x^2}$, $x = \pm 2$, $1 - \frac{4}{x^2} > 0$, $\frac{x^2}{4} > 1$,

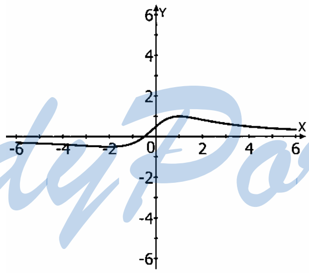
$x^2 > 4$, $|x| > 2$,

при $|x| \geq 2$ - возрастает

$|x| \leq 2$ - убывает

$x = -2 - \min$, $x = 2 - \max$, асимптоты – $y = x$, $x = 0$.

921. а)



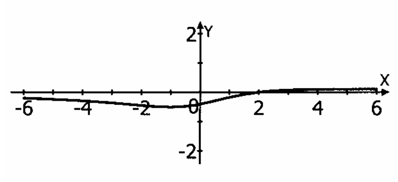
$y = \frac{2x+1}{x^2+2}$, $y' = \frac{4x^2 + 4x - 4x^2 - 2x}{(x^2+2)^2} = \frac{2x}{(x^2+2)^2}$,

$y' > 0$ при $x^2 + x - 4 < 0$,

при $x \in \left(\frac{-1-\sqrt{17}}{2}; \frac{-1+\sqrt{17}}{2}\right)$ функция растет, при $x \in \left(\frac{-1-\sqrt{17}}{2};\frac{-1+\sqrt{17}}{2}\right)$ убывает,

асимптота – $y = 0$.

б)



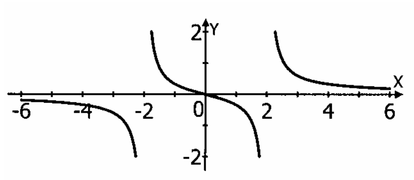
$y = \frac{x-2}{x^2+5}$, $y' = \frac{x^2+5-2x^2+4x}{(x^2+5)^2} = \frac{-x^2+4x+5}{(x^2+5)^2}$,

$y' > 0$ при $x2 - 4x - 5 < 0$; $x \le -1$; $x = 5$,

при $x \in [-1;5]$ – возрастает, при $x \le -1, x \ge 5$ – убывает

$x = -1 - \min$, $x = 5 - \max$, асимптота – $y = 0$.

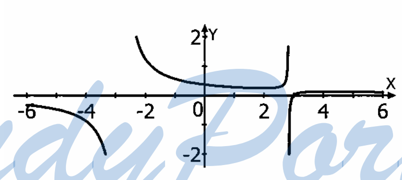
922. а)



$y = \frac{x}{x^2-4}$, $y' = \frac{x^2-4-2x^2}{(x^2-4)^2} = -\frac{x^2+4}{(x^2-4)^2}$,

убывает при всех $x \ne \pm 2$, асимптота – $x = \pm 2$, $y = 0$.

б)



$y = \frac{x-3}{x^2-8}$, $y' = \frac{x^2-8-2x^2+6x}{(x^2-8)^2} = \frac{-x^2+6x-8}{(x^2-8)^2}$,

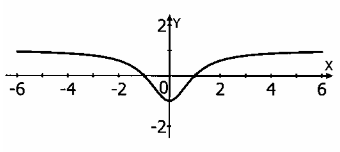
$y' > 0$ при $x^2 - 6x + 8 < 0$, $x = 4, x = 2$,

при $x \in [2;2\sqrt{2}) \cup (2\sqrt{2};4]$ – возрастает,

при $x \le 2, x \ge 4$ – убывает,

$x = 2 - \min$, $x = 4 - \max$, асимптоты: $x = \pm 2\sqrt{2}$, $y = 0$.

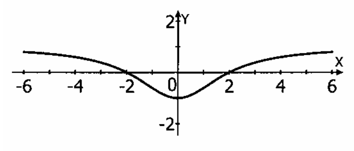
923. а)



$$y=\frac{x^2-1}{x^2+1},\quad y'=\frac{2x^3+2x-2x^3+2x}{(x^2+1)^2}=\frac{4x}{(x^2+1)^2},$$

$x \geq 0$ – возрастает, $x \leq 0$ – убывает, $x=0$ – min.

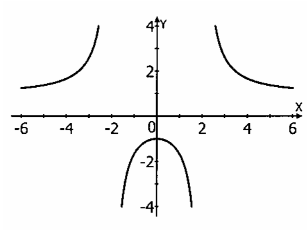
б)



$$y=\frac{x^2-4}{x^2+4},\quad y'=\frac{2x^3+2x-2x^3+2x}{(x^2+1)^2}=\frac{4x}{(x^2+1)^2},$$

$x \geq 0$ – возрастает, $x \leq 0$ – убывает, $x=0$ – min.

924. а.)



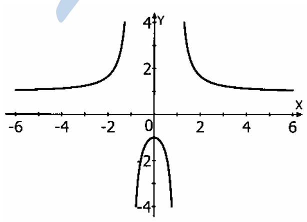
$$y=\frac{x^2+4}{x^2-4},\quad y'=\frac{2x^3-8x-2x^3-8x}{(x^2-4)^2}=\frac{-16x}{(x^2-4)^2},$$

$x \in [0;2) \cup (2;+\infty)$ -убывает

$x \in (-\infty;-2) \cup (2;0]$ -возрастает

$x=0$ – max, асимптоты: $y=0$; $x=\pm 2$.

б)



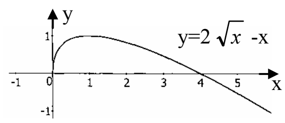
$y = \frac{x^2+1}{x^2-1}$, $y' = \frac{2x^3-2x-2x^3-2x}{(x^2-1)^2} = \frac{-4x}{(x^2-1)^2}$,

$x \in [0;1) \cup (1;+\infty)$ -убывает

$x \in (-\infty;-1) \cup (-1;0]$ -возрастает

$x = 0 - \max$, асимптоты: $y = 0$; $x = \pm 1$.

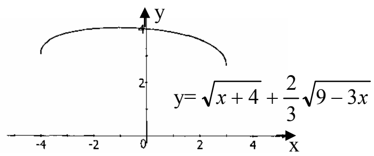
925. а)



$y = 2\sqrt{x} - x$, $y' = \frac{1}{\sqrt{x}} - 1$,

при $x \ge 1$ – убывает, при $x \in [0;1]$ – возрастает, $x = 1 - \max$.

б)

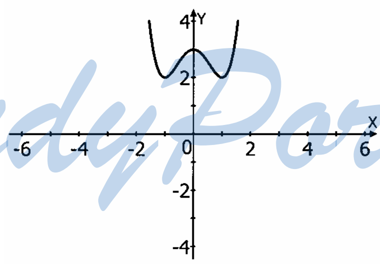


$y = \sqrt{x+4} + \frac{2}{3}\sqrt{9-3x}$, $y' = \frac{1}{2\sqrt{x+4}} - \frac{1}{\sqrt{9-3x}} = \frac{\sqrt{9-3x} - 2\sqrt{x+4}}{2\sqrt{x+4}\sqrt{9-3x}}$,

$\sqrt{9-3x} > 2\sqrt{x+4}$, $9-3x > 4x+16$, $7x < -7$, $x < -1$,

при $x \in [-4;-1]$ – возрастает, при $x \in [-1;3]$ – убывает, $x = -1 - \max$.

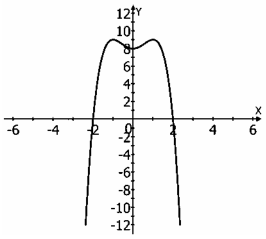
926. а)



б) Количество корней в данном уравнении – это количество пересечений графиков $y = x^4 - 2x^2 + 3$ и $y = a$.

Из рисунка вилно, что такой случай имеет место, когда прямая $y = a$ касается графика функции в точке $(0;\ y(0))$ $y(0) = 3$, следовательно, $a=3$.

927. а)



б) Чтобы уравнение не имело корней, необходимо, чтобы прямая лежала выше графика функции $y = -x^4 + 2x^2 + 8$.

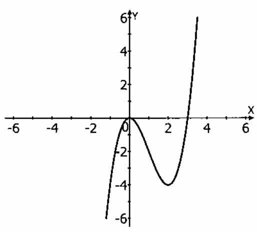
Найдем точки максимума: $y' = -4x^3 + 4x = 4x(1 - x^2) = 0$,

$x = 0$ – точка минимума, $x = \pm 1$ – точки максимума,

$y(1) = y(-1) = -1 + 2 + 8 = 9$.

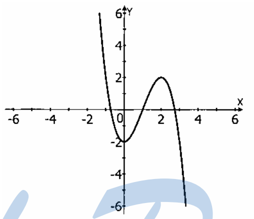
Следовательно, $a > 9$.

928. а)



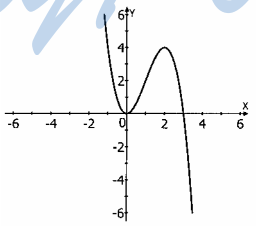
3 корня

б)



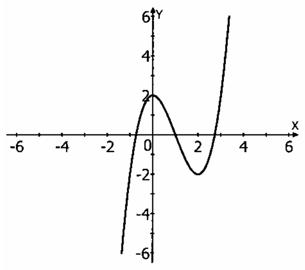
1 корень

в)



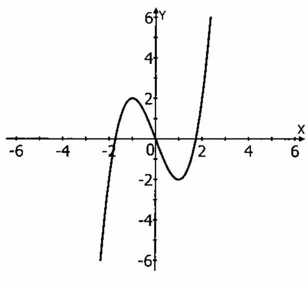
3 корня

г)



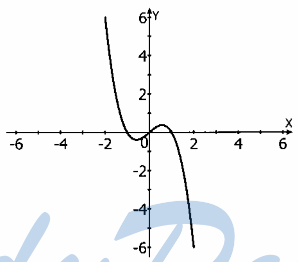
1 корень

929. а)



при $|a| > 2$

б)



$a \in (-2; 2)$

930. а) $x^3 + 5 = 15 - x$, $y_1 = x^3 + 5$, $y_2 = 15 - x$,

$y_1' = 3x^2 + 5$, возрастает на R

$y_2' = -1$, убывает на R

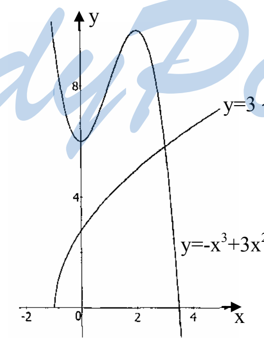
$\Rightarrow$ только 1 корень $x = 2$;

б) $x^5 + 3x^3 + 7x - 11 = 0$,

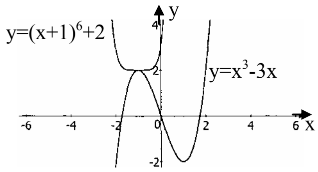
$y' = 5x^4 + 9x^2 + 7$, возрастает на R

$\Rightarrow$ 1 корень $x = -1$

в) $2x^5+3x^3=17-12x$, $2x^5+3x^3+12x-17=0$
$y'=10x^4+9x^2+12$, возрастает на R
$\Rightarrow$ 1 корень $x=-1$
г) $x^5+4x^3+8x-13=0$,
$y'=5x^4+12x^2+8$, возрастает на R
$\Rightarrow$ 1 корень $x=-1$
931. а) $\sin 5x-2\cos x-8x-x^5+2=0$, $y_1=\sin 5x-2\cos x-8x$,
$y_2=x^5-2$, $y_1'=5\cos 5x+2\sin x-8$ - убывает на R
$y_2^1=5x^4+8$ - возрастает на R $\Rightarrow$ только одно решение $x=0$.
б) $4\cos 3x+5\sin\frac{x}{2}+15x=4-x^3$, $y_1=4\cos 3x+5\sin\frac{x}{2}+15x$, $y_2=4-x^3$,
$y_1'=-12\sin 3x+\frac{5}{2}\sin\frac{x}{2}+15$ - возрастает на R
$y_2'=4-3x^2$ - убывает на R $\Rightarrow$ только одно решение $x=0$
932. а) $3\cos\frac{\pi x}{2}+5\sin\frac{\pi x}{2}+18x=46-x^5-22x^3$, $y_1=3\cos\frac{\pi x}{2}+5\sin\frac{\pi x}{2}+18x$
$y_2=46-x^5-22x^3$, $y_1'=-\frac{3\pi}{2}\sin\frac{\pi x}{2}+\frac{5\pi}{2}\cos\frac{\pi x}{2}+18$, возрастает на R;
$y_2'=-5x^4-66x^2$, убывает на R $\Rightarrow$ 1 корень $x=1$;
б) $2\sin\frac{\pi}{2}x-2\cos\pi x-8x=x^5-50$, $y_1=2\sin\frac{\pi}{2}x-2\cos\pi x-8x$,
$y_2=x^5-50$, $y_1'=\pi\cos\frac{\pi x}{2}+2\pi\sin\pi x-8$, убывает на R;
$y_2'=5x^4$, возрастает на R $\Rightarrow$ 1 корень $x=2$
933. а) $x=3$



б) $x = -1$



§ 36. Применение производной для отыскания наибольших и наименьших значений велечин

934. а) $y = 3x - 6$, $x \in [-1;4]$

$y' = 3 \Rightarrow$ функция растет на R

$x_{min} = -1$, $y(-1) = -9$, $y_{min} = -9$;

$x_{max} = 4$, $y(-1) = 6$, $y_{max} = 6$.

б) $y = -\frac{8}{x}$, $x \in \left[\frac{1}{4};8\right]$

$y' = \frac{8}{x^2} > 0 \Rightarrow$ функция растет при $x \in \left[\frac{1}{4};8\right]$

$x_{min} = \frac{1}{4}$, $y(\frac{1}{4}) = -32$, $y_{min} = -32$;

$x_{max} = 8$, $y(8) = -1$, $y_{max} = -1$.

в) $y = -\frac{1}{2}x + 4$, $x \in [-2;6]$

$y' = -\frac{1}{2} \Rightarrow$ функция убывает на R

$x_{min} = 6$, $y(6) = 1$, $y_{min} = 1$;

$x_{max} = -2$, $y(-2) = 5$, $y_{max} = 5$.

г) $y = \frac{3}{x}$, $x \in [0,3;2]$

$y' = -\frac{3}{x^2} < 0 \Rightarrow$ функция убывает при [0,3; 2]

$x_{max} = 0,3$, $y(0,3) = 10$, $y_{max} = 10$;

$x_{min} = 2$, $y(2) = \frac{3}{2}$, $y_{min} = \frac{3}{2}$.

935. а) $y = x^2 - 8x + 19, \qquad x \in [-1;5]$

$y' = 2x - 8, \qquad x = 4$ - точка минимума;

$y(4) = 16 - 32 + 19 = 3$; $\quad y(-1) = 1 + 8 + 19 = 28$;

$y(5) = 25 - 40 + 19 = 4$; $\quad y_{max} = 28, \qquad y_{min} = 3.$

б) $y = x^2 + 4x - 3, \qquad x \in [0;2]$

$y' = 2x + 4, \qquad x = -2, \qquad -2 \in [0;2];$

$y(0) = -3$, $\quad y(2) = 4 + 8 - 3 = 9$; $\quad y_{max} = 9, \qquad y_{min} = -3.$

в) $y = 2x^2 - 8x + 6, \qquad x \in [-1;4];$

$y' = 4x - 8, \qquad x = 2; \quad y(2) = 8 - 16 + 6 = -2;$

$y(-1) = 2 + 8 + 6 = 16$; $\quad y(4) = 32 - 32 + 6 = 6$;

$y_{max} = 16, \qquad y_{min} = -2.$

г) $y = -3x^2 + 6x - 10, \qquad x \in [-2;9]; \quad y' = -6x + 6, \qquad x = 1;$

$y(1) = -3 + 6 - 10 = -7; \quad y(-2) = -12 - 12 - 10 = -34;$

$y(9) = -243 + 54 - 10 = -199; \quad y_{max} = -7, \qquad y_{min} = -199.$

936. а) $y = 2\sin x, \qquad x \in \left[-\frac{\pi}{2};\pi\right]; \quad y' = 2\cos x;$

$x = -\frac{\pi}{2}, \qquad x = \frac{\pi}{2}; \qquad y_{max} = 2, \qquad y_{min} = -2.$

б) $y = -2\cos x, \qquad x \in \left[-2\pi;-\frac{\pi}{2}\right]$

$y_{max} = 2,\ y_{min} = -2$.

в) $y = 6\cos x, \qquad x \in \left[-\frac{\pi}{2};0\right]; \qquad y_{max} = 6,\ y_{min} = 0$.

г) $y = -\frac{1}{2}\sin x, \qquad x \in \left[-\frac{\pi}{2};\frac{\pi}{2}\right]; \qquad y_{max} = \frac{1}{2},\ y_{min} = -\frac{1}{2}$.

937. а) $y = tgx, \qquad x \in \left[-\frac{\pi}{3};-\frac{\pi}{6}\right]; \qquad y_{max} = -\frac{\sqrt{3}}{3},\ y_{min} = -\sqrt{3}$.

б) $y = -3tgx, \qquad x \in \left[\pi;\frac{4\pi}{3}\right]; \qquad y_{min} = -3\sqrt{3},\ y_{max} = 0$.

в) $y = -2tgx, \qquad x \in \left[0;\frac{\pi}{6}\right]; \qquad y_{max} = 0,\ y_{min} = -\frac{2\sqrt{3}}{3}$.

г) $y = \frac{1}{2}tgx, \qquad x \in \left[-\pi;-\frac{3\pi}{4}\right]; \qquad y_{max} = \frac{1}{2}, \quad y_{min} = 0$.

938. а) $y=\sqrt{x}$, $x\in[0;9]$; $y_{max}=3$, $y_{min}=0$.

б) $y=\sqrt{-x}$, $x\in[-4;0]$; $y_{max}=2$, $y_{min}=0$.

в) $y=-\sqrt{x}$, $x\in[4;16]$; $y_{max}=-2$, $y_{min}=-4$.

г) $y=-\sqrt{-x}$, $x\in[-9;-4]$; $y_{max}=-2$, $y_{min}=-3$.

939. а) $y=12x^4$, $x\in[-1;2]$; $y_{max}=192$, $y_{min}=0$.

б) $y=-6x^5$, $x\in[0,1;2]$

$y_{max}=-\frac{6}{100000}=-\frac{3}{50000}$, $y_{min}=-192$.

в) $y=-3x^7$, $x\in[0;1]$; $y_{max}=0$, $y_{min}=-3$.

г) $y=\frac{x^4}{9}$, $x\in[-1;3]$; $y_{max}=9$, $y_{min}=0$.

940. $y=\sin x$;

а) $\left[0;\frac{2\pi}{3}\right]$; $y_{max}=1$, $y_{min}=0$.

б) $\left[2\pi;\frac{8\pi}{3}\right]$; $y_{max}=1$, $y_{min}=0$.

в) $\left[-2\pi;-\frac{4\pi}{3}\right]$; $y_{max}=1$, $y_{min}=0$.

г) $\left[6\pi;\frac{26\pi}{3}\right]$; $y_{max}=1$, $y_{min}=-1$.

941. $y=x^3-9x^2+24x-1$; $y'=3x^2-18x+24=0$;

$x^2-6x+8=0$; $x=4$, $x=2$;

$y_{max}=y(2)=8-36+48-1=19$;

$y_{min}=y(4)=64-144+96-1=15$.

а) $[-1;3]$;

$y_{max}=y(2)=19$; $y_{min}=y(-1)=-1-9-24-1=-35$.

б) $[3;6]$;

$y(3)=27-81+72-1=17$; $y(6)=35$; $y_{max}=35$; $y_{min}=15$.

в) $[-2;3]$;

$y_{max}=y(2)=19$, $y_{min}=y(-2)=-8-36-48-1=-93$.

г) $[3;5]$;

$y_{max}=y(5)=125-225+120-1=19$, $y_{min}=y(4)=15$.

942. $y = x^3 + 3x^2 - 45x - 2$; $y' = 3x^2 + 6x - 45 = 0$;

$x^2 + 2x - 15 = 0$; $x = -5$, $x = 3$;

$y_{max} = y(-5) = -125 + 75 + 225 - 2 = 173$,

$y_{min} = y(3) = 27 + 27 - 135 - 2 = -83$.

а) $[-6;0]$; $y_{max} = 173$; $y_{min} = -2$.

б) $[1;2]$; $y_{max} = -43$, $y_{min} = -72$.

в) $[-6;-1]$; $y_{max} = 173$, $y_{min} = 45$.

г) $[0;2]$; $y_{max} = -2$, $y_{min} = -72$.

943. $y = x^3 - 9x^2 + 15x - 3$; $y' = 3x^2 - 18x + 15 = 0$;

$x^2 - 6x + 5 = 0$; $x = 5$, $x = 1$;

$y_{max} = y(1) = 1 - 9 + 15 - 3 = 4$, $y_{min} = y(5) = 125 - 225 + 75 - 3 = -28$.

а) $[0;2]$; $y_{max} = 4$, $y_{min} = -3$.

б) $[3;6]$; $y_{max} = -12$, $y_{min} = -28$.

в) $[-1;3]$; $y_{max} = 4$, $y_{min} = -28$.

г) $[2;7]$; $y_{max} = 4$, $y_{min} = -28$.

944. $y = x^4 - 8x^3 + 10x^2 + 1$; $y' = 4x^3 - 24x^2 + 20x = 0$;

$4x(x^2 - 6x + 5) = 0$; $x = 0$, $x = 5$, $x = 1$; $y(0) = 1$,

$y_{max} = y(1) = 1 - 8 + 10 + 1 = 4$, $y_{min} = y(5) = 625 - 1000 + 250 + 1 = -124$.

а) $[-1;2]$; $y_{max} = 10$, $y_{min} = -7$.

б) $[1;6]$; $y_{max} = 4$, $y_{min} = -124$.

в) $[-2;3]$; $y_{max} = 121$, $y_{min} = -44$.

г) $[-1;7]$; $y_{max} = 148$, $y_{min} = -124$.

945. $y = x + \frac{4}{x-1}$; $y' = 1 - \frac{4}{(x-1)^2} = 0$; $(x-1)^2 = 4$; $x = -1$, $x = 3$;

$y_{max} = y(3) = 3 + 2 = 5$, $y_{min} = y(-1) = -1 + \frac{4}{-2} = -1 - 2 = -3$.

а) $[2;4]$; $y_{max} = 6$, $y_{min} = 5$.

б) $[-2;0]$; $y_{max} = -3$, $y_{min} = -4$.

946. а) $y = \operatorname{ctg} x + x$ $x \in \left[\frac{\pi}{4}; \frac{3\pi}{4}\right]$; $y' = -\frac{1}{\sin^2 x} + 1 \le 0$;

$y_{max} = 1 + \frac{\pi}{4}$, $y_{min} = -1 + \frac{3\pi}{4}$.

б) $y = 2\sin x - x \quad x \in [0;\pi]$; $y' = 2\cos x - 1$;

$x = \pm\frac{\pi}{3} + \pi n$, $x = \frac{\pi}{3}$; $y_{max} = y\left(\frac{\pi}{3}\right) = \sqrt{3} - \frac{\pi}{3}$; $y_{min} = -\pi$.

в) $y = 2\cos x + x$, $x \in \left[-\frac{\pi}{2};\frac{\pi}{2}\right]$;

$y' = -2\sin x + 1$; $\sin x = \frac{1}{2}$; $x = \frac{\pi}{6}$;

$y_{max} = y\left(\frac{\pi}{6}\right) = \sqrt{3} + \frac{\pi}{6}$; $y_{min} = -\frac{\pi}{2}$.

г) $y = tgx - x \quad x \in \left[0;\frac{\pi}{3}\right]$; $y' = \frac{1}{\cos^2 x} - 1$;

$x = \pi n$, $x = 0$; $y_{max} = y\left(\frac{\pi}{3}\right) = \sqrt{3} - \frac{\pi}{3}$; $y_{min} = y(0) = 0$.

947. а) $y = x^3 - 2x^2 + 1$, $[0,5;+\infty)$; $y' = 3x^2 - 4x = 0$; $x(3x-4) = 0$;

$x = 0$, $x = \frac{4}{3}$; y_{max} не существует; $y_{min} = y(\frac{4}{3}) = \frac{64}{27} - \frac{32}{9} + 1 = -\frac{5}{27}$.

б) $y = x - 2\sqrt{x}$, $[0;+\infty)$; $y' = 1 - \frac{1}{\sqrt{x}}$; $x = 1$;

y_{max} не существует, $y_{min} = y(1) = 1 - 2 = -1$.

в) $y = \frac{1}{5}x^5 - x^2$, $(-\infty;1]$; $y' = x^4 - 2x = 0$; $x(x^3 - 2) = 0$;

$x = 0$, $x = \sqrt[3]{2}$; $y_{max} = y(0) = 0$; y_{min} не существует;

г) $y = \frac{x^4}{x^4+1} = 1 - \frac{1}{x^4+1}$, $x \in R$; $y' = \frac{4x^3}{(x^4+1)^2} = 0$,

y_{max} не существует; $y_{min} = y(0) = 0$.

948. а) $y = x + \frac{1}{x}$, $(-\infty;0)$; $y' = 1 - \frac{1}{x^2}$; $x = \pm 1$;

$y_{max} = y(-1) = -1 - 1 = -2$; y_{min} не существует;

б) $y = \frac{3x}{x^2+3}$, $[0;+\infty)$; $y' = \frac{3x^2 + 9 - 6x^2}{(x^2+3)^2} = -3 \cdot \frac{x^2-3}{x^2+3}$; $x = \pm\sqrt{3}$.

$y_{max} = y(\sqrt{3}) = \frac{3\sqrt{3}}{3+3} = \frac{\sqrt{3}}{2}$; $y_{min} = y(0) = 0$.

в) $y = -2x - \frac{1}{2x}$, $(0;+\infty)$; $y' = -2 + \frac{1}{2x^2} = 0$; $4x^2 = 1$; $x = \pm\frac{1}{2}$;

$y_{max} = y\left(\frac{1}{2}\right) = -1 - 1 = -2$; y_{min} не существует.

г) $y=\sqrt{2x+6}-x$, $[-3;+\infty)$; $y'=\frac{1}{\sqrt{2x+6}}-1=0$; $\sqrt{2x+6}=1$;

$x=-\frac{5}{2}$; $y_{max}=y\left(-\frac{5}{2}\right)=1+\frac{5}{2}=3{,}5$, y_{min} не существует.

949. а) $\begin{cases} a+b=24 \\ ab=\max \end{cases}$; $\begin{cases} a=24\text{-}b \\ 24b\text{-}b^2=y \end{cases}$;

$y'=24-2b$;

$b=12$, $y(12)=144$;

$b=12$, $a=12$.

б) $ab=484$; $a=\frac{484}{b}$; $\frac{484}{b}+b=y$; $y'=1-\frac{484}{b^2}$;

$b=22$, $a=22$.

950. а) $a-b=10$; $a=10+b$; $10b+b^2=y$; $y'=2b+10$;

$b=-5$, $a=5$.

б) $a-b=98$; $a=98+b$; $b^2+98b=y$; $y'=2b+98$;

$b=-49$, $a=49$.

951. а) $a(a+36)=y$;

$a^2+36a=y$; $y'=2a+36$;

$a=-18$, $b=-18+36=18$.

б) $a(a-28)=y$; $a^2-28a=y$; $y'=2a-28$;

$a=14$, $b=-14$.

952. а) $\begin{cases} a+b=3 \\ y=3a+b^3 \end{cases}$; $\begin{cases} a=3-b \\ y=3a+b^3 \end{cases}$;

$y=9-3b+b^3$; $y'=3b^2-3$;

$b=\pm1$, но т.к. по условию $b>0$, то

$b=1$, $a=2$.

б) $\begin{cases} a+b=5 \\ y=ab^3 \end{cases}$; $\begin{cases} a=5-b \\ y=ab^3 \end{cases}$;

$y=5b^3-b^4$;

$y'=15b^2-4b^3=b^2(15-4b)$;

$b=0$, $b=\frac{15}{4}$, но т.к. по условию $b>0$, то $b=\frac{15}{4}$, $a=\frac{5}{4}$.

953. а) $\begin{cases} 2a+2b=56 \\ ab=y \end{cases}$; $\begin{cases} a=28\text{-}b \\ 28b\text{-}b^2=y \end{cases}$; $y'=28-2b$; $2b=28$;

$b=14$, $a=14$.

б) $\begin{cases} a+b=36 \\ y=ab \end{cases}$; $\begin{cases} a=36-b \\ y=ab \end{cases}$; $36b-b^2=y$; $y'=36-2b$;

$b=18,\quad a=18$.

954. а) $\begin{cases} a+b=100 \\ y=ab \end{cases}$; $\begin{cases} a=100-b \\ y=ab \end{cases}$; $y=100b-b^2$; $y'=100-2b$;

$b=50,\quad a=50$.

б) $\begin{cases} a+b=120 \\ ab=y \end{cases}$; $\begin{cases} a=120-b \\ ab=y \end{cases}$; $y=120b-b^2$; $y'=120-2b$;

$b=60,\quad a=60$.

955. а) $\begin{cases} ab=16 \\ 2a+2b=y \end{cases}$; $\begin{cases} a=\frac{16}{b} \\ 2a+2b=y \end{cases}$; $y=\frac{32}{b}+2b$; $y'=2-\frac{32}{b^2}$;

$b=4,\quad a=4$.

б) $\begin{cases} ab=64 \\ 2a+2b=y \end{cases}$; $\begin{cases} a=\frac{164}{b} \\ 2a+2b=y \end{cases}$; $y=\frac{128}{b}+2b$; $y'=2-\frac{128}{b^2}$;

$b=8,\quad a=8$.

956. $\begin{cases} ab=2500 \\ 2a+2b=y \end{cases}$; $\begin{cases} a=\frac{2500}{b} \\ 2a+2b=y \end{cases}$; $y=\frac{5000}{b}+2b$; $y'=2-\frac{5000}{b^2}$;

$b=50,\quad a=50$.

957. KD = DM = x;

$BH1 = \frac{3\sqrt{2}}{2}$;

$DH_2 = \frac{x\sqrt{2}}{2}$ (H_1 и H_2 – точки пересечения BD с PE и KM соответственно)

$S = \frac{1}{2}(PE + KM)\cdot H_1H_2 = \frac{1}{2}(x\sqrt{2} + 3\sqrt{2})(8\sqrt{2} - \frac{3\sqrt{2}}{2} - \frac{x\sqrt{2}}{2})=$

$= \frac{1}{2}(39 + 10x - x2)$;

$S' = \frac{1}{2}(10 - 2x) = 0$; $\qquad x = 5$;

$S = \frac{1}{2}(39 + 50 - 25) = 32$.

958. а) $y=\sqrt{1+\cos 2x}$, $\left[-\frac{\pi}{2};\frac{\pi}{2}\right]$;

$y_{max}=\sqrt{2}$, $y_{min}=0$.

б) $y=\sqrt{1+\sin x}$, $\left[0;\frac{\pi}{2}\right]$; $y_{max}=\sqrt{2}$, $y_{min}=1$.

в) $y=\sqrt{1-\sin 2x}$, $[0;\pi]$; $y_{max}=\sqrt{2}$, $y_{min}=0$.

г) $y=\sqrt{1+\cos x}$, $\left[-\frac{\pi}{2};0\right]$; $y_{max}=\sqrt{2}$, $y_{min}=0$.

959. а) $y=2-3\sin x+4\cos x$;

$y=2-5\sin(x+\alpha)$; $y_{max}=7$, $y_{min}=-3$.

б) $y=3\sin x-4\cos x+1$;

$y=5\sin(x-\alpha)+1$; $y_{max}=6$, $y_{min}=-4$.

960. а) $y=x^4+8x^3+24x^2+32x+21$, $[-3;0]$;

$y'=4x^3+24x^2+48x+32=0$; $x^3+6x^2+12x+8=0$;

$(x+2)(x^2-2x+4+6x)=0$; $(x+2)(x^2+4x+4)=(x+2)^3=0$; $x=-2$;

$y(-2)=16-64+96-64+21=133$; $y(-3)=81-216+216-96+21=6$;

$y(0)=21$; $y_{min}=y(-2)=5$, $y_{max}=y(6)=21$.

б) $y=x^4-4x^3+6x^2-4x-9$, $[0;4]$;

$y'=4x^3-12x^2+12x-4=0$; $x^3-3x^2+3x-1=0$;

$(x-1)^3=0$; $x=1$; $y_{max}=y(4)=71$, $y_{min}=y(1)=1-4+6-4-9=-10$.

961. а) $y=x^2-5|x|+6$, $[0;4]$;

на этом промежутке $x\ge 0 \Rightarrow y=x^2-5x+6$;

$y'=2x-5$; $x=\frac{5}{2}$;

$y_{max}=y(0)=6$; $y_{min}=y\left(\frac{5}{2}\right)=\frac{25}{4}-\frac{25}{2}+6=-\frac{25}{4}+6=-\frac{1}{4}$.

б) $y=x^2-5|x|+6$, $[-5;0]$;

на этом промежутке $x\le 0 \Rightarrow y=x^2+5x+6$;

$y'=2x+5$; $x=-\frac{5}{2}$;

$y_{max}=y(0)=6$; $y_{min}=y\left(-\frac{5}{2}\right)=\frac{25}{4}-\frac{25}{2}+6=-\frac{1}{4}$.

в) $y=x^2+8|x|+7$, $[1;5]$;

на этом промежутке $x\ge 0 \Rightarrow y=x^2+8x+7$;

$y'=2x+8$; $x=-4$ - не подходит;

$y_{max}=y(5)=25+40+7=72$;

$y_{min}=y(1)=16$.

г) $y = x^2 + 8|x| + 7, \qquad [-8;-2]$;

на этом промежутке $x \le 0 \Rightarrow \quad y = x^2 - 8x + 7$; $y' = 2x - 8$;

$x = 4$ - не подходит;

$y_{max} = y(-8) = 64 + 64 + 7 = 135$; $y_{min} = y(-2) = 4 + 32 + 7 = 43$.

962. а) $y = x^3 - 3x, \qquad (-\infty;0]$;

$y' = 3x^2 - 3$; $x = -1$; $y_{max} = y(-1) = -1 + 3 = 2$; y_{min} - не существует .

б) $y = x^3 - 3x, \qquad [0;+\infty)$; $y_{min} = y(1) = -2$; y_{max} - не существует .

963. а) $y = \frac{x}{x^4+3}, \qquad [0;+\infty)$;

$y' = \frac{x^4 + 3 - 4x^4}{(x^4+3)^2} = \frac{3 - 3x^4}{(x^4+3)^2}$; $x = \pm 1$; $y_{max} = y(1) = \frac{1}{4}$; $y(0) = 0 - \min$

б) $y = \frac{x}{x^4+3}, \qquad (-\infty;0]$; $y_{min} = y(-1) = -\frac{1}{4}$; $y_{max} = y(0) = 0.$

964. а) $y = \sin^2\frac{x}{2}\sin x, \qquad [-\pi;0]$;

$y = \frac{1}{2}\sin x - \frac{1}{2}\cos x \sin x = \frac{1}{2}\sin x - \frac{1}{4}\sin 2x$;

$y' = \frac{1}{2}\cos x - \frac{1}{2}\cos 2x = 0$; $\sin\frac{3x}{2}\sin\frac{x}{2} = 0$;

$x = 2\pi n, \qquad x = \frac{2\pi n}{3}$; $x = -\frac{2\pi}{3}$;

$y\left(-\frac{2\pi}{3}\right) = -\frac{3}{4}\cdot\frac{\sqrt{3}}{2} = -\frac{3\sqrt{3}}{8} - y_{min}$; $y_{max} = 0$.

б) $y = \cos^2\frac{x}{2}\cos x, \qquad [0;\pi]$; $y' = -\cos\frac{x}{2}\cdot\sin\frac{x}{2}\cos x - \cos^2\frac{x}{2}\sin x = 0$;

$\cos\frac{x}{2}\cdot(\sin\frac{x}{2}\cos x + \cos\frac{x}{2}\sin x) = 0$; $\cos\frac{x}{2}\cdot\sin\frac{3x}{2} = 0$;

$x = \pi + 2\pi n$, $\qquad x = \frac{2\pi k}{3}$;

$y(0) = 1$; $y(\pi) = 0$; $y\left(\frac{2\pi}{3}\right) = \frac{1}{4}\cdot\left(-\frac{1}{2}\right) = -\frac{1}{8}$; $y_{min} = -\frac{1}{8}$, $y_{max} = 1$

965. а) $y = x^2 - 4x + 5 + |1 - x|, \qquad [0;4]$;

1) $x \ge 1$; $y = x^2 - 3x + 4$; $y' = 2x - 3 = 0$; $x = \frac{3}{2}$;

$y(1) = 1 - 4 + 5 + (1 - 1) = 2$; $y\left(\frac{3}{2}\right) = \frac{9}{4} - 6 + 5 + \left(1 - \frac{3}{2}\right) = \frac{7}{4}$; $y(4) = 8$;

2) $x \le 1$; $y = x^2 - 5x + 6$; $y' = 2x - 5 = 0$; $x = \frac{5}{2}$ - не подходит; $y(0) = 6$;

$y_{min} = \frac{7}{4}$; $y_{max} = 8$.

б) $y = |x^3 - 1| - 3x$, $[-1;3]$;

1) $x \ge 1$; $y = x^3 - 3x - 1$; $y' = 3x^2 - 3$; $x = 1$; $y(1) = -3$; $y(3) = -17$;

2) $x \le 1$; $y = 1 - 3x - x^3$; $y' = -3 - 3x^2$; $y(1) = -3 = y_{max}$; $y(-1) = 5$;

$y_{min} = -3$; $y_{max} = 17$.

966. а) $y = 2x - \sqrt{16x - 4}$, $x \in \left[\frac{1}{4};\frac{17}{4}\right]$;

$y' = 2 - \frac{8}{\sqrt{16x-4}}$; $x = \frac{5}{4}$; $y\left(\frac{5}{4}\right) = \frac{5}{2} - 4 = -\frac{3}{2}$;

$y\left(\frac{1}{4}\right) = \frac{1}{2}$; $y\left(\frac{17}{4}\right) = \frac{17}{2} - 8 = \frac{1}{2}$; $y \in \left[-\frac{3}{2};\frac{1}{2}\right]$.

б) $y = 2\sqrt{x-1} - \frac{1}{2}x$, $x \in [1;10]$;

$y' = \frac{1}{\sqrt{x-1}} - \frac{1}{2}$; $x = 5$;

$y(5) = 4 - \frac{5}{2} = \frac{3}{2}$; $y(1) = -\frac{1}{2}$; $y(10) = 6 - 5 = 1$; $y \in \left[-\frac{1}{2};\frac{3}{2}\right]$.

967. а) $y = x\sqrt{x+2}$; $y' = \sqrt{x+2} + \frac{x}{2\sqrt{x+2}} = \frac{2x+4+x}{2\sqrt{x+2}} = \frac{3x+4}{2\sqrt{x+2}}$;

$x = -\frac{4}{3}$; $y\left(-\frac{4}{3}\right) = -\frac{4}{3} \cdot \sqrt{-\frac{4}{3} + 2} = -\frac{4\sqrt{6}}{9}$; $y \in \left[-\frac{4}{9}\sqrt{6};+\infty\right)$.

б) $y = x\sqrt{1-2x}$; $y' = \sqrt{1-2x} - \frac{x}{\sqrt{1-2x}} = \frac{1-2x-x}{\sqrt{1-2x}} = \frac{1-3x}{\sqrt{1-2x}}$; $x = \frac{1}{3}$;

$y\left(\frac{1}{3}\right) = \frac{1}{3} \cdot \sqrt{1 - \frac{2}{3}} = \frac{1}{3\sqrt{3}}$; $y \in \left(-\infty;\frac{\sqrt{3}}{9}\right]$.

968. а) $y = x\sqrt{x+a}$; $y' = \sqrt{x+a} + \frac{x}{2\sqrt{x+a}} = \frac{3x+2a}{2\sqrt{x+a}}$; $x = -\frac{2a}{3}$;

$y\left(-\frac{2a}{3}\right) = -\frac{2a}{3} \cdot \sqrt{\frac{a}{3}} = -6\sqrt{3}$; $a\sqrt{a} = 27$;

$a = 27^{\frac{2}{3}} = 9$.

б) $y=(a-x)\sqrt{x}$; $y'=-\sqrt{x}+\frac{a-x}{2\sqrt{x}}=\frac{a-x-2x}{2\sqrt{x}}=\frac{a-3x}{2\sqrt{x}}$; $x=\frac{a}{3}$;

$y\left(\frac{a}{3}\right)=\frac{2a}{3}\cdot\sqrt{\frac{a}{3}}=10\sqrt{5}$; $a\sqrt{a}=15\sqrt{15}$; $a=15$.

969. а) $a_9=1$;

$\begin{cases}a_1+8d=1\\(a_1+3d)(a_1+6d)(a_1+7d)=y\end{cases}$; $\begin{cases}a_1=1-8d\\(1-5d)(1-2d)(1-d)=y\end{cases}$;

$y=(1+10d^2-7d)(1-d)=1-d+10d^2-10d^3-7d+7d^2=$

$=1-8d+17d^2-10d^3$; $y'=-8+34d-30d^2=0$; $15d^2-17d+4=0$;

$D=289-240=49$; $d_1=\frac{17+7}{30}=\frac{4}{5}$; $d_2=\frac{1}{3}$;

$y\left(\frac{4}{5}\right)=(1-4)\left(1-\frac{8}{5}\right)\left(1-\frac{4}{5}\right)=-3\cdot\left(-\frac{3}{5}\right)\frac{1}{5}=\frac{9}{25}$;

$y\left(\frac{1}{3}\right)=\left(1-\frac{5}{3}\right)\left(1-\frac{2}{3}\right)\left(1-\frac{1}{3}\right)=-\frac{2}{3}\cdot\frac{1}{3}\cdot\frac{2}{3}=-\frac{4}{27}$; $d=\frac{4}{5}$.

б) $\begin{cases}a_1+d=6\\(a_1+2d)a_1(a_1+5d)=y\end{cases}$; $\begin{cases}a_1=6-d\\(6+d)(6-d)(6+4d)=y\end{cases}$;

$y=(36-d^2)(6+4d)=216+144d-6d^2-4d^3$;

$y'=144-12d-12d^2=0$; $d^2+d-12=0$;

$d_1=3$; $d_2=-4$; $y(3)=9\cdot3\cdot18=486$; $y(-4)=2\cdot10\cdot(-10)=-200$; $d=-4$.

970. а) $y_1=2x^2$ $y_2=4x$

Длина отрезка равна

$4x-2x^2=f(x)$, $x\in[0;2]$;

$f'(x)=4-4x$; $x_0=1$; $f(1)=4-2=2$;

б) $y_1=x^2$; $y_2=-2x$;

Длина отрезка равна

$-2x-x^2=f(x)$, $x\in[-2;0]$; $f'(x)=-2-2x$; $x_0=-1$;

$f(-1)=2-1=1$.

971. а) $y=x^2$, $A(0;1{,}5)$; $\sqrt{(0-x)^2+(1{,}5-y)^2}=f(x)$;

$f(x)=\sqrt{x^2+(1{,}5-x^2)^2}==\sqrt{x^2+\frac{9}{4}-3x^2+x^4}=\sqrt{x^4-2x^2+\frac{9}{4}}$

$$f'(x)=\frac{4x^3-4x}{2\sqrt{x^4-2x^2+\frac{9}{4}}}=2\cdot\frac{x^3-x}{\sqrt{x^4-2x^2+\frac{9}{4}}};$$

$x=0,\quad x=\pm1;\quad f(0)=\frac{3}{2};\quad f(1)=\sqrt{1+\frac{1}{4}}=\frac{\sqrt{5}}{2}=f(-1);\quad (1;1),(-1;1)$

б) $y=\sqrt{x}$, $\qquad A(4{,}5;0)$;

$$f(x)=\sqrt{(4{,}5-x)^2+y^2}=\sqrt{\frac{81}{4}-9x+x^2+x}=\sqrt{\frac{81}{4}-8x+x^2};$$

$$f'(x)=\frac{2x-8}{2\sqrt{\frac{81}{4}-8x+x^2}};\quad x=4;\qquad y(4)=2;\qquad (4;2).$$

972. $S=\frac{1}{2}\left(15+2\sqrt{225-h^2}+15\right)h=$

$=\left(15+\sqrt{225-h^2}\right)h$ (здесь h – высота трапеции);

$$S'=15+\sqrt{225-h^2}-\frac{h^2}{\sqrt{225-h^2}};\quad 15\cdot\sqrt{225-h^2}+225-2h^2=0;$$

$$50625-225h^2=50625-900h^2+4h^4;\quad 4h^4-675h^2=0;\quad h^2(4h^2-675)=0;$$

$$h^2=\frac{675}{4};\quad a=15+2\sqrt{225-\frac{675}{4}}=15+2\cdot\frac{15}{2}=30.$$

973. а) Пусть α - угол между основанием и боковой стороной x - сторона прямоугольника, которая совпадает с высотой, y – его другая стороной.

Тогда $tg\alpha=5=\frac{x}{80-y}$; $80-y=\frac{x}{5}$; $y=80-\frac{x}{5}$; $S=80x-\frac{x^2}{5}$;

$S'=80-\frac{2x}{5}$;

$x=200$, но $x\in(0;100]\Rightarrow\ x=100$, $\qquad y=60$; $\ S=6000$.

б) $a=24$, $\ b=8$, $\ h=12$.

Пусть α - угол между большим основанием трапеции и ее боковой стороной, x –сторона прямоугольника, которая совпадает с высотой, y – его другая сторона.

Тогда $tg\alpha=\frac{3}{4}=\frac{x}{24-y}$; $24-y=\frac{4x}{3}$; $y=24-\frac{4x}{3}$; $S=24x-\frac{4x^2}{3}$;

$S'=24-\frac{8x}{3}$; $x=9$, $\qquad y=12$; $S=108$.

974. а) Пусть x – сторона прямоугольника, лежащая на AB, y – его другая сторона, $\alpha = \angle DCB$.

Тогда $tg\alpha = \frac{7-5}{9-3} = \frac{2}{6} = \frac{1}{3} = \frac{x}{9-y}$; $x = 3 - \frac{1}{3}y$; $y = -3x + 9$;

$S = -3x^2 + 9x$; $S' = -6x + 9$; $x = \frac{3}{2}$; $y = -\frac{9}{2} + 9 = \frac{9}{2}$

но $AE \cdot AB = 21 \Rightarrow S_{max} = 21$.

б) $a = 7$, $b = 18$, $c = 3$, $m = 1$;

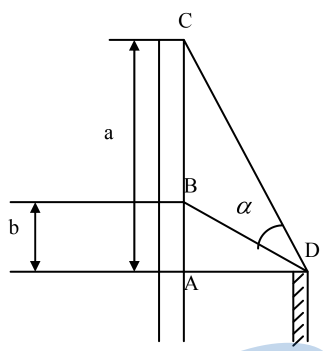
Пусть x – сторона прямоугольника, лежащая на AB, y – его другая сторона, $\alpha = \angle DCB$.

Тогда $tg\alpha = \frac{7-1}{18-3} = \frac{2}{5} = \frac{x}{18-y}$; $18 - y = \frac{5}{2}x$; $y = 18 - \frac{5}{2}x$;

$S = 18x - \frac{5}{2}x^2$; $S' = 18 - 5x$; $x = \frac{18}{5}$; $y = 9$; $S = 32{,}4$;

$AE \cdot AB = 21 \Rightarrow S_{max} = 32{,}4$.

975.



$AC = a$; $AB = b$;

$CB = AC - AB = a - b$; $AD = x$; $BD = \sqrt{x^2 + b^2}$; $CD = \sqrt{x^2 + a^2}$.

По теореме косинусов

$$(a-b)^2 = x^2 + b^2 + x^2 + a^2 - 2\sqrt{\left(x^2+b^2\right)\left(x^2+a^2\right)}\cos\alpha;$$

$$2x^2 + 2ab = 2\sqrt{\left(x^2+b^2\right)\left(x^2+a^2\right)}\cos\alpha;$$

$$\cos\alpha = \frac{x^2 + ab}{\sqrt{\left(x^2+b^2\right)\left(x^2+a^2\right)}}.$$

$$f(x) = \frac{x^2 + ab}{\sqrt{\left(x^2+b^2\right)\left(x^2+a^2\right)}} = \frac{x^2 + ab}{\sqrt{x^4 + x^2\left(a^2+b^2\right) + b^2a^2}};$$

$$f'(x)=\frac{2x\sqrt{x^4+x^2\left(a^2+b^2\right)+b^2a^2}-\dfrac{(x^2+ab)(4x^3+2x(a^2+b^2))}{2\sqrt{x^4+x^2\left(a^2+b^2\right)+b^2a^2}}}{\left(x^2+b^2\right)\left(x^2+a^2\right)}=0;$$

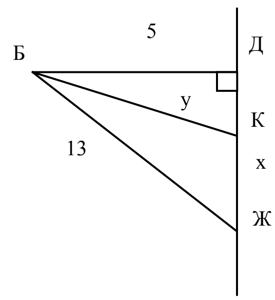
$$2x\sqrt{\left(x^2+b^2\right)\left(x^2+a^2\right)}=x(x^2+ab)\frac{2x^2+a^2+b^2}{\sqrt{\left(x^2+b^2\right)\left(x^2+a^2\right)}};$$

$$2(a^2b^2+x^2a^2+x^2b^2+x^4)=2x^4+2x^2ab+x^2a^2+x^2b^2+a^3b+ab^3;$$

$$2ab(ab-x^2)+a^2(x^2-ab)+b^2(x^2-ab)=0;$$

$(x^2-ab)(a-b)^2=0$; $x=\sqrt{ab}$; α_{max} при $x=\sqrt{ab}$.

976.



$ЖД=12$;

x – расстояние, которое пешеход пройдет по дороге;

y – расстояние, которое пешеход пройдет по лесу;

Суммарное время: $t=\dfrac{x}{5}+\dfrac{y}{3}$;

$DK=12-x$; $y=\sqrt{25+(12-x)^2}$;

$$t'=\frac{1}{5}+\frac{2x-24}{2\cdot 3\sqrt{x^2-24x+169}}=\frac{1}{5}+\frac{x-12}{3\sqrt{x^2-24x+169}}=0;$$

$\dfrac{3}{5}\sqrt{x^2-24x+169}=12-x$; $9x^2-9\cdot 24x+9\cdot 169=25\left(144-24x+x^2\right)$;

$16x^2-384x+2079=0$; $x_1=8{,}25$; $x_2=15{,}75$ – не подходит;

$x=\dfrac{33}{4}$; $y=\sqrt{25+3{,}75^2}=\dfrac{25}{4}$;

$t=\dfrac{x}{5}+\dfrac{4}{5}=\dfrac{33}{20}+\dfrac{25}{12}=\dfrac{56}{15}\approx 3$ часа 44 минуты.

977. $V = x^2y = 32$ (y – высота бака, x – длина стороны его основания);

$S = x^2 + 4xy = x^2 + \frac{128}{x}$; $S'(x) = 2x - \frac{128}{x^2} = 0$;

$x^3 = 64$; $x_0 = 4$; $y_0 = 2$.

Ответ: 4 дм; 4 дм; 2 дм.

978. $V = x^2y = 343$ (y – высота бака, x – длина стороны его основания);

$S = 2x^2 + 4xy = 2x^2 + \frac{1372}{x}$; $S' = 4x - \frac{1372}{x^2} = 0$; $x_0 = 7$; $y_0 = 7$.

Ответ: 7 м; 7 м; 7 м.

979. $V = 6x^2y = 576$ (y – высота короба, 2x и 3x – длины сторон его основания);

$S = 12x^2 + 6xy + 4xy = 12x^2 + \frac{960}{x}$; $S'(x) = 24x - \frac{960}{x^2} = 0$;

$x^3 = 40$; $x_0 = 2\sqrt[3]{5}$; $y_0 = \frac{576}{6} \cdot \frac{1}{4\sqrt[3]{25}} = 24\frac{\sqrt[3]{5}}{5}$;

Ответ: $4\sqrt[3]{5}$м; $6\sqrt[3]{5}$м; $\frac{24\sqrt[3]{5}}{5}$м.

980. $V = \sqrt{(d^2 - x^2)}x$ (x – длина бокового ребра призмы);

$V' = d^2 - 3x^2 = 0$; $x = \frac{d}{\sqrt{3}} = \frac{\sqrt{3}}{3}d$.

981. $a = 2\sqrt{p^2 - h^2}$; $S_{осн} = 4\left(p^2 - h^2\right)$;

$V = \frac{1}{3} \cdot 4\left(p^2 - h^2\right)h$; $V'(h) = \frac{4}{3}\left(p^2 - 3h^2\right) = 0$; $h = \frac{f}{\sqrt{3}} = \frac{p\sqrt{3}}{3}$.

982. $2h + 2x = p$ (h – высота цилиндра, x – его диаметр);

$V = \pi\left(\frac{x}{2}\right)^2 h = \pi\left(\frac{p - 2h}{2}\right)^2 h$;

$V'(h) = \left(\frac{\pi}{4}\left(p^2 - 4ph + 4h^2\right)h\right)' = \frac{\pi}{4}\left(p^2 - 8ph + 12h^2\right) = 0$;

$12h^2 - 8ph + p^2 = 0$; $h_{1,2} = \frac{8p \pm 4p}{24}$; $h_1 = \frac{p}{6}$; $h_2 = \frac{p}{2}$;

h_2 не подходит, т.к. $2h_2 = p \Rightarrow x = 0$, чего не может быть $\Rightarrow$ $h = \frac{p}{6}$

983. $S = 2\pi Rh + 2\pi R^2$ - площадь боковой поверхности; $V = \pi R^2 h$;

$h = \frac{V}{\pi R^2}$; $S = \frac{2V}{R} + 2\pi R^2$; $S'(R) = -2\frac{V}{R^2} + 4\pi R = 0$; $R^3 = \frac{V}{2\pi}$; $R = \sqrt[3]{\frac{V}{2\pi}}$.